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On the generality of the topological theory of visual shape perception

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Abstract. This study used a series of six closely related experiments to examine whether individuals use topological structures to discriminate figures. Strict control was exerted over the selection of stimuli, which were a specific type of randomly generated lined figures that can be classified using isomorphic sets defined by graph theory. Any two figures within an isomorphic set possessed the same topological structure. The experiments described here used a same/different discrimination task with simultaneously presented pairs of figures: (a) identical pairs (Id pairs), in which each pair of figures had the same topological and superficial properties; (b) nonidentical and isomorphic pairs (Iso pairs), in which each pair had the same topological but different superficial properties; and (c) nonidentical and nonisomorphic pairs (Noniso pairs), in which each pair had different topological properties. Within these experiments I varied the conditions related to the intersecting line segments, presentation of points defining each figure, figure complexity, stimulus aspect ratios, and the parity of the total line-segment lengths between the figures in each pair. These variations showed that the latencies for making accurate discriminations were shorter for Noniso pairs than for Iso pairs, suggesting that individuals are sensitive to topology when distinguishing figures.

Keywords: topology, isomorphism, graph invariants, same/different task

1 Introduction

Researchers have long sought to answer the question of whether people use structural information such as the overall connectedness of line segments in addition to the segments' location when they identify two-dimensional figures. Findings have shown that structural properties—such as connectedness and closures (Palmer 1978; Treisman and Souther 1985), line terminators (Julesz 1981), inside/outside relationships (Treisman and Gormican 1988), and symmetry (Palmer and Hemenway 1978; Pashler 1990), or in more general terms orientation and coordinate invariant descriptions (Corballis 1988; Eley 1982; Takano 1989) and landmark features (Hochberg and Gellman 1977)—are crucial in identifying two-dimensional figures. However, going beyond these arguments regarding the detection of a figure's individual structural properties, cognitive scientist Lin Chen most explicitly claimed that the topological structures of a given figure can be directly perceived.

Specifically, Chen (1982) proposed that the visual system is sensitive to global topological information, and that our visual system is capable of detecting a figure's deep structural features regardless of how the figure's component parts are superficially configured. Chen (2005) classified two contrasting lines of thinking in his study of perception: (a) the early feature analytic position, which assumes that perceptual processing proceeds from local feature detection to global object recognition, and (b) the early holistic registration position, which assumes that the encoding of a topological structure is followed by local processing. As Chen recognized, this topological hypothesis poses a fundamentally divergent position regarding whether or not the visual system is sensitive to topological information.

Previously, Chen and his colleagues published a series of studies designed to elucidate topological perception. In one of these studies (Chen 1982) the participants were briefly presented with three pairs of figures and were asked to judge whether the two figures in a pair were the same or different. Pair 1 consisted of a solid square and a solid circle, pair 2 a solid triangle and a solid circle, and pair 3 a ring and a solid circle (figure 1). Although all of the paired figures were different in shape, the two figures in pair 1 and pair 2 were topologically equivalent, whereas the figures in pair 3 were topologically different. By adjusting the figures' illumination levels to keep the overall probability-of-difference reporting at 50%, Chen showed that the probability of reporting that the figures in pair 3 were different was far larger than with the other pairs, indicating that the participants were sensitive to topological differences.

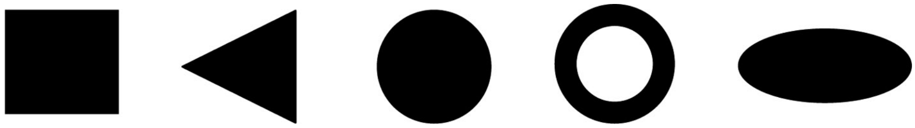


Figure 1. Some schematic examples of Chen's figures. The figures are (a) a solid square, (b) a solid triangle, (c) a solid circle, (d) a ring, and (e) a solid ellipse. The figures were drawn by myself.

