Lorentz Symmetry Violation in the Fermion Number Anomaly with the Chiral Overlap Operator

Hiroki Makino and <u>*Okuto Morikawa*</u> (Kyushu Univ.) arXiv:1609.08376 [hep-lat]

1. Introduction

- QFT has succeeded as the **perturbation** theory.
- For vectorlike gauge theories, the **lattice regularization** gives a non-perturbative definition (e.g., Lattice QCD).
- ***No definition of non-perturbative chiral gauge theories yet***

A proposal of Grabowska and Kaplan [2015, 2016]

➢ A 5d domain-wall lattice formulation of chiral gauge theories

➢ A 4d lattice formulation based on the **chiral overlap operator** (which is derived from the above domain-wall formulation)

The **fermion number anomaly** in this formulation has important phenomenological implications. [Okumura & Suzuki 2016]

We discuss <u>the continuum limit of this fermion number anomaly.</u>

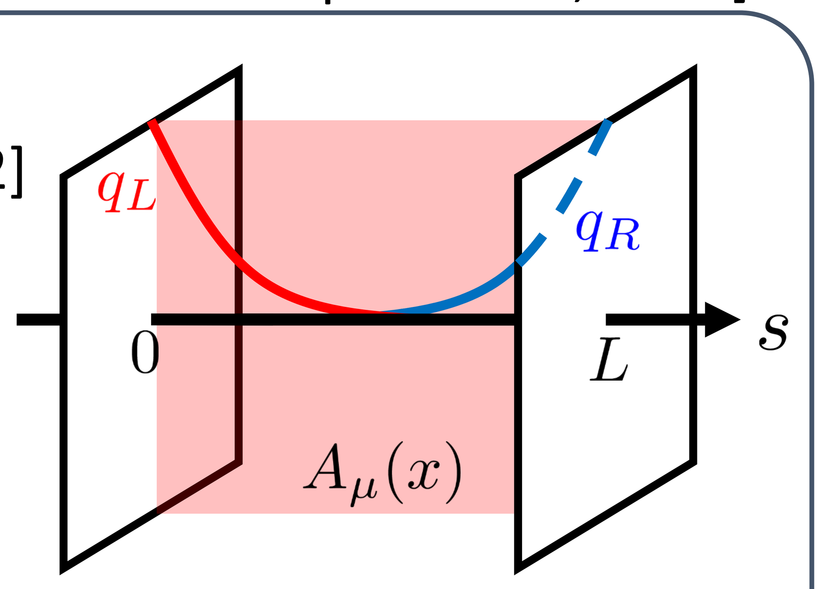
2. Chiral Lattice Formulation

[Grabowska & Kaplan 2015, 2016]

Domain-wall setup

[Kaplan 1992]

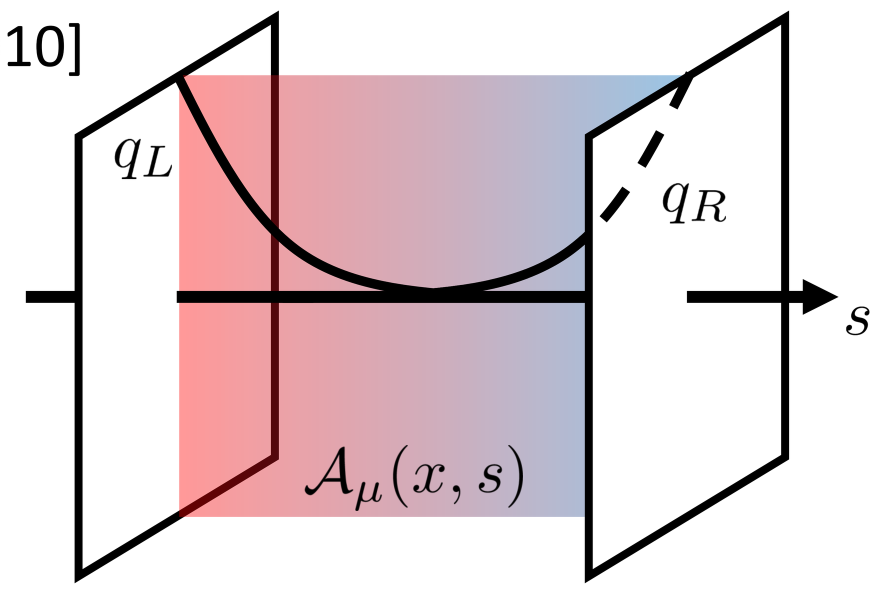
5d lattice space (x, s) with an extra dimension s



- Chiral zero modes are localized near two 4d domain-walls.
- To make the wave function overlap vanish, $L \to \infty$.

