Disturbance Evaluation Circuit in Quantum Measurement

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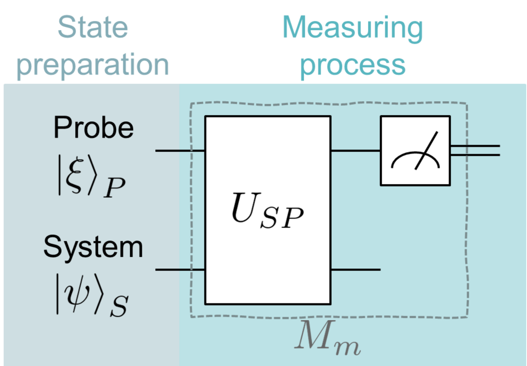
1. Introduction

■ **Measuring process (indirect measurement model) :** $M = (\mathcal{K}, \xi, U, M)$

Quantum Root-Mean-Square (QRMS) Error / Disturbance [1-6]



	Settings	
	Measured System S	Probe System P
Hilbert space	$\mathcal{H}$	$\mathcal{K}$
Initial state	$\vert\psi\rangle$	$\vert\xi\rangle$
Measuring interaction (Unitary op.)	U on $\mathcal{H}\otimes\mathcal{K}$	
Observable	A, B	M

Measured Observable : $A(0) = A \otimes I$ Disturbed Observable : $B(0) = B \otimes I$

Meter Observable : $M(0) = I \otimes M$ Time evolution : $M(t) = U^\dagger(I \otimes M)U$, $B(t) = U^\dagger(B \otimes I)U$

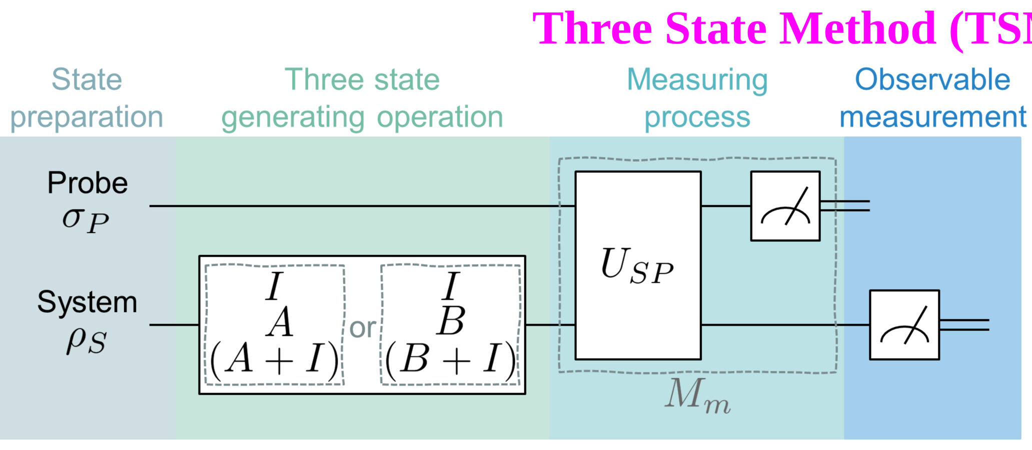
Operator-based error measure : $\varepsilon_O(A) := \|[M(t) - A(0)]\,|\psi\rangle\,|\xi\rangle\|$

disturbance measure : $\eta_O(B) := \|[B(t) - B(0)]\,|\psi\rangle\,|\xi\rangle\|$

How to obtain error & disturbance ?