However, this claim was later rebutted by Rubin and Kanwisher (1985), who found that the addition of a controlled-stimulus luminous flux to Chen's previous experiment confounded the topological factor data. In turn, Chen (1990) argued against their rebuttal by suggesting that the visibilities of stimuli in the Rubin and Kanwisher study were confounded by the luminous flux.

Chen (1985) also tested his theory of topological perception in the arena of apparent motion. In that experiment two static displays were sequentially presented to participants. The first display contained a standard figure shown at the center of the field of view, and the second display contained two test figures that were located at the right and the left of the center of the field of view, respectively. For each pair of test figures, one was topologically equivalent to the standard and the other was topologically different from the standard. Participants were asked to choose which direction, rightward or leftward, that a standard figure appeared to have moved. To secure optimal apparent motion, the duration of the first display and the intervals between the first and the second displays were adjusted for each participant. Participants exhibited a clear tendency to choose the direction of motion from the standard figures to their topologically equivalent figures.

Todd et al (1998) employed a match-to-a-sample task to examine stereoscopic perception of lined figures. A display containing three figures (ie a 'standard figure' at the center and 'test figures' to the left and right) was presented at each trial. In each trial one of the test figures was designated as a target and the other test figure was designated as a foil, but which of these figures was the target was not revealed to the participant. The three-dimensional (3-D) Euclidean structure of the target was identical to that of the standard, but the foil had a different Euclidean structure from that of the standard. The three properties by which the geometrical structures could vary between the standards and the foils were controlled as follows: (a) in the topological condition the two figures had different topological structures related to the states (ie the presence or absence) of an intersection of line segments in 3-D space; (b) in the affine condition the foil and the standard had an identical state of an intersection, and thus had the same topological structure in 3-D space; (c) in the Euclidean condition the states of an intersection, and thus the topological structures in 3-D space, were not controlled between the two test figures. Participants were required to decide which test figure had the same 3-D structure as the standard figure, regardless of their orientations. The authors found that error rates and the latencies to make choices were both smallest in the topological condition, intermediate in the affine condition, and largest in the

Euclidean condition. They reported that differences in topological structures were easiest to recognize.

That same year Hecht and Bader (1998) examined stimulus selections under more strictly controlled conditions in which the figures presented to participants consisted of line segments whose formations were controlled by the number of components (ie maximally connected line segments), the number of closed components, and the number of included components (ie components inside of a given closed component). Examples of these stimulus figures are shown in figure 2.

