

# Renormalon in $SU(N)$ QCD(adj.) on compactified spacetime

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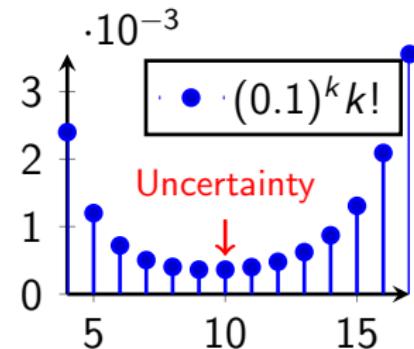
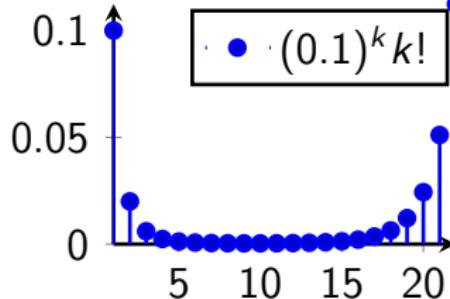
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# Factorial growth of perturbation series

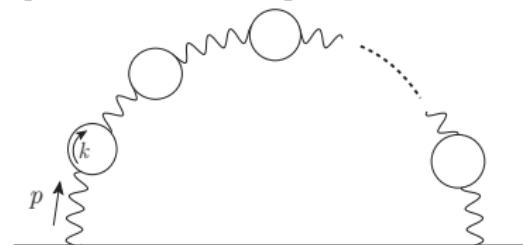
- In QM/QFT, perturbative expansions are divergent series
- Typically,  $F(\lambda) = \sum_{k=0}^{\infty} c_k \lambda^k$ ,  $c_k \sim k!$



- A source of  $k!$  in QFT: **renormalon** [*'t Hooft 1979*]

▶ Amplitude of a single Feynman diagram  $\sim \beta_0^k k!$

( $\beta_0$ : one-loop coefficient of the beta function)



# Borel resummation and perturbative ambiguity

- Borel resummation: summing divergent asymptotic series

$$f(g^2) \sim \sum_{k=0}^{\infty} f_k \left( \frac{g^2}{16\pi^2} \right)^{k+1} \quad \text{with } f_k \sim \beta_0^k k!$$

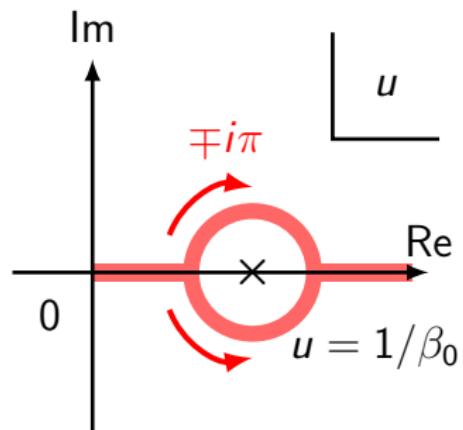
↓ Borel transform

$$B(u) \equiv \sum_{k=0}^{\infty} \frac{f_k}{k!} u^k = \frac{1}{1 - \beta_0 u} \quad (\text{Pole singularity at } u = 1/\beta_0).$$

- The Borel sum is given by

$$f(g^2) \equiv \int_0^{\infty} du B(u) e^{-16\pi^2 u/g^2}.$$

- $\beta_0 > 0 \rightarrow$  ill-defined due to the pole  
 $\Rightarrow$  Imaginary ambiguity  $\sim \pm e^{-16\pi^2/(\beta_0 g^2)}$



# Resurgence theory and renormalon puzzle

- Consistency of quantum theories → Resurgence structure

$$\delta_{\text{renormalon}} \sim e^{-16\pi^2/(\beta_0 g^2)} \xleftrightarrow{\text{Cancellation}} \text{Nonperturbative effect such as instanton}$$