Combining the gradient flow (gauge covariant smearing)

- Flow time $\tau(s)$:a monotonically increasing function of s with $\tau(0) = 0, \tau(L) = \infty$
- Flow equation [Lüscher 2010] ~ diffusion equation $\partial_\tau \mathcal{A}_\mu(x, s) = D_\nu \mathcal{F}_{\nu\mu}$
- Boundary condition $\mathcal{A}_\mu(x, s = 0) = A_\mu(x)$



Gradient flow

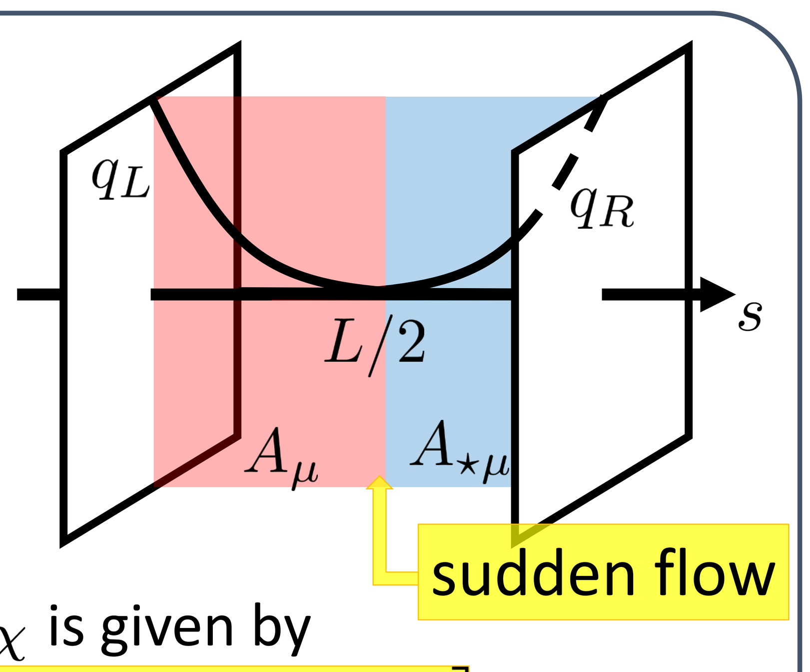
➢ The physical modes in A_μ damps exponentially as the flow time increases.

$$A_{\star\mu}(x) \equiv \mathcal{A}_\mu(x, s = L)$$

➢ The right-handed fermion would **decouple** from A_μ.

Chiral overlap operator

In the model figured on the right, taking the limit $L \to \infty$, we can obtain a 4d effective theory $\mathcal{L} = \bar{\psi}(x)\mathcal{D}_\chi\psi(x)$ in a closed form.



The **chiral overlap operator** $\mathcal{D}_\chi$ is given by

$$a\mathcal{D}_\chi = 1 + \gamma_5 \left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon\epsilon_\star + 1} (1 - \epsilon) \right]$$

where we have used the sign functions

$$\epsilon = H_w[A]/\sqrt{H_w[A]^2}, \quad \epsilon_\star = H_w[A_\star]/\sqrt{H_w[A_\star]^2}$$

of the hermitian Wilson-Dirac operator

$$H_w = \gamma_5 \left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) - \frac{1}{2}a\nabla_\mu\nabla_\mu^* - m \right].$$

∇_μ, ∇_μ^*: forward and backward gauge covariant lattice derivatives

3. Fermion Number Anomaly

Ginsparg-Wilson relation: $\gamma_5\mathcal{D}_\chi + \mathcal{D}_\chi\gamma_5 = a\mathcal{D}_\chi\gamma_5\mathcal{D}_\chi$

Modifications

- γ_5 : $\hat{\gamma}_5 \equiv \gamma_5(1 - a\mathcal{D}_\chi)$
- Chiral projection operators: $\hat{P}_\pm \equiv (1 \pm \hat{\gamma}_5)/2$
- Chiral components: $\hat{P}_-\psi_L(x) = \psi_L(x), \bar{\psi}_L(x)P_+ = \bar{\psi}_L(x),$ $\hat{P}_+\psi_R(x) = \psi_R(x), \bar{\psi}_R(x)P_- = \bar{\psi}_R(x).$

Decomposition $\bar{\psi}\mathcal{D}_\chi\psi = \bar{\psi}_L\mathcal{D}_\chi\psi_L + \bar{\psi}_R\mathcal{D}_\chi\psi_R$

Fermion number anomaly

Consider the fermion number $U(1)$ transformation

$$\psi_L \to e^{i\theta}\psi_L, \bar{\psi}_L \to e^{-i\theta}\bar{\psi}_L.$$

Fermion number anomaly on the lattice associated with the *left-handed fermion* [Okumura & Suzuki 2016]

$$\mathcal{A}_L^{(a)}(x) \equiv \langle \partial_\mu j_{L\mu}(x) \rangle = \mathrm{tr}\left[\hat{P}_-(x, x) - P_+\delta(x, x)\right] = -\,\mathrm{tr}\,\hat{\gamma}_5(x, x)/2$$