■ **Previous experimental evaluation methods for error & disturbance**

Three State Method (TSM) [7-10]



Quantum Instrument or Measurement Operators :

$$\boldsymbol{M} : \{M_m\},\ \mathcal{I}(m)\rho = M_m \rho M_m^\dagger$$

$$E_m := M_m^\dagger M_m,\ \textstyle\sum_m E_m = I$$

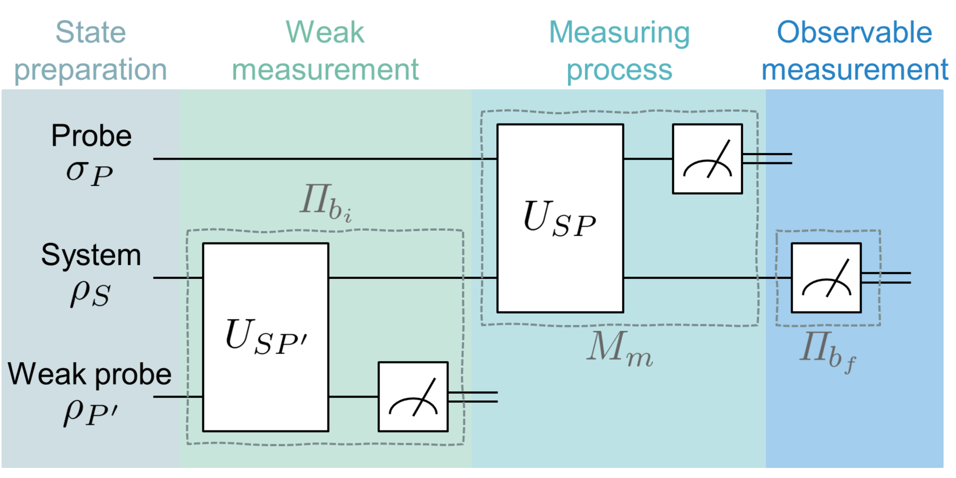
$$O_B = \textstyle\sum_m M_m^\dagger B M_m,$$

$$O_B^{(2)} = \textstyle\sum_m M_m^\dagger B^2 M_m$$

Equivalent expression

$$\eta_O^2(B) = \langle\psi|O_B^{(2)}|\psi\rangle + \langle\psi|O_B|\psi\rangle + \langle B\psi|O_B|B\psi\rangle + \langle\psi|B^2|\psi\rangle - \langle(B+I)\psi|O_B|(B+I)\psi\rangle$$

Weak Measurement Method (WMM) [11-16]



Weak joint probability (WJP) :

$$p_w(b_i, b_f) = p_w(b_i|b_f)p(b_f) = \mathrm{Re}\,\mathrm{Tr}\{\Pi_{b_f} U \Pi_{b_i}(\rho\otimes\sigma)U^\dagger\} = \mathrm{Re}\,\langle U^\dagger \Pi_{b_f} U \Pi_{b_i}\rangle$$

Weak valued probability

a.k.a. Weak conditional probability :

$$\langle \Pi_b \rangle_w = \frac{\mathrm{Tr}\{\Pi_{b_f} U \Pi_{b_i}(\rho\otimes\sigma)U^\dagger\}}{\mathrm{Tr}\{\Pi_{b_f} U (\rho\otimes\sigma)U^\dagger\}}$$

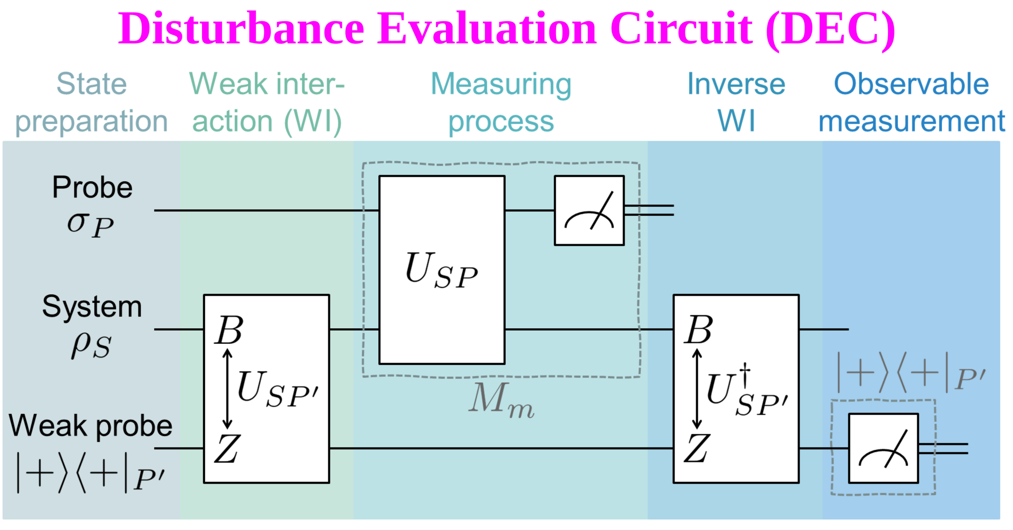
Representation of disturbance using WJP :

