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Estimation of Blackbody Zeeman Shift in $^1S_0$–$^3P_0$ Transition Frequencies

Masatoshi Kajita*

National Institute of Information and Communications Technology, Koganei, Tokyo 184-8795, Japan

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The blackbody radiation Zeeman (BBRZ) shift in the $^1S_0$–$^3P_0$ transition frequencies of ^{87}Sr and ^{171}Yb atoms and an $^{115}In^+$ ion was theoretically estimated with the temperature of the surroundings between 50 and 300 K. The BBRZ shift was confirmed to be negligible small also after the measurement uncertainty of 10^{-19} is attained. Its dependence on temperature is more complicated than that for hyperfine transition frequencies in the microwave region.

1. Introduction

The measurement uncertainties of time and frequency were reduced drastically using atomic clocks, but they cannot be zero because the atomic transition frequencies are shifted by atomic motion (Doppler shift) or interaction with electric and magnetic fields (Stark and Zeeman shifts). For example, blackbody radiation (BBR) induces a frequency shift at an electric field of $\langle E(t)^2\rangle^{1/2} = (T/(300\,\mathrm{K}))^2 \times 8.3\,\mathrm{V/cm}$, where T is the thermodynamic temperature of the surroundings. To measure the transition frequencies with uncertainties lower than 10^{-14}, the Stark shift induced by BBR (BBRS shift) is required to be estimated and corrected to the measured frequency. There are many theoretical and experimental studies on this issue.[1–8] Using ^{87}Sr and ^{171}Yb atoms trapped in an optical lattice with kinetic energy of a few μK, a measurement uncertainty of the $^1S_0$–$^3P_0$ transition frequencies on the order of 10^{-18} was obtained after suppression of the uncertainty of the BBRS shift.[9–12] Moreover, a change in the height by several centimeters became detectable from the gravitational redshift.

BBR also generates a magnetic field of $\langle B(t)^2\rangle^{1/2} = (T/(300\,\mathrm{K}))^2 \times 0.027\,\mathrm{G}$, which induces the Zeeman (BBRZ) shift. As the BBRZ shift is much smaller than the BBRS shift, there are only a few theoretical studies of the BBRZ shift in the hyperfine transition frequencies of alkali atoms and alkali-like ions.[6,13] Porsev et al. estimated the BBRZ shift ^{87}Sr$^1S_0$–$^3P_0$ transition frequency with $T = 300$ K to be five orders smaller than the BBRS shift.[7] However, measurement uncertainties lower than 10^{-19} might be obtained using $^1S_0$–$^3P_0$ transition frequencies of alkali-earth atoms in the near future,[14] and it is also useful to estimate frequency shifts of less than 10^{-18} more in detailed with different conditions. In this paper, we estimate the BBRZ shift in the $^1S_0$–$^3P_0$ transition frequencies of ^{87}Sr and ^{171}Yb atoms and an $^{115}In^+$ ion[15] with $T = 50$–300 K.

2. BBR Stark Shift

The BBR energy per volume in terms of the electric and magnetic fields at each frequency component is given by

$$\rho(\nu)\,d\nu = \varepsilon_0\langle E(\nu,t)^2\rangle\,d\nu = \frac{1}{\mu_0}\langle B(\nu,t)^2\rangle\,d\nu = \frac{8\pi h\nu^3}{c^3[\exp(h\nu/k_BT)-1]}\,d\nu, \tag{1}$$

where ν is the frequency component of BBR, h is the Planck constant, c is the speed of light, and k_B is the Boltzmann constant. The peak frequency of the BBR is proportional to T and it is 31 THz with $T = 300$ K. The BBRS shift in the 0-state induced by the couplings of the electric dipole force with i-states ($i = 1, 2, 3, \ldots$) is given by

$$\Delta\epsilon_S(0) = \sum d_{0i}^2 \int \frac{\langle E(\nu,t)^2\rangle\nu_{0i}}{h(\nu_{0i}^2-\nu^2)}\,d\nu = \sum d_{0i}^2 \int \frac{\rho(\nu)\nu_{0i}}{\varepsilon_0 h(\nu_{0i}^2-\nu^2)}\,d\nu, \tag{2}$$

where $\nu_{0i} = (\epsilon_0 - \epsilon_i)/h$ (ϵ_j: the energy at the j-state) and d_{0i} is the matrix element of the electric dipole moment between the 0- and i-states. BBRS is induced by the coupling with other electronic states and $|\nu_{0i}| \gg \nu$. The BBRS shift is proportional to $\int\rho(\nu)\,d\nu$ and the fractional BBRS shift in the Cs hyperfine transition frequency was obtained to be $-1.71\times10^{-14}(T\,(\mathrm{K})/300)^4$.[5] The detailed calculation of BBRS shift in the $^1S_0$–$^3P_0$ transition frequencies of neutral atoms and ions have been performed by Porsev et al.[7] and Safronova et al.,[8] respectively.

