One-loop perturbative coupling of A and $A_\star$ through the chiral overlap operator

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- H. Makino, O. M., and H. Suzuki, to appear in PTEP [arXiv:1704.04862 [hep-lat]].

Chiral gauge theory on the lattice

- No defintion of chiral gauge theories on the lattice.
- Proposal of Grabowska–Kaplan [2015, 2016]
  - 5D domain-wall (DW) lattice formulation.
  - 4D lattice formulation based on Chiral overlap operator.
- Subtlety associated with topological charge [Okumura–Suzuki, Makino–Morikawa].
- We discuss gauge-breaking effects even for anomaly-free cases.

Chiral DW fermion [Grabowska–Kaplan 2015]

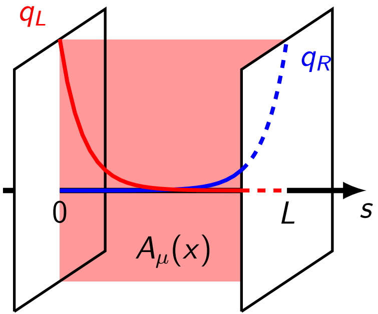
- 5D lattice space (x, s) with 4D DWs [Callan–Harvey, Kaplan]
  - Chiral zero modes are localized near two 4D DWs.
- Gradient flow [Narayanan–Neuberger, Lüscher]

$$\partial_\tau B_\mu(x, s) = D_\nu G_{\nu\mu}(x, s)$$

  - Flow time $\tau(s)$; Boundary condition $B_\mu(x, s = 0) = A_\mu(x)$.
- Manifestly gauge invariant.
- B_μ goes to pure gauge.

$$A_{\star\mu}(x) \equiv B_\mu(x, s = L) = g^{-1}(x)\partial_\mu g(x)$$

- RH fermion would couple only to gauge mode.



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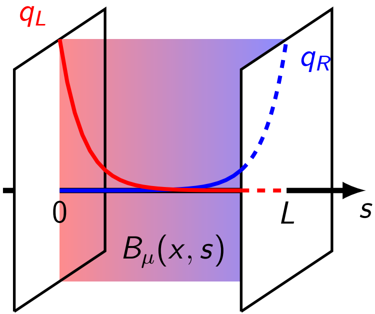


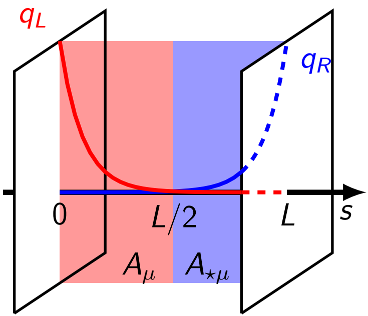
Chiral overlap operator [Grabowska–Kaplan 2016]

- DW formulation
  $\rightarrow$ 4D eff. theory [Neuberger, Vranas, Kikukawa–Noguchi]
- Assuming abrupt transition (sudden flow),

$$a\hat{\mathcal{D}}_\chi = 1 + \gamma_5 \left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon \epsilon_\star + 1} (1 - \epsilon) \right].$$

$$\epsilon = \frac{H_w[A]}{\sqrt{H_w[A]^2}}, \quad \epsilon_\star = \frac{H_w[A_\star]}{\sqrt{H_w[A_\star]^2}},$$

$$\gamma_5 H_w = \frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - \frac{1}{2} a \nabla_\mu \nabla^*_\mu - m.$$



Properties of $\hat{\mathcal{D}}_\chi$ [Grabowska–Kaplan 2016]

1. Ginsparg–Wilson relation

$$\gamma_5\hat{\mathcal{D}}_\chi + \hat{\mathcal{D}}_\chi\gamma_5 = a\hat{\mathcal{D}}_\chi\gamma_5\hat{\mathcal{D}}_\chi.$$

2. Classical continuum limit of $\hat{\mathcal{D}}_\chi$

$$am\hat{\mathcal{D}}_\chi = \gamma_\mu D_\mu(A)\frac{1-\gamma_5}{2} + \gamma_\mu D_\mu(A_\star)\frac{1+\gamma_5}{2} + \mathcal{O}(a),$$

where $D_\mu(A) = \partial_\mu + A_\mu$.
A couples only to LH fermion; $A_\star$ couples only to RH fermion.
In the tree level, A and $A_\star$ decouple from each other.