- Resurgence theory for ODE and QM
- But, it is not clear what cancels renormalon...
  - ▶  $S = S_I/\beta_0$  (instanton  $S_I = 8\pi^2/g^2$ )? If exists,  $\delta_{\text{NP}} \sim e^{-2S_I/\beta_0}$
- A possible candidate [Argyres–Ünsal '12, ...]:  
Bion [Ünsal '07] (fractional instanton/anti-instanton pair)  
on  $\mathbb{R}^{d-1} \times S^1$  with  $\mathbb{Z}_N$ -twisted boundary conditions (BC)

$$\varphi^A(x, x_d + 2\pi R) = e^{2\pi i m_A R} \varphi^A(x, x_d), \quad m_A R = \begin{cases} A/N & \text{for } A \neq N \\ 0 & \text{for } A = N \end{cases}$$

- $S_B = S_I/N \leftarrow$  similar  $N$  dependence ( $\beta_0 = \frac{11}{3}N$  for  $SU(N)$  GT)
- Renormalon on  $S^1$  compactified spacetime?

# Renormalon analysis on $\mathbb{R}^3 \times S^1$

- Renormalon analysis for 4D  $SU(N)$  QCD(adj.) on  $\mathbb{R}^3 \times S^1$   
( $N = 2, 3$  [Anber-Sulejmanbasic '14],  $\forall N$  [Ashie-O.M.-Suzuki-Takaura '20])
- (Usually) Renormalon can appear from

$$g^2(\mu^2) \int \frac{d^4 p}{(2\pi)^4} \frac{(p^2)^\alpha}{1 - \Pi(p^2)} \quad \Pi(p^2) \sim g^2(\mu^2) \ln p^2 \quad \text{as } p^2 \rightarrow 0$$

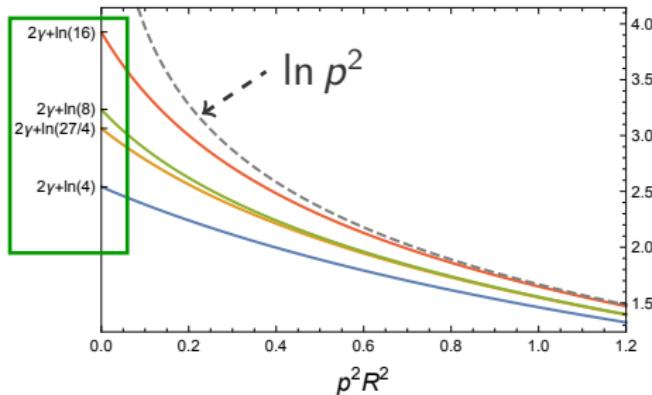
- ▶ Logarithmic factor in vacuum polarization is crucial
- ▶ Coupling expansion:  $g^{2(k+1)} \int_p (p^2)^\alpha (\ln p^2)^k \rightarrow g^{2(k+1)} k!$

- 1-loop analysis on  $\mathbb{R}^3 \times S^1$

$$L(p^2 \rightarrow 0) = -\frac{m_{sc}^2}{p^2} + \text{const.}$$

$$T(p^2 \rightarrow 0) = \text{const.}$$

- No factorial growth occurs  
→ No renormalons

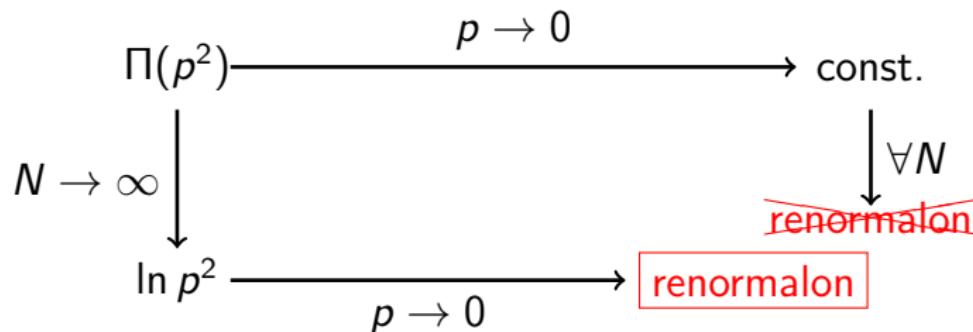


# Subtlety of large $N$ limit on $\mathbb{R}^3 \times S^1$

- Naive (apparent)  $1/N$  expansion

$$\ln p^2 + \underbrace{\frac{1}{N} f_{\text{finite}}(p^2)}_{-\ln p^2 + \dots \text{ as } p \rightarrow 0} \xrightarrow{p \rightarrow 0} \text{const.}$$

- Does there exist renormalons in  $N \rightarrow \infty$ ?