$$p_w(\delta b) = \sum_{b_i} p_w(b_i, b_f = b_i + \delta b) = \sum_b \mathrm{Re}\,\langle U^\dagger \Pi_{b+\delta b} U \Pi_b\rangle$$

$$\Rightarrow \sum_{\delta b} \delta b^2 p_w(\delta b) = \sum_{b,b'} (b'-b)^2\,\mathrm{Re}\,\langle U^\dagger \Pi_{b'} U \Pi_b\rangle = \langle [B(t) - B(0)]^2\rangle$$

2. Proposed method

Disturbance Evaluation Circuit (DEC)



	Settings		
	Probe System P	Measured System S	Weak Probe P'
Initial state	σ	ρ	$\vert+\rangle\langle+\vert$
Interaction	Arbitrary measuring interaction U_{SP}		Weak interaction $U_{SP'} = \exp(-i\theta B\otimes Z)$, $\theta \ll 1$
Observable	M	A, B	$\Pi_+^X := \vert+\rangle\langle+\vert$

$$\langle \Pi_+^X\rangle_{P'}(\mathrm{out}) = \sum_m \mathrm{Tr}\{(I\otimes\Pi_+^X)U_{SP'}^\dagger M_m U_{SP'}(\rho\otimes\Pi_+^X)U_{SP'}^\dagger M_m^\dagger U_{SP'}\} \approx 1 + \theta^2 \sum_m \mathrm{Tr}\{[M_m, B]\rho[M_m^\dagger, B]\}$$

Decoherence Disturbance

$$\langle \Pi_+^X\rangle_{P'}(\mathrm{in}) - \langle \Pi_+^X\rangle_{P'}(\mathrm{out}) = 1 - \langle \Pi_+^X\rangle_{P'}(\mathrm{out}) \approx \theta^2 \sum_m \mathrm{Tr}\{[M_m, B]\rho[M_m, B]^\dagger\}$$

$$\eta_O^2(B) = \sum_m \mathrm{Tr}\{[M_m, B]\,\rho\,[M_m, B]^\dagger\} = \frac{\langle \Pi_+^X\rangle_{P'}(\mathrm{in}) - \langle \Pi_+^X\rangle_{P'}(\mathrm{out})}{\theta^2}$$

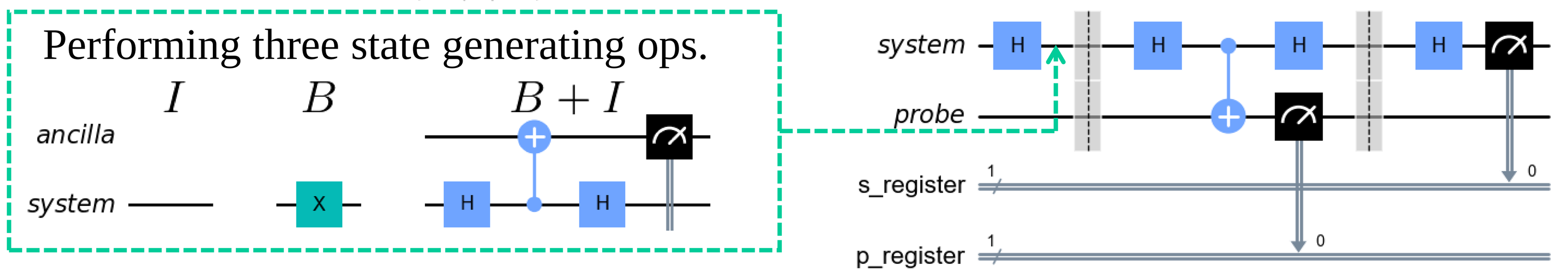
$$\Leftrightarrow \quad \varepsilon_O^2(A) := \sum_m \|M_m(m-A)\,|\psi\rangle\|^2, \quad \eta_O^2(B) := \sum_m \|[M_m, B]\,|\psi\rangle\|^2 \quad [17]$$