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- Investigate how the decoupling between A and $A_\star$ is modified under radiative corrections.

One-loop effective action

- Fermion one-loop effective action

$$\ln \mathcal{Z}[A, A_\star] = \ln \int \prod_x \left[d\psi(x) d\bar{\psi}(x) \right] \exp\left[-a^4 \sum_x \bar{\psi}(x) \hat{\mathcal{D}}_\chi \psi(x) \right]$$

- Infinitesimal variations δ_A and $\delta_{A\star}$

$$\delta_A A_\mu \neq 0, \qquad \delta_A A_{\star\mu} = 0,$$
$$\delta_{A\star} A_{\star\mu} \neq 0, \qquad \delta_{A\star} A_\mu = 0.$$

- Gauge transformation δ^ω

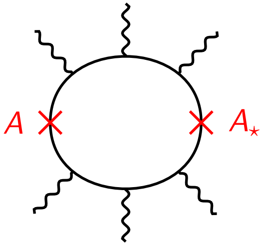
$$\delta^\omega_A A_\mu(x) = \partial_\mu \omega(x) + [A_\mu(x), \omega], \qquad \delta^\omega_A A_{\star\mu}(x) = 0,$$
$$\delta^\omega_{A\star} A_{\star\mu}(x) = \partial_\mu \omega(x) + [A_{\star\mu}(x), \omega], \qquad \delta^\omega_{A\star} A_\mu(x) = 0.$$

- Gauge invariance $(\delta^\omega_A + \delta^\omega_{A\star}) \ln \mathcal{Z}[A, A_\star] = 0$

One-loop perturbative coupling

- Let us focus on a variation of effective action:

$$\delta_A \delta_{A\star} \ln \mathcal{Z}[A, A_\star] \xrightarrow{a\to 0} -\int d^4x\, \mathcal{L}(A, A_\star; \delta A, \delta_\star A_\star) \neq 0.$$



This implies breaking of BRS symmetry!

- Using gauge invariance, setting $A_\star = 0$ ($\because A_\star$: pure gauge),

$$\delta_A \delta_A^\omega \ln \mathcal{Z}[A, 0] \xrightarrow{a\to 0} \int d^4x\, \mathcal{L}(A, A_\star = 0; \delta A, \delta_\star^\omega A_\star|_{A\star=0}).$$

- $\delta_A \delta_A^\omega \ln \mathcal{Z}[A, 0]$ does not vanish even for anomaly-free cases.
- Thus, $\ln \mathcal{Z}[A, 0]$ is not invariant under gauge variation δ_A^ω.

Discussion with $A_\star = 0$ gauge

- Functional integral over A ($A_\star$ is given from A)

$$\langle \mathcal{O}[A] \rangle = \int \mathcal{D}A \, \mathcal{Z}[A, A_\star] \, e^{-S_G[A]} \, \mathcal{O}[A].$$

- Faddeev–Popov determinant: $\Delta[A_\star] \int \mathcal{D}g \, \delta(A_\star^g) = 1$.
  Inserting this into functional integral,

$$\int \mathcal{D}g \int \mathcal{D}A \, \underline{\Delta[A_\star] \, \delta(A_\star^g)} \mathcal{Z}[A, A_\star] \, e^{-S_G[A]} \, \mathcal{O}[A]$$

$$\propto \int \mathcal{D}A \, \underline{\Delta[A_\star] \, \delta(A_\star)} \, \mathcal{Z}[A, 0] \, e^{-S_G[A]} \, \mathcal{O}[A].$$

- Faddeev–Popov ghosts and Nakanishi–Lautrup field

$$S_{c\bar{c}} + S_{\mathrm{gf}} = \int d^4x \left\{ \bar{c}_\mu^a \left[\partial_\mu c^a + f^{abc} A_{\star\mu}^b c^c \right] - i B_\mu^a A_{\star\mu}^a \right\}.$$

- $S_{c\bar{c}}$, S_{gf}, S_G and $\mathcal{O}[A]$ are BRS invariant; In $\mathcal{Z}[A, 0]$ is not.