- ▶ Large  $N \rightarrow$  volume independence of integrand
- Definition of renormalon  $\stackrel{\text{Incompatible?}}{\longleftrightarrow} 1/N$  expansion

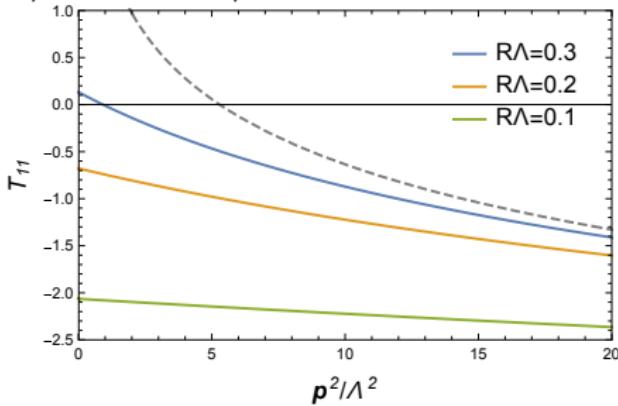
# “Renormalon precursor” and decompactification

- $\delta_{\text{renormalon}}|_{\mathbb{R}^3 \times S^1} = 0$  for  $\forall R$ , but  $\delta_{\text{renormalon}}|_{\mathbb{R}^4} = e^{-16\pi^2/(\beta_0 g^2)}$
- Can renormalon on  $\mathbb{R}^4$  emerge under  $R \rightarrow \infty$ ?
- Reconsider

$$g^2(\mu^2) \int \frac{d^3 p}{(2\pi)^4} \frac{1}{2\pi R} \sum_{p_3 \in \mathbb{Z}/R} \frac{(p^2)^\alpha}{1 - \Pi(p^2)}.$$

→ Pole singularity at  $1 - \Pi(p^2)|_{\mu^2} = \Pi(p^2)|_{\mu^2=\Lambda^2} = 0$

- For  $p_3 \lesssim \Lambda$ ,  
ambiguities of  $\int d^3 p$  can arise  
("Renormalon precursor")
- $R \rightarrow \infty$ ,  $\text{pole}|_{\mathbb{R}^3 \times S^1} \rightarrow \text{pole}|_{\mathbb{R}^4}$   
then,  $\delta|_{\mathbb{R}^4}$  emerge!



► On  $\mathbb{R}^4$ , ambiguity of  $\int d^4 p = \text{ambiguity in Borel sum}$

# Summary

- (Our) Current understanding on resurgence structure in QFT

	PT	Semi-classical object
$\mathbb{R}^4$	$\delta_{\text{PFD}} \sim e^{-16\pi^2/g^2}$	$\delta_{\text{instanton}} \sim e^{-2S_I}$
$\mathbb{R}^4$	$\delta_{\text{renormalon}} \sim e^{-16\pi^2/(\beta_0 g^2)}$	?
$\mathbb{R}^3 \times S^1$	$\delta'_{\text{PFD}} \sim e^{-16\pi^2/(Ng^2)} [14\text{pSA-2}]$	$\delta_{\text{bion}} \sim e^{-2S_I/N}$
$\mathbb{R}^3 \times S^1$	$\delta_{\text{renormalon}} = 0$	No

- ▶ PFD: Proliferation of Feynman Diagrams
- ▶  $S_I = 8\pi^2/g^2$
- There remains renormalon puzzle (renormalon precursor also)
- Helpful in giving a unified understanding on resurgence