→ the continuum limit $\lim_{a\to 0} \mathcal{A}_L^{(a)}(x)$

Properties of $\mathcal{A}_L^{(a)}$

- Symmetry under the exchange of A and $A_\star$ in parity-odd/even parts

$$\mathcal{A}_L^{(a)} = \mathcal{A}_L^{(a)\mathrm{odd}} + \mathcal{A}_L^{(a)\mathrm{even}} \to \begin{array}{l} \mathcal{A}_L^{(a)\mathrm{odd}}[A_\star, A] = +\mathcal{A}_L^{(a)\mathrm{odd}}[A, A_\star] \\ \mathcal{A}_L^{(a)\mathrm{even}}[A_\star, A] = -\mathcal{A}_L^{(a)\mathrm{even}}[A, A_\star] \end{array}$$

- When $A_\star = A$, $\mathcal{A}_L^{(a)}(x)[A, A] = \mathrm{tr}\,\epsilon(x, x)/2$
- Integral over spacetime $a^4 \sum_x \mathcal{A}_L^{(a)}(x) = \frac{1}{2}a^4 \sum_x \mathrm{tr}\,\epsilon(x, x)$ and $\frac{1}{2}\mathrm{tr}\,\epsilon(x, x) \xrightarrow{a\to 0} -\frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\,\mathrm{tr}\left[F_{\mu\nu}F_{\rho\sigma}\right]$

Result

By explicit calculation, we can find $(\mathcal{D}_\rho = \partial_\rho + [A_\rho, \cdot])$

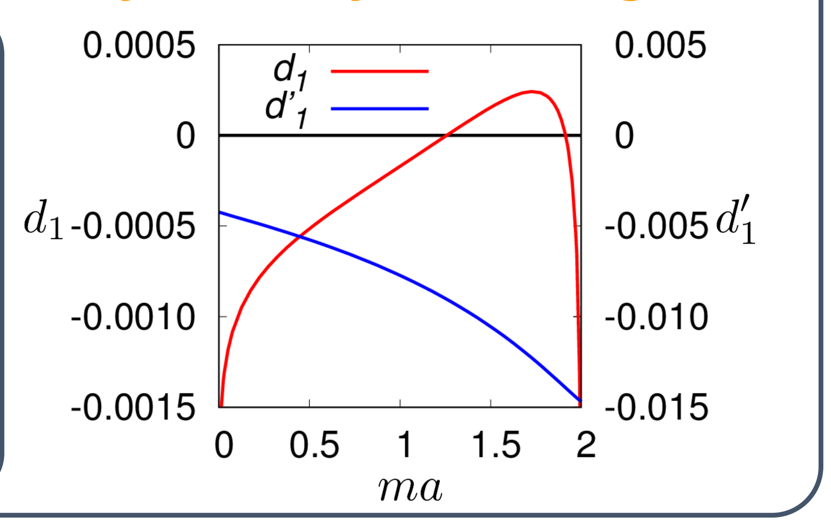
$$\lim_{a\to 0} \mathcal{A}_L^{(a)} = -\frac{1}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}\left\{ \mathrm{tr}\left[F_{\mu\nu}F_{\rho\sigma} + F_{\star\mu\nu}F_{\star\rho\sigma}\right] - \frac{2}{3}\partial_\mu\,\mathrm{tr}\left[C_\nu\mathcal{D}_\rho C_\sigma + C_\nu\mathcal{D}_{\star\rho}C_\sigma\right] \right\} + \underline{d_1}\partial_\mu\,\mathrm{tr}\left[C_\mu C_\nu C_\nu\right] + \underline{d_1'}\partial_\mu\,\mathrm{tr}\left[C_\mu C_\mu C_\mu\right]$$

Lorentz Symmetry Violating Term

Adjoint variable

$$C_\mu(x) \equiv A_{\star\mu}(x) - A_\mu(x)$$

transforms as the adjoint representation under the gauge transformation.



4. Discussion

In this chiral lattice formulation,

- $d_1'\partial_\mu\,\mathrm{tr}\left[C_\mu^3\right] \propto$ the gauge anomaly coefficient $\mathrm{tr}\,T^a\{T^b, T^c\}$.
- Then, for the fermion number $U(1)$, anomaly-free → only ☐ contributes.
- For more general $U(1)$ charges, anomaly-free → $d_1'\partial_\mu\,\mathrm{tr}\left[C_\mu^3\right]$ does not necessarily vanish.

Lorentz symmetry and **gauge anomaly** are linked in this way.

In the sudden flow model, the right-handed fermion seems not to successfully decouple, so we may have to choose a gradual flow scenario.