

Identification of perturbative ambiguity canceled against bion

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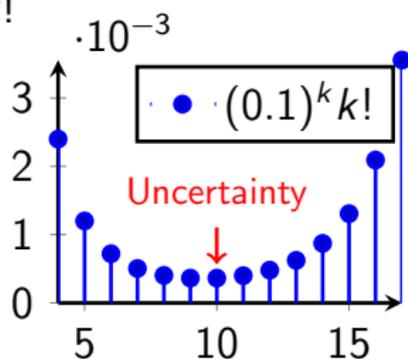
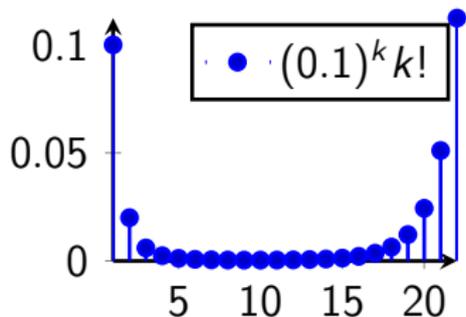
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2020/9/14 日本物理学会 2020 年秋季大会

- PLB **807** (2020) 135570 [arXiv:2003.04759 [hep-th]]

Factorial growth of perturbation series

- In QM/QFT, perturbative expansions are divergent series
- Typically, $F(\lambda) = \sum_{k=0}^{\infty} c_k \lambda^k$, $c_k \sim k!$



1 Proliferation of Feynman diagrams

Ground state energy

Zeeman effect $\sim (-1)^k (2k)!$

Stark effect $\sim (2k)!$

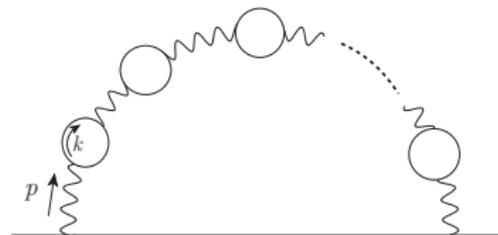
$V(\phi) \sim \phi^3$ $\sim \Gamma(k + 1/2)$

$V(\phi) \sim \phi^4$ $\sim (-1)^k \Gamma(k + 1/2)$

Double well $\sim k!$

Periodic cosine $\sim k!$

2 Renormalon [t Hooft '79]



$\sim \beta_0^k k!$

Borel resummation and perturbative ambiguity

- Borel resummation: summing divergent asymptotic series

$$f(g^2) \sim \sum_{k=0}^{\infty} f_k \left(\frac{g^2}{16\pi^2} \right)^{k+1} \quad \text{with } f_k \sim a^k k! \text{ as } k \rightarrow \infty$$

⇓ Borel transform

$$B(u) \equiv \sum_{k=0}^{\infty} \frac{f_k}{k!} u^k = \frac{1}{1-au} \quad (\text{Pole singularity at } u = 1/a).$$

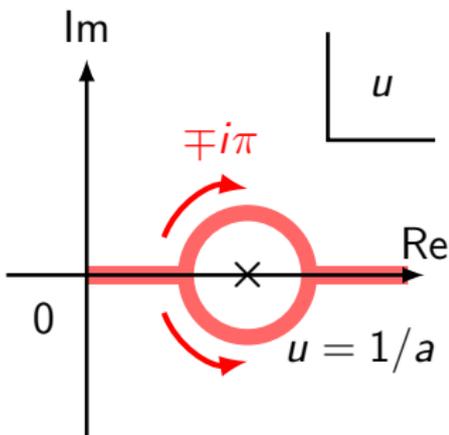
- The Borel sum is given by

$$f(g^2) \equiv \int_0^{\infty} du B(u) e^{-16\pi^2 u/g^2}.$$

- $a < 0$ (alternating series) \rightarrow convergent

- $a > 0 \rightarrow$ ill-defined due to the pole

\Rightarrow Imaginary ambiguity $\sim \pm e^{-16\pi^2/(ag^2)}$



Resurgence theory and semi-classical picture

- Resurgence structure

$$\delta_{\text{PT}} \sim e^{-16\pi^2/(ag^2)} \overset{\text{Cancellation}}{\longleftrightarrow} \text{Nonperturbative effects}$$

① Instanton (one-instanton action $S_I = 8\pi^2/g^2$)

② Bion [Ünsal '07]: fractional instanton/anti-instanton pair

on $\mathbb{R}^{d-1} \times S^1$ with \mathbb{Z}_N -twisted boundary conditions (BC)

	PT	Semi-classical object
\mathbb{R}^4	$\delta_{\text{PFD}} \sim e^{-16\pi^2/g^2}$	$\delta_{\text{instanton}} \sim e^{-2S_I}$
\mathbb{R}^4	$\delta_{\text{renormalon}} \sim e^{-16\pi^2/(\beta_0 g^2)}$	
$\mathbb{R}^3 \times S^1$		$\delta_{\text{bion}} \sim e^{-2S_B} = e^{-2S_I/N}$

(E.g., $\beta_0 = \frac{11}{3}N$ for $SU(N)$ GT)

- Reading out nonperturbative effects from PT: Resurgence theory

Resurgence theory and semi-classical picture

- Resurgence structure

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- Reading out nonperturbative effects from PT: Resurgence theory
- What about QFT?
 - Conjecture: renormalon \leftrightarrow Bion [Argyres–Ünsal '12, ...]
 - But, no renormalon on $\mathbb{R}^3 \times S^1 \rightarrow$ Conjecture [16pSA-4]