Explicit forms of $\mathcal{L}$ and $\ln \mathcal{Z}$

- Relation between $\mathcal{L}$ and $\ln \mathcal{Z}$

$$-\delta_{A}\delta_{A\star} \ln \mathcal{Z}[A, A_\star] \xrightarrow{a\to 0} \int d^4x\, \mathcal{L}(A, A_\star; \delta A, \delta_\star A_\star).$$

- Expand around $a = 0$ and simplify in terms of

$$\bar{A}_\mu \equiv \frac{1}{2}(A_\mu + A_{\star\mu}) \quad \begin{cases} \bar{D}_\mu & = \partial_\mu + [\bar{A}_\mu, \cdot] \\ \bar{F}_{\mu\nu} & = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu + [\bar{A}_\mu, \bar{A}_\nu], \end{cases}$$

$$C_\mu \equiv A_{\star\mu} - A_\mu.$$

C_μ transforms as adjoint representation under $\delta^\omega_A + \delta^\omega_{A\star}$.

Parity-odd part of $\mathcal{L} \sim \delta\delta \ln \mathcal{Z}$

$$
-\frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\,\mathrm{tr}\Bigg[\left(\bar{F}_{\mu\nu}+\frac{1}{12}[C_\mu, C_\nu]\right)\{\delta A_\rho, \delta_\star A_{\star\sigma}\} \\
-\frac{1}{3}C_\mu\left(\{\delta A_\nu, \bar{D}_\rho\delta_\star A_{\star\sigma}\}+\{\delta_\star A_{\star\nu}, \bar{D}_\rho\delta A_\sigma\}\right)\Bigg]
$$

- This is proportional to gauge anomaly coefficient $\mathrm{tr}\, T^a\{T^b, T^c\}$.

Parity-even, Lorentz-preserving part of $\mathcal{L} \sim \delta\delta \ln \mathcal{Z}$