3. Simulation

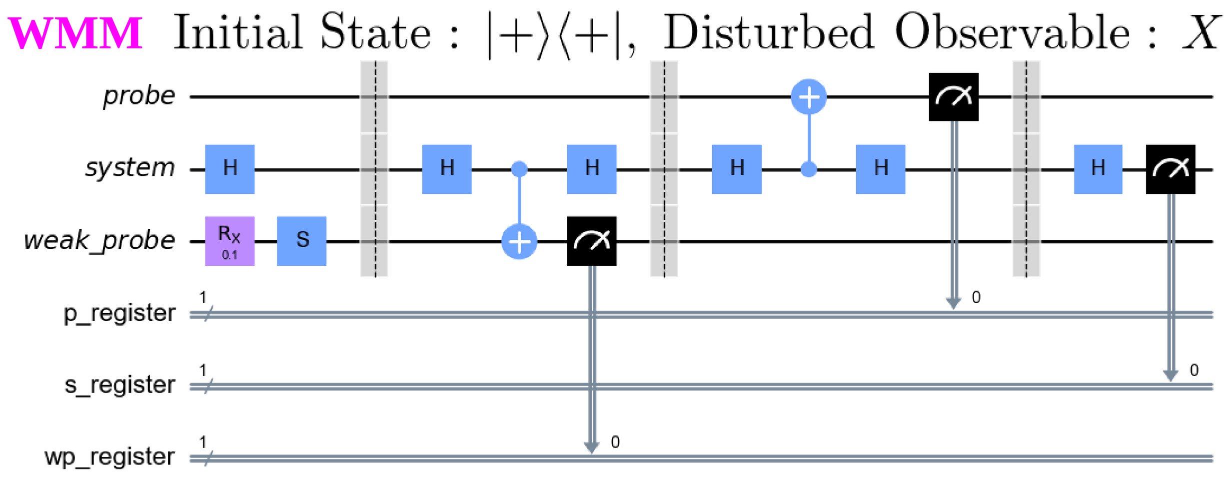
■ **Experiments on a QASM simulator**

TSM Initial State : $|+\rangle\langle+|$, Disturbed Observable : X

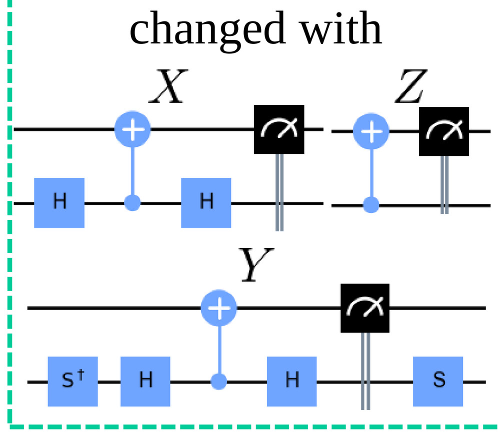
Performing three state generating ops.