3. BBR Zeeman Shift

The BBRZ shift in the 0-state induced by the couplings of the magnetic dipole force with i-states ($i = 1, 2, 3, \ldots$) is given by

$$\Delta\epsilon_Z(0) = \sum \mu_{0i}^2 \int \frac{\langle B(\nu,t)^2\rangle\nu_{0i}}{h(\nu_{0i}^2-\nu^2)}\,d\nu = \sum \mu_{0i}^2 \int \frac{\mu_0\rho(\nu)\nu_{0i}}{h(\nu_{0i}^2-\nu^2)}\,d\nu, \tag{3}$$

where μ_{0i} is the matrix element of the magnetic dipole moment between the 0- and i-states. μ_{0i} is given by the sum of $g_X\mu_BM_X$ for angular momentums X (= electron orbital angular momentum L, electron spin S, and nuclear spin I). Here, μ_B is the Bohr magneton ($\mu_B/h = 1.4$ MHz/G), g_X is the g-factor ($g_L = 1$, $g_S = 2.003$, $g_I < 10^{-3}$), and M_X is the $0-i$ matrix element of the component of X parallel to the magnetic field. The BBRZ shift in the hyperfine transition frequencies of alkali atoms is induced only by the coupling between the hyperfine states. $|\nu_{0i}|$ is in the microwave region ($|\nu_{0i}| \ll \nu$), and $\int\rho(\nu)/\nu^2\,d\nu$ is proportional to T^2. The Zeeman shift on the Cs clock transition frequency $^2S_{1/2}$ $(F, M_F) = (3,0) - (4,0)$ is induced by the coupling between hyperfine states (F: hyperfine state). Using $[M_S, M_I]$ states, the (3,0) and (4,0) states are given by $([1/2,-1/2] - [-1/2,1/2])/\sqrt{2}$ and $([1/2,-1/2] + [-1/2,1/2])/\sqrt{2}$, respectively. $\mu_{01} = \mu_B(g_S - g_I)/2$ is then derived, and the

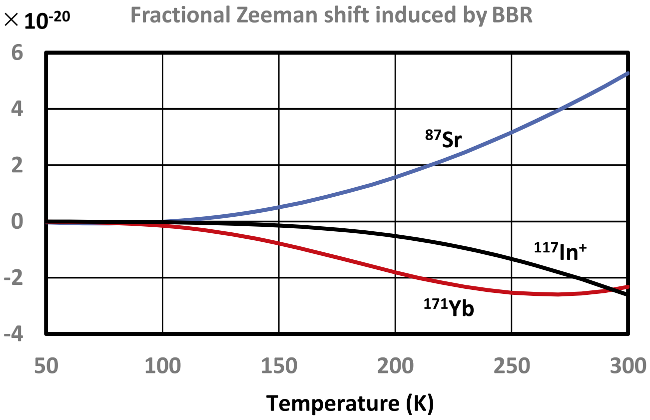

Fig. 1. (Color online) Dependence of the fractional blackbody-radiation Zeeman shift in the $^1S_0$–$^3P_0$ transition frequencies at the temperatures between 50 and 300 K.

fractional BBRZ shift in the Cs clock transition frequency was estimated by Itano et al. and Han et al. to be $-1.30\times 10^{-17}(T\ (\mathrm{K})/300)^2$ [6] and $-1.94\times 10^{-17}(T\ (\mathrm{K})/300)^2$,[13] respectively. The BBRZ shift is negligible for the Cs clock with a current measurement uncertainty of 10^{-16},[16] but it may be significant when a lower uncertainty is attained in future.

For the $^1S_0$–$^3P_0$ transition frequencies ν_c of ^{87}Sr (429 THz[9,10]) and ^{171}Yb (518 THz[11,12]) atoms and an $^{115}\mathrm{In}^+$ (1265 THz[15]) ion, the BBRZ shift is induced by the energy shift of the $^3P_0$ $M_J=0$ state $[\Delta\epsilon_Z(^3P_0)]$ by the coupling with the $^3P_1$ $M_J=0$ state. Then $\nu_{0i}=-\nu_{fs}$, where ν_{fs} is the transition frequency between the $^3P_0$ $M_J=0$ and $^3P_1$ $M_J=0$ states (5.6 THz for ^{87}Sr,[9,10] 20 THz for ^{171}Yb,[11,12] and 39 THz for $^{115}\mathrm{In}^+$ [15]). Using the $[M_L, M_S]$ states (here, we ignore the effect of the nuclear spin), the $^3P_0$ $M_J=0$ and $^3P_1$ $M_J=0$ states are given by $([0,0]-[1,-1]-[-1,1])/\sqrt{3}$ and $([1,-1]-[-1,1])/\sqrt{2}$, respectively. The BBRZ shift was estimated by numerical integration of Eq. (3) taking $\mu_{01}=\mu_B(g_S-g_L)(2/3)^{1/2}$. Figure 1 shows the dependence of the fractional BBRZ shift on temperature. Porsev et al. estimated the BBRZ shift in the ^{87}Sr transition frequency with $T=300$ K to be 2.4×10^{-5} Hz (fractional shift of 5.6×10^{-20}),[7] which is consistent with the present calculation (5.3×10^{-20}). The dependence of the BBRZ shift on temperature is more complicated than that for the BBRS shift, because the effects from the BBR frequency components with $\nu<\nu_{fs}$ (giving a negative shift) and $\nu>\nu_{fs}$ (giving a positive shift) are both significant. The fractional BBRZ shift in the $^1S_0$–$^3P_0$ transition frequencies is on the order of 10^{-20} with $T=300$ K and it is less than 2×10^{-21} with $T<100$ K.

4. Conclusion

The uncertainties of the order of 10^{-19} might be attained with the $^1S_0$–$^3P_0$ transition frequencies of alkali-earth atoms or alkali-earth like ions. Therefore, it is useful to confirm that the BBRZ shift in the $^1S_0$–$^3P_0$ transition frequencies is negligible also after the measurement uncertainty of 10^{-19} is attained. The fractional BBRZ shift is less than 10^{-19} (10^{-21}) with $T=300$ K (<90 K), which corresponds to the change of the gravitational redshift by the change of the hight of 1 mm (10 µm).

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*kajita@nict.go.jp

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