$$
\begin{aligned}
&\frac{f_0}{a^2} \operatorname{tr} \delta A_\mu \delta_\star A_{\star\mu} \\
&+ (-3f_1/2 + f_2/2 - f_3/2) \operatorname{tr}[(\bar{D}_\mu \delta A_\mu) C_\nu \delta_\star A_{\star\nu} - C_\mu \delta A_\mu (\bar{D}_\nu \delta_\star A_{\star\nu})] \\
&- (f_1/2 + f_2/2 - 3f_3/2) \operatorname{tr}[C_\mu (\bar{D}_\nu \delta A_\mu) \delta_\star A_{\star\nu} - \delta A_\mu C_\nu (\bar{D}_\mu \delta_\star A_{\star\nu})] \\
&- (f_1/2 + f_2/2 - 3f_3/2) \operatorname{tr}[C_\nu \delta A_\mu (\bar{D}_\mu \delta_\star A_{\star\nu}) - (\bar{D}_\nu \delta A_\mu) C_\mu \delta_\star A_{\star\nu}] \\
&+ (-7f_1/2 + f_2/2 + f_3/2) \operatorname{tr}[(\bar{D}_\mu C_\mu) \delta A_\nu \delta_\star A_{\star\nu} - \delta A_\nu (\bar{D}_\mu C_\mu) \delta_\star A_{\star\nu}] \\
&- (3f_1/2 - f_2/2 + f_3/2) \operatorname{tr}[\delta A_\mu C_\mu (\bar{D}_\nu \delta_\star A_{\star\nu}) - C_\nu (\bar{D}_\mu \delta A_\mu) \delta_\star A_{\star\nu}] \\
&+ (13f_1 - 3f_2 - 3f_3) \operatorname{tr}(\bar{D}_\mu \delta A_\mu)(\bar{D}_\nu \delta_\star A_{\star\nu}) \\
&+ (9f_1 - 3f_2 - f_3) \operatorname{tr}(\bar{D}_\mu \delta A_\nu)(\bar{D}_\mu \delta_\star A_{\star\nu}) \\
&+ (-19f_1 + 5f_2 + 5f_3) \operatorname{tr}(\bar{D}_\nu \delta A_\mu)(\bar{D}_\mu \delta_\star A_{\star\nu}) \\
&+ (11f_1/6 - f_2/6 - 7f_3/6) \operatorname{tr} C_\mu \delta A_\nu C_\mu \delta_\star A_{\star\nu} \\
&+ (-13f_1/6 + 11f_2/6 - 7f_3/6) \operatorname{tr}(C_\mu \delta A_\mu C_\nu \delta_\star A_{\star\nu} + C_\nu \delta A_\mu C_\mu \delta_\star A_{\star\nu}) \\
&+ (-5f_1/12 + 19f_2/12 - 17f_3/12) \operatorname{tr}(C_\nu C_\mu \delta A_\mu \delta_\star A_{\star\nu} + \delta A_\mu C_\mu C_\nu \delta_\star A_{\star\nu}) \\
&+ (19f_1/12 - 5f_2/12 - 5f_3/12) \operatorname{tr}(C_\mu C_\nu \delta A_\mu \delta_\star A_{\star\nu} + \delta A_\mu C_\nu C_\mu \delta_\star A_{\star\nu}) \\
&+ (-17f_1/12 + 19f_2/12 - 11f_3/12) \operatorname{tr}(C_\mu C_\mu \delta A_\nu \delta_\star A_{\star\nu} + \delta A_\nu C_\mu C_\mu \delta_\star A_{\star\nu})
\end{aligned}
$$

Parity-even, Lorentz-violating part of $\mathcal{L} \sim \delta\delta \ln \mathcal{Z}$

$$
\begin{aligned}
&\frac{3}{2}\left(9f_1 - f_2 - f_3 - \frac{f_4}{2} - \frac{f_5}{2}\right)\operatorname{tr}[(\bar{D}_\nu C_\nu)\delta A_\nu \delta_\star A_{\star\nu} - \delta A_\nu(\bar{D}_\nu C_\nu)\delta_\star A_{\star\nu}] \\
&\quad - \left(9f_1 - f_2 - f_3 - \frac{f_4}{2} - \frac{f_5}{2}\right)\operatorname{tr}(\bar{D}_\nu \delta A_\nu)(\bar{D}_\nu \delta_\star A_{\star\nu}) \\
&\quad + \left(\frac{47f_1}{2} - \frac{7f_2}{2} - \frac{7f_3}{2} + \frac{f_4}{4} - \frac{7f_5}{4}\right)\operatorname{tr} C_\nu \delta A_\nu C_\nu \delta_\star A_{\star\nu} \\
&\quad + \left(\frac{67f_1}{4} - \frac{11f_2}{4} - \frac{11f_3}{4} + \frac{5f_4}{8} - \frac{11f_5}{8}\right) \\
&\qquad\qquad \times \operatorname{tr}(C_\nu C_\nu \delta A_\nu \delta_\star A_{\star\nu} + \delta A_\nu C_\nu C_\nu \delta_\star A_{\star\nu})
\end{aligned}
$$

(coefficients $f_i = f_i(am)$)

Gauge anomaly

- Integrating $\int d^4x\, \mathcal{L} \sim \delta\delta \ln \mathcal{Z}$ with A, we can obtain
  1. gauge anomaly (parity-odd)
  2. effective action: $\ln \mathcal{Z}$ (parity-even).

$$
\begin{aligned}
&\delta_A^\omega \ln \mathcal{Z}[A,0]\big|_{\text{parity-odd}} \\
&= \frac{1}{24\pi^2}\epsilon_{\mu\nu\rho\sigma}\int d^4x\ \mathrm{tr}\left[(\partial_\mu\omega)\left(A_\nu\partial_\rho A_\sigma + \frac{1}{2}A_\nu A_\rho A_\sigma\right)\right]
\end{aligned}
$$