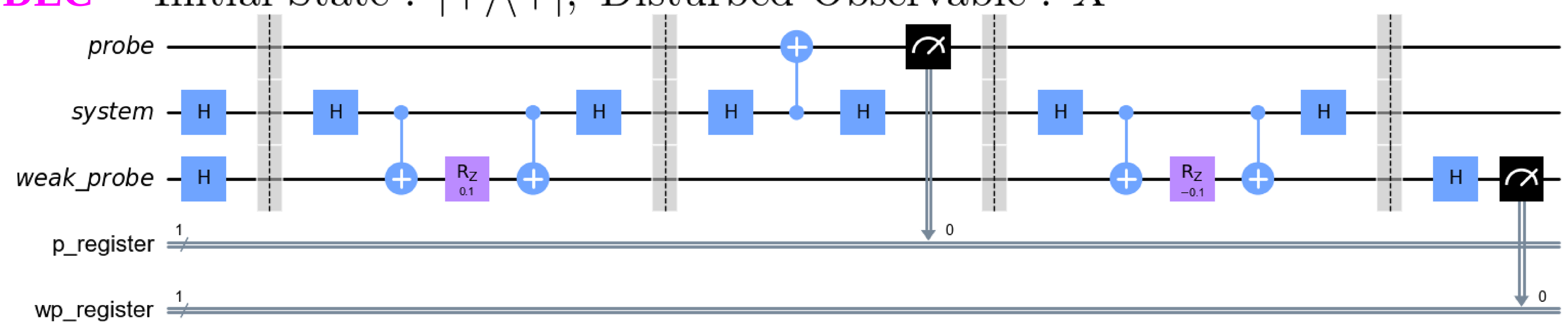
WMM Initial State : $|+\rangle\langle+|$, Disturbed Observable : X



Measurements can be changed with



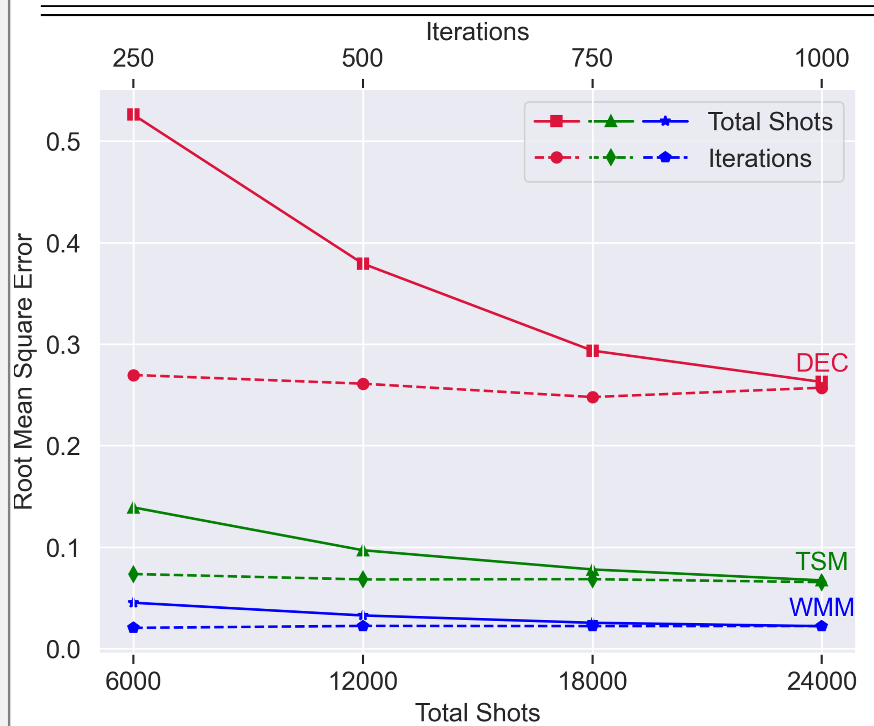
DEC Initial State : $|+\rangle\langle+|$, Disturbed Observable : X