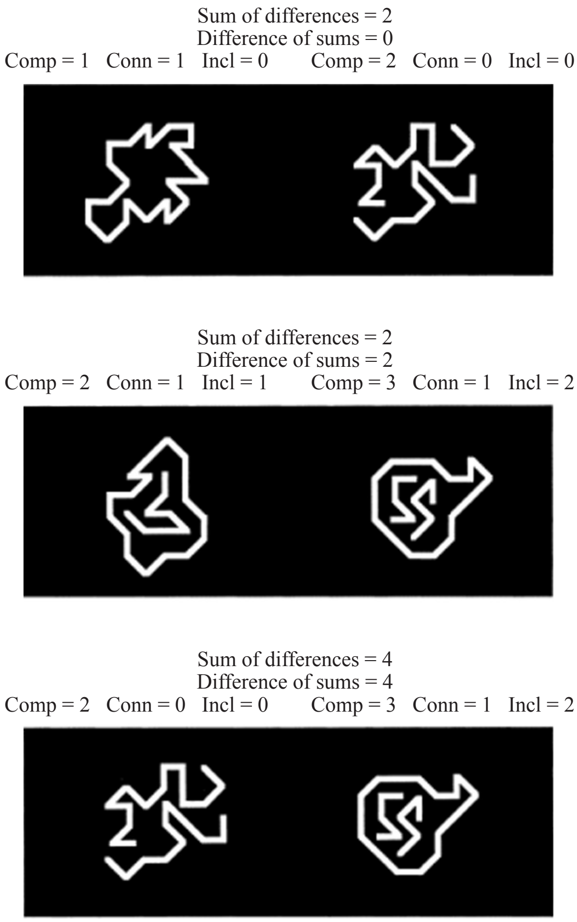


Figure 2. Some of the stimulus figures used in Hecht and Bader (1998). Here, the topological properties were the number of components (Comp), number of connected components (Conn), and number of included components (Incl) for each figure. To describe the topological differences between a given pair of figures, two scores were devised. The sum of the differences is defined as the sum of the absolute differential values of the three properties of Comp, Conn, and Incl. The difference of sums score is the absolute differential value of the respective sums of the three properties between the two figures of a pair. (Reproduced with permission from Elsevier.)

Three types of figures were generated according to these three properties. In each trial three figures of a specific type were simultaneously presented, and participants were asked to point out the figure that was not identical to the other two. Across all three types of figures, response latencies decreased as the difference in the property values between nonidentical pairs of figures increased, indicating that participants were sensitive to topological differences when perceiving figures.

More recently, Zhang et al (2009) employed a backward-masking paradigm to measure the visibilities of textured targets of an S-like figure and a ring figure in different background textures with variable target-mask stimulus onset asynchronies (SOAs). The two figures were topologically different in terms of whether or not they had a hole. The results of this study consistently showed that participants more accurately detected the presence of the ring figure than the S-like figure with small SOAs, where a mask should strongly interfere with the visibility of a target, supporting the authors' hypothesis that if a topological property, as a global property, is fundamental to visual perception, then the topological differences between a textured ring and a background texture could be more easily discriminated than the local feature difference of a textured S from a background texture.

In Chen and Zhou (1997) three figures flanked by two digits were simultaneously presented to each participant as a tachistoscopic presentation. In each trial the three figures of the experimental condition remained the same: a solid triangle, a ring, and a solid ellipse (figures 1b, 1d, 1e). The participants' primary task was to verbally report the two flanking digits, and their secondary task was to report the shapes of the three figures. It was assumed that the three figures were perceived preattentively because the participants' attention was mainly directed to the digits. The authors asserted that an illusory conjunction had occurred, and that the participants saw nonexistent hollow triangles and/or hollow ellipses in some of the trials. Here, an illusory conjunction indicated an erroneous combination of preattentively detected primitive features resulting in the perception of a nonexistent property (Treisman and Gelade 1980). According to Chen and Zhou (1997), if the participants incorrectly detected that there was a hole inside of a solid triangle or a solid ellipse, this would indicate that hollowness is a primitive property.

Chen (1982) considered that topology concerns the nature of a given figure as a whole rather than the respective properties of its components. Hence, his topological hypothesis presupposes prior access to global information over local features (eg Navon 1977). In a similar experiment Han et al (1999) used a compound stimulus figure either of an arrow or a triangle consisting of either arrows or of triangles at the component level to examine the global precedence effect. In the orientation-discrimination task used in that study participants were instructed to discriminate the left/right directions of the stimulus figure or figures either at the component level or at the compound level. In the closure-discrimination task used in that study participants judged whether the closure (ie a triangle) was present or not, at either the compound or the component level. In both tasks the latencies of the participants were shorter following compound-level instructions than component-level instructions, indicating the presence of a global feature bias.