Gauge *non-invariant* effective action

$$
\begin{aligned}
&\ln \mathcal{Z}[A,0]\big|_{\text{parity-even}} \\
&= \int d^4x \ \mathrm{tr}\Bigg[\frac{f_0}{2a^2}A_\mu A_\mu \\
&\quad + \tfrac{1}{2}(-13f_1 + 3f_2 + 3f_3)A_\mu\partial_\mu\partial_\nu A_\nu \\
&\quad + (5f_1 - f_2 - 2f_3)(A_\mu\partial_\nu\partial_\nu A_\mu - A_\mu A_\nu \partial_\mu A_\nu + A_\mu A_\nu \partial_\nu A_\mu) \\
&\quad + \tfrac{2}{3}(f_1 + f_2 - 2f_3)A_\mu A_\mu A_\nu A_\nu + \tfrac{1}{12}(-11f_1 + f_2 + 7f_3)A_\mu A_\nu A_\mu A_\nu \\
&\quad + \tfrac{1}{2}\left(9f_1 - f_2 - f_3 - \tfrac{f_4}{2} - \tfrac{f_5}{2}\right)A_\mu\partial_\mu\partial_\mu A_\mu \\
&\quad + \tfrac{1}{12}\left(57f_1 - 9f_2 - 9f_3 + \tfrac{3f_4}{2} - \tfrac{9f_5}{2}\right)A_\mu A_\mu A_\mu A_\mu\Bigg]
\end{aligned}
$$

- Even for anomaly-free cases, these terms do not generally vanish, and should be removed by local counterterms.
  → ~~Non-perturbative~~ formulation.

Summary

- Grabowska–Kaplan lattice formulation of chiral gauge theories.
  - Analytical model: abrupt transition.
  - Tree-level decoupling between A and $A_\star$.

- We focused on one-loop effective coupling between A and $A_\star$.
- Effective coupling $\neq 0$ even for anomaly-free cases.
- This means BRS non-invariance and need of local counterterms!

- *Improvement of $\hat{\mathcal{D}}_\chi$?*
- *Abrupt transition $\to$ gradual flow scenario?*

Backup: Coefficients f_i

$$f_0(am) \equiv \int_p \left(-\frac{1}{4t} - \frac{s_\rho^2}{4t} - \frac{cc_\rho}{4t} \right),$$

$$f_1(am) \equiv \int_p \left(\frac{1}{64t^2} - \frac{c_\rho c_\sigma}{128t} + \frac{s_\rho^2 s_\sigma^2}{32t^2} \right),$$

$$f_2(am) \equiv \int_p \left(-\frac{c_\rho c_\sigma}{32t} + \frac{7s_\rho^2 s_\sigma^2}{64t^2} + \frac{cs_\rho^2 c_\sigma}{32t^2} + \frac{c^2 c_\rho c_\sigma}{64t^2} \right),$$

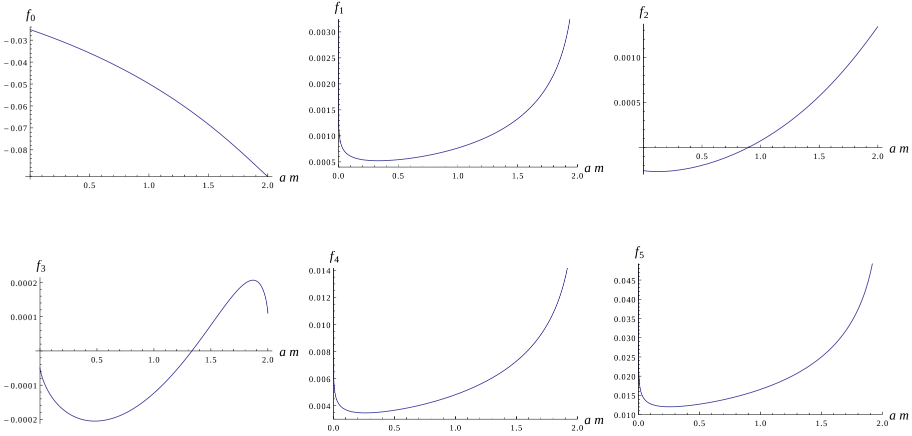
$$f_3(am) \equiv \int_p \left(-\frac{c_\rho c_\sigma}{32t} + \frac{3s_\rho^2 s_\sigma^2}{32t^2} - \frac{s_\rho^2}{32t^2} - \frac{cc_\rho}{32t^2} \right),$$