Our work: Enhancement of PFD ambiguity

- What cancels bion ambiguities?
- Answer: **PFD-type ambiguity** (cf. [Anber–Sulejmanpasic '14])

$$\delta_{\text{PFD}}|_{\mathbb{R}^4} \sim e^{-16\pi^2/g^2} \stackrel{?}{\leftrightarrow} \delta_{\text{bion}}|_{\mathbb{R}^3 \times S^1} \sim e^{-2S_B} = e^{-2S_I/N}$$

- **Enhancement of PFD upon compactification** [O.M.–Takaura '20]

$$\text{PFD: } k! \rightarrow N^k k!$$

$$\delta_{\text{PFD}} \sim e^{-16\pi^2/g^2} \rightarrow \delta'_{\text{PFD}} \sim e^{-16\pi^2/(Ng^2)}$$

- Keys (similar to Linde problem ['80] in finite-temperature QFT):
 - ① S^1 compactification \rightarrow IR divergences
 - ② twisted BC \rightarrow twist angles as IR regulators
- Let us study the $\mathbb{C}P^N$ model on $\mathbb{R} \times S^1$
 - ▶ Bion calculus [Fujimori–Kamata–Misumi–Nitta–Sakai '18]

2-dimensional $\mathbb{C}P^N$ model with twisted BC

- 2D $\mathbb{C}P^{N-1}$ model ($A = 1, 2, \dots, N$)

$$S = \frac{1}{g^2} \int d^2x \left[\partial_\mu \bar{z}^A \partial_\mu z^A - j_\mu j_\mu + f(\bar{z}^A z^A - 1) \right]$$

where $j_\mu = (1/2i) \bar{z}^A \overleftrightarrow{\partial}_\mu z^A$, f is an auxiliary field ($\bar{z}^A z^A = 1$)

▶ # of vacuum bubble diagrams $\sim 4^k \Gamma(k + 1/2)$

- \mathbb{Z}_N -twisted BC:

$$z^A(x_1, x_2 + 2\pi R) = e^{2\pi i m_A R} z^A(x_1, x_2), \quad m_A R = \begin{cases} A/N & \text{for } A \neq N \\ 0 & \text{for } A = N \end{cases}$$

- Amplitude of a diagram (F^{2k} : $2k$ th-order polynomial, $f_0 = \langle f \rangle$)

$$\xrightarrow{p_2=0, A=1} \frac{V_2(g^2)^k}{(2\pi R)^{k+1}} \int \left(\prod_{i=1}^{k+1} \frac{dp_{i,1}}{2\pi} \right) \frac{F^{2k}(p_{i,1}, m_A)}{\prod_{i=1}^{2k} [q_{i,1}^2 + m_A^2 + f_0]}$$

IR divergence in massless limit $m_A^2 + f_0 \rightarrow 0$ ($\int d^2p \rightarrow \int dp$)

IR structure and enhancement mechanism

- $m_A^2 + f_0 = 1/(NR)^2 + f_0$ works as an IR regulator

- ▶ $m_A^2 \gg f_0$

$$\sim \frac{V_2}{R^2} \frac{1}{(m_A R)^{k-1}} \left(\frac{g^2}{4\pi}\right)^k \xrightarrow{A=1} \frac{V_2}{R^2} \frac{1}{N} \left(\frac{Ng^2}{4\pi}\right)^k \quad \text{Enhancement!}$$

- ▶ $m_A^2 \ll f_0$

$$\sim \frac{V_2}{R^2} \sum_{\alpha \geq 0} \frac{(m_A R)^\alpha}{(f_0 R^2)^{(k+\alpha-1)/2}} \left(\frac{g^2}{4\pi}\right)^k \quad \text{~~Enhancement~~}$$

- Dependence on $NR\Lambda$ (Λ : dynamical scale)

- ▶ $NR\Lambda \ll 1$ (Bion calculus is valid)

$$\sqrt{f_0} R \sim \frac{g^2}{4\pi} \Rightarrow [m_A^2 = \mathcal{O}(g^0)] \gg [f_0 = \mathcal{O}(g^4)] \quad \text{Enhancement!}$$

- ▶ $NR\Lambda \gg 1$ (Large N)

$$f_0 \sim \Lambda^2 \Rightarrow \frac{m_A^2}{f_0} = \frac{1}{(NR\Lambda)^2} \ll 1 \quad \text{~~Enhancement~~}$$

Summary and discussion

- Resurgence structure on $\mathbb{R}^{d-1} \times S^1$

	PT	Semi-classical object
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$\mathbb{R}^3 \times S^1$	$\delta'_{\text{PFD}} \sim e^{-16\pi^2/(Ng^2)}$	$\delta_{\text{bion}} \sim e^{-2S_I/N}$
$\mathbb{R}^3 \times S^1$	$\delta_{\text{renormalon}} = 0$ [16pSA-4]	No

- Enhancement phenomenon is consistent with bion
- Borel singularities at $m_A R = A/N = \underbrace{1/N}_{1\text{-bion}}, \underbrace{2/N}_{2\text{-bion}}, \dots$
- $NRA \gg 1$ [Ishikawa-O.M.-Shibata-Suzuki '20]
vs $NRA \ll 1$ [Fujimori-Kamata-Misumi-Nitta-Sakai '18]

$$E_{\text{large } N} \sim N^{-1}(\Lambda R)^{-2} \delta \epsilon^2 \quad \Rightarrow f_0 = \Lambda^2 \text{ in denominator}$$

$$\updownarrow \quad \quad \quad \downarrow (m_A^2 \text{ vs } f_0)$$

$$\text{Im } E_{1\text{-bion}} \sim \pm N(\Lambda R)^2 \delta \epsilon^2 \quad \Leftarrow \text{enhancement of } N$$

- Helpful in giving a unified understanding on resurgence