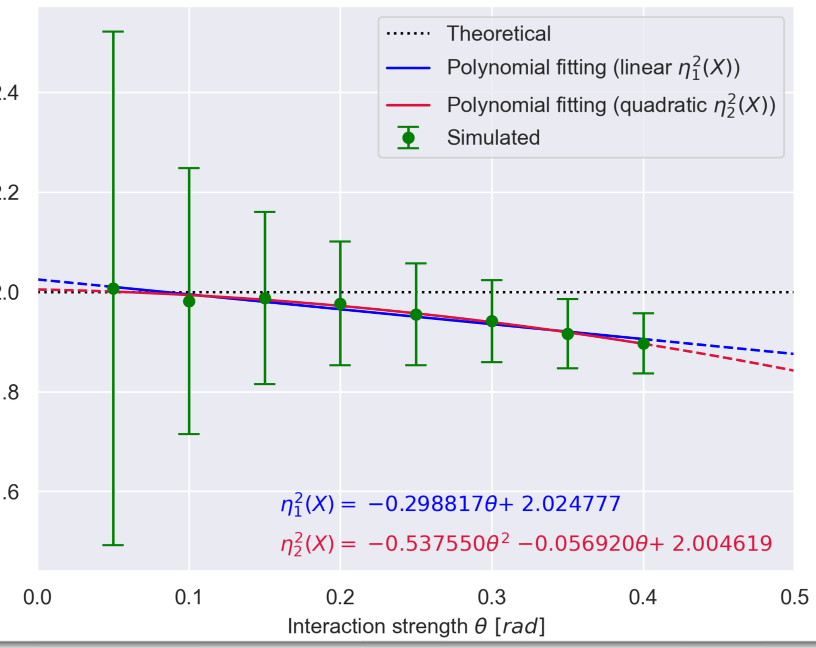


Total Shots (num. of measurements) : 24000, Iterations (num. of trials) : 1000

Measurement Operators	Theoretical Values	TSM
$M_{\pm1}^X = (I \pm X)/2$	0	$0.0 \pm 0\ (0, 0)$
$M_{\pm1}^Y = (I \pm Y)/2$	2	$1.998 \pm 0.06840\ (-1.847\times10^{-3},\ 0.06843)$
$M_{\pm1}^Z = (I \pm Z)/2$	2	$1.998 \pm 0.06726\ (-2.336\times10^{-3},\ 0.06730)$

WMM ($\theta = 0.1$[rad])	DEC ($\theta = 0.1$[rad])
$0.00009782 \pm 0.002269\ (9.782\times10^{-5},\ 0.002271)$	$0.0 \pm 0\ (0, 0)$
$1.999 \pm 0.02155\ (-6.970\times10^{-4},\ 0.02157)$	$1.999 \pm 0.2500\ (-1.367\times10^{-3},\ 0.2501)$
$1.999 \pm 0.02228\ (-7.211\times10^{-4},\ 0.02229)$	$1.992 \pm 0.2606\ (-8.000\times10^{-3},\ 0.2607)$





↖ Each simulation result is the mean of iterations : Mean ± SD (Bias, RMSE).

Mean : $m = \dfrac{1}{n}\displaystyle\sum_{j=1}^n a_j$

Standard Deviation (SD) :

$$\sigma = \left[\frac{1}{n}\sum_{j=1}^n (a_j - m)^2\right]^{1/2}$$

Theoretical value : a, Bias : $b = m - a$,

Root Mean Square Error (RMSE) :

$$e = \left[\frac{1}{n}\sum_{j=1}^n (a_j - a)^2\right]^{1/2},\ e^2 = \sigma^2 + b^2.$$

DEC can be expected to improve accuracy by increasing Total Shots. Also, TSM and WMM are less dependent on Total Shots and Iterations, allowing robust accuracy.

In regions where θ is small, the variance is large. For large values of θ, the data deviate from the theoretical values. Taking the zero limit of θ allows us to find a near-ideal disturbance by extrapolation.

4. Conclusion

In this study, we proposed an experimental evaluation method for disturbance in quantum measurement. Our method **directly** and **quantitatively** determines **the amount of disturbance** **by measuring the decoherence of the auxiliary system (weak probe system)**. A quantum circuit for that purpose was constructed and the validity of the proposed method was verified by a concrete measurement model by simulation. For future works, it is important to conduct the experiments including non-obvious POVMs and confirm the effectiveness of quantum error mitigation.

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