$$f_4(am) \equiv \int_p \left(\frac{1}{96t} + \frac{s_\rho^2}{96t} + \frac{cc_\rho}{96t} + \frac{1}{16t^2} \right),$$

$$f_5(am) \equiv \int_p \left(\frac{1}{16t} + \frac{c_\rho c_\sigma}{32t} + \frac{7}{32t^2} - \frac{c^2}{32t^2} + \frac{cc_\rho}{16t^2} + \frac{s_\rho^2}{32t^2} \right).$$

$$s_\mu = \sin p_\mu, \qquad c_\mu = \cos p_\mu, \qquad c = \sum_\mu (c_\mu - 1) + am, \qquad t = \sum_\mu s_\mu^2 + c^2.$$

Backup: Coefficients f_i



Backup: Is $A_\star$ pure gauge?

- Gradient flow

$$\partial_t B_\mu(x,t) = D_\nu G_{\nu\mu}(x,t), \qquad B_\mu(x,t=0) = A_\mu(x).$$

- In abelian theory, solution to flow eq. is given by

$$B_\mu(x,t) = \int d^4y \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \left[\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) e^{-tp^2} + \frac{p_\mu p_\nu}{p^2} \right] A_\nu(y)$$

$$\xrightarrow{t\to\infty} g(x)^{-1}\partial_\mu g(x)\text{: Pure gauge,}$$

$$g(x) = \exp\left[-\int d^4y \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2} \partial_\mu A_\mu(y) \right].$$

- *Assumption*: topologically trivial → We can take $A_\star = 0$ gauge.

Backup: Chiral overlap operator (gradual flow)

- Dirac operator of Grabowska–Kaplan

$$a\mathcal{D}_\chi = \lim_{L\to\infty}\left[1 + \gamma_5 \frac{1 - \prod_{s=L}^{1} T(s)}{1 + \prod_{s=L}^{1} T(s)}\right].$$

- Transfer matrix

$$T(s) = \begin{pmatrix} B_s^{-1} & -B_s^{-1}C_s \\ -C_s^\dagger B_s^{-1} & B_s + C_s^\dagger B_s^{-1} C_s \end{pmatrix},$$

where

$$B_s = 1 + a_5\left(-\frac{a}{2}\nabla_{\mu,s}\nabla^*_{\mu,s} - m\right), \quad C_s = a_5 \sigma_\mu \frac{1}{2}(\nabla_{\mu,s} + \nabla^*_{\mu,s}).$$

Backup: Lattice modifications

- Modified γ_5

$$\hat{\gamma}_5 \equiv \gamma_5(1 - a\hat{\mathcal{D}}_\chi),$$

which satisfies

$$(\hat{\gamma}_5)^2 = 1, \quad \hat{\mathcal{D}}_\chi \hat{\gamma}_5 = -\gamma_5 \hat{\mathcal{D}}_\chi.$$

- Using modified chiral projection operators

$$\hat{P}_\pm = \frac{1}{2}(1 \pm \hat{\gamma}_5),$$

the chiral components are defined as

$$\hat{P}_- \psi_L(x) = \psi_L(x), \qquad \bar{\psi}_L P_+ = \bar{\psi}_L(x),$$
$$\hat{P}_+ \psi_R(x) = \psi_R(x), \qquad \bar{\psi}_R P_- = \bar{\psi}_R(x).$$