Eidels et al (2008) investigated the role of topological structure in the search for target elements with the use of a singly presented stimulus figure. In their first experiment participants were asked to decide whether target elements were present in singly presented stimulus figures. The target element was either a right angle facing to the left (ie the mirror image of the letter L) or a diagonal line running from the lower left to the upper right (ie a right diagonal). Four stimulus figures were prepared: (a) a triangle consisting of the mirror image of the letter L and a right diagonal (ie a left triangle); (b) an arrow consisting of the mirror image of the letter L and a left diagonal (ie a right arrow); (c) an arrow consisting of the letter L and a right diagonal (ie a left arrow); and (d) a triangle consisting of the letter L and a left diagonal (ie a right triangle). Of these, the left triangle contained two target elements, the right and the left arrows contained one target element, and the right triangle contained no target elements. It is noteworthy that the two triangles were as mutually topologically equivalent as the two arrows, but between the left triangle and the two types of arrow stimuli, all of which required positive responses, the topological structures were different. These structural differences suggest that if the stimuli are processed on a feature-by-feature basis, the latencies should be smaller for the stimuli having two targets than for the stimuli having only one target (ie a redundancy gain). However, the results of this study show that processing occurred more quickly for the single-target conditions than for the redundant target condition, indicating that the topological factor along with the response assignments of the experiment superseded the redundancy gain. In their second experiment (Eidels et al 2008) the target was either the mirror image of the letter L or a left diagonal line, as in experiment 1. However, in the formations of the stimulus figures, the diagonal line was connected at the midpoint rather than the endpoint of the letter L or the mirror image of the letter L, thus making all four stimuli topologically equivalent to one another. The results of this experiment indicated that the redundancy gain reappeared, as is the case with many visual search studies.

The presence of a configural superiority effect could also suggest sensitivity to topological structures. The configural superiority effect is said to occur when an improvement in target detection is observed by adding context elements to target elements. Pomerantz (2003), taking an example of a feature search for a left diagonal among three right diagonal distracters in a display, explained that adding a subsequent irrelevant display consisting of four letter L elements would make the target in the initial display far easier to spot. This was because the configurations of the elements in the initial and the subsequent displays emerged and the configured arrows and a triangle are easily discriminated. That is, the difference in the directions of the diagonal lines is not easily discriminated, but the topological difference between closure (ie a triangle) and openness (ie arrows) is easily discriminated.

From a different standpoint, Bedford (2001) claimed that the identity of two shapes can be better determined by their geometric structures than by metric measures, such as their similarity, spatial frequencies, and ecological optics. By explaining Kline's *transformation* approach, Bedford stated that the properties of geometry that remain unchanged by a group of transformations are the properties of that geometry. Five geometries can be ordered from the most specific (lowest level) to the most general (highest level) based on the number of properties left unchanged: Euclidean, similarity, affine, projective, and topology. According to Bedford, the more general a transformation, the more the properties of the original form are altered, and the less likely it is that the two shapes before and after the transformation will be judged to refer to the same object.

Wagemans and his colleagues (1994) have investigated the perception of figures in non-Euclidean geometries. Wagemans et al employed same/different discriminations of simultaneously presented pairs of polygons. Ten polygons were randomly generated as original polygons. In a 'same' pair belonging to a subset of level 1 one polygon was affine transformed once, whereas the other remained untransformed from the same original polygon. One polygon of a 'same' pair of a subset of level 2 was affine transformed twice, whereas the other one was not transformed. One polygon of a 'same' pair of a subset of level 3 was affine transformed three times. For pairs belonging to the total set, the pairs were generated by any combination of the three levels of transformations. Therefore, the sizes of the respective subsets were incrementally ordered from level 1 to level 3, and the size of the total set was the largest among all sets. Participants were asked to decide whether two figures of a pair presented were the same or different regardless of any affine transformations. The latencies for the same responses were shorter for the level 1 subset than for the level 2 subset, and shorter for the level 2 subset than for the level 3 subset, but not significantly different between the level 3 subset and the total set. According to Wagemans et al, the hypothesis of easier discriminations for figures belonging to smaller subsets was confirmed, even for sets defined by affine transformations.

Kukkonen et al (1996) asked participants to decide whether a given pair of figures was the same regardless of affine transformation. They used dot patterns consisting of four points and closed figures consisting of four line segments, which spanned the four points. In both types of figures two rules concerning the generation of patterns were specified. Under the parallelism rule, the positions of first three points were randomly specified and a fourth point was positioned randomly at one of fifteen possible locations along the invisible diagonal bisector defined by the first three points. If the fourth point was positioned at the center of the fifteen possible locations, this yielded a parallel arrangement of the points. Under the collinearity rule, the points were generated in the same way as under the parallelism rule except that the center of the fifteen locations of the fourth point was at the midpoint of an invisible line segment drawn between the two endpoints already specified. As for lined figures, depending on the positions of the fourth selected point, figures had the shapes of triangles, quadrilaterals having a convexity or a concavity. Here, it must be noted that parallelism is invariant under affine transformations and collinearity is invariant under the projective transformations. As to the figures made by the parallelism rule, for both types of figures performance with the same pairs was best when the figures were parallel; it deteriorated slightly as the figures became less parallel, and improved again as they were made less parallel still. As to the figures made using the collinearity rule, for both types of figures participants were successful in detecting the same pairs when they were collinear; performance worsened slightly as the patterns were made slightly concave or convex, and then improved again as they were made more strongly concave or convex. The authors claimed that the properties of collinearity and parallelism and convexity and concavity could provide strong cues to identifying and discriminating random figures.

Wagemans et al (1997) employed a match-to-a-sample task to examine whether participants perceived that perspectively transformed figures or projectively transformed figures were better matched to standard figures. Here, perspective transformations are a subset of projective transformations. The standard figures were irregular pentagons with the positions of the vertices chosen pseudo-randomly. According to the nature of the samples, three types of triplets were generated: (a) one sample was perspectively transformed from a standard figure and the other sample was perspectively transformed from a nonstandard figure; (b) one was projectively transformed from a standard figure, and the other was projectively transformed from a nonstandard figure; and (c) one was perspectively transformed and the other projectively transformed from a standard figure. The following results were obtained: perspective transformed versions from the standard figures were predominantly chosen for the triplets of (a), projective transformed versions from the standards were predominantly chosen for the triplets of (b), and perspectively transformed versions were preferred to projectively transformed versions for the triplets of (c), although the preference for the perspective transformations decreased with increasing perspective slants and decreasing projective slants. The authors concluded that the visual system does not deal with perspective transformations in a categorically different way from more general projective transformations.

1.1 *(6 point, n line) figures and topological perception*

As Pomerantz (2003) noted, a change to a single aspect of a figure may change many other aspects as well. To understand the role of a specific aspect of a figure in human shape perception, we sometimes require a broader knowledge of the stimulus figures beyond the specific aspect of a researcher's interest.

I have previously studied a type of randomly generated figure called a (6 point, *n* line) figure or (6, *n*) figure (eg Kanbe 2001, 2008, 2009). A (6, *n*) figure is a figure having *n* line segments that connect *n* pairs of vertices in an invisible regular hexagon, respectively. Concerning (6, *n*) figures, various information derived from graph theory is available, including isomorphic sets of figures. Here, a graph is defined by a set of points and line segments that span the pairs of points. If the line segments connecting *n* pairs of points of one figure identically correspond with the line segments connecting *n* pairs of points in another figure, irrespective of the locations of the points, the two figures are said to be isomorphic to each other [for a more precise definition see Harary (1969)]. If the property of a figure is invariant to any other isomorphic figures, the property is called a graph invariant, or, by Chen's usage, a topological property. For example, the number of line segments incident with a given point is called a degree. A point with a degree equal to 0 is called an isolated point. A point with a degree equal to 1 is called an endpoint. The maximum degree of a figure, the number of isolated points in a figure, and the number of endpoints in a figure are examples of graph invariants. Figures belonging to the same isomorphic set cannot be distinguished by any graph invariants. That is, isomorphism specifies the set of figures that are topologically equivalent, and thus provides the mathematical basis of the topological hypothesis.

For the purpose of studying the perception of two-dimensional figures on a firm basis, I calculated graph invariants and other figural indices on every (6, *n*) figure, with $n = 1$–6, on the condition that the locations of the six points defining each figure were at the vertices of a regular hexagon. The values of graph invariants and other locational and orientational information of various aspects of each figure were stored in a database that was initially built in MS-DOS and is now accessible by Windows-compatible FORTRAN programs. Changing the locations of the points could concomitantly change the values of locational and orientational indices but does not change the values of graph invariants.

Using a stored set of 12 graph invariants (eg number of cycles, maximum degree, number of isolated points, number of endpoints), every (6, *n*) figure from $n = 1$–6 can be classified into isomorphic sets; and 55 variable locations, orientations, and other indices (eg location of the point having maximum degree, orientation formed by endpoints, number of intersecting line segments) for every figure within an isomorphic set can be distinguished from all other figures, with the exception of 10 combinations of figures for $n = 4$ and four combinations for $n = 5$.

Some results provided in Kanbe (2009) have relevance for the topological hypothesis. In that study (6, 5) figures were used as stimuli to investigate the roles of endpoints and of closures in figural recognition. Because *closure* is a poorly defined term, I will henceforth use *cycle*, which is a graph theory term. Here, a cycle is a closed alternating sequence of points and line segments with a number of distinct points $\geqslant 3$ [for a more precise definition, see Harary (1969)]. In the third experiment of my study the latencies of the same/different judgments were compared between pairs of figures that had only one cycle and pairs of figures that had three cycles (figure 3). The obtained latencies were shortest for the pairs of figures that were different but had three cycles each. It is noteworthy that all of the (6, 5) figures that have three cycles belong to the same isomorphic set, within which graph invariants cannot distinguish figures from one another. Hence, the results of this study may not be consistent with the topological hypothesis, suggesting that further examinations of Chen's topological theory are warranted.

Figure 3. Example of a nonidentical pair of figures that have three cycles each, as used in Kanbe (2009). Smaller cycles inside of a larger cycle and the larger cycle itself were both counted in the number of cycles.

For (6, *n*) figures there are a total of 15 figures that all belong to one isomorphic set for (6, 1) figures. There are 105 figures that constitute two isomorphic sets for (6, 2) figures, 455 figures and five isomorphic sets for (6, 3) figures, 1365 figures and nine isomorphic sets for (6, 4) figures, 3003 figures and 15 isomorphic sets for (6, 5) figures, and 5005 figures and 21 isomorphic sets for (6, 6) figures.

The present study used (6, 3) figures and (6, 5) figures as stimuli. Concerning the (6, 3) figures, the number of figures constituting each isomorphic set is 60 for isomorphic set 1, 20 for set 2, 180 for set 3, 180 for set 4, and 15 for set 5. Here, the code numbers attached to the isomorphic sets are only used to distinguish the sets and do not imply any specific order. The figures shown in figure 4 are representative examples of the five isomorphic sets.

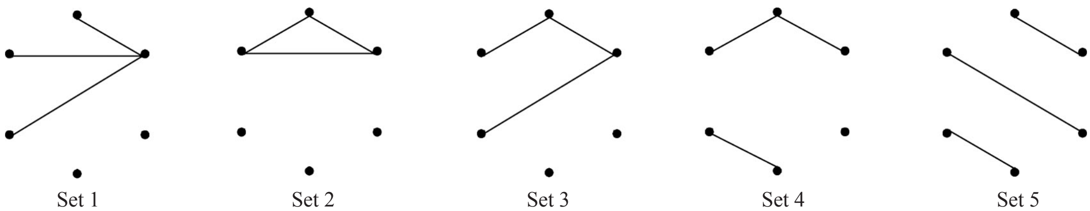


Figure 4. Representative example figures belonging to the five isomorphic sets of (6, 3) figures. The code numbers (1–5) attached to the isomorphic sets do not imply any specific order.

For the (6, 5) figures, the sizes of the 15 isomorphic sets are 6, 180, 120, 90, 360, 360, 180, 90, 360, 360, 360, 60, 45, 72, and 360. The figures shown in figure 5 are representative of the 15 isomorphic sets.

Using the (6, *n*) figures, the topological structure of the stimulus figures can be unambiguously classified according to their isomorphic sets, and the complexity of stimulus figures can be controlled by the number of lines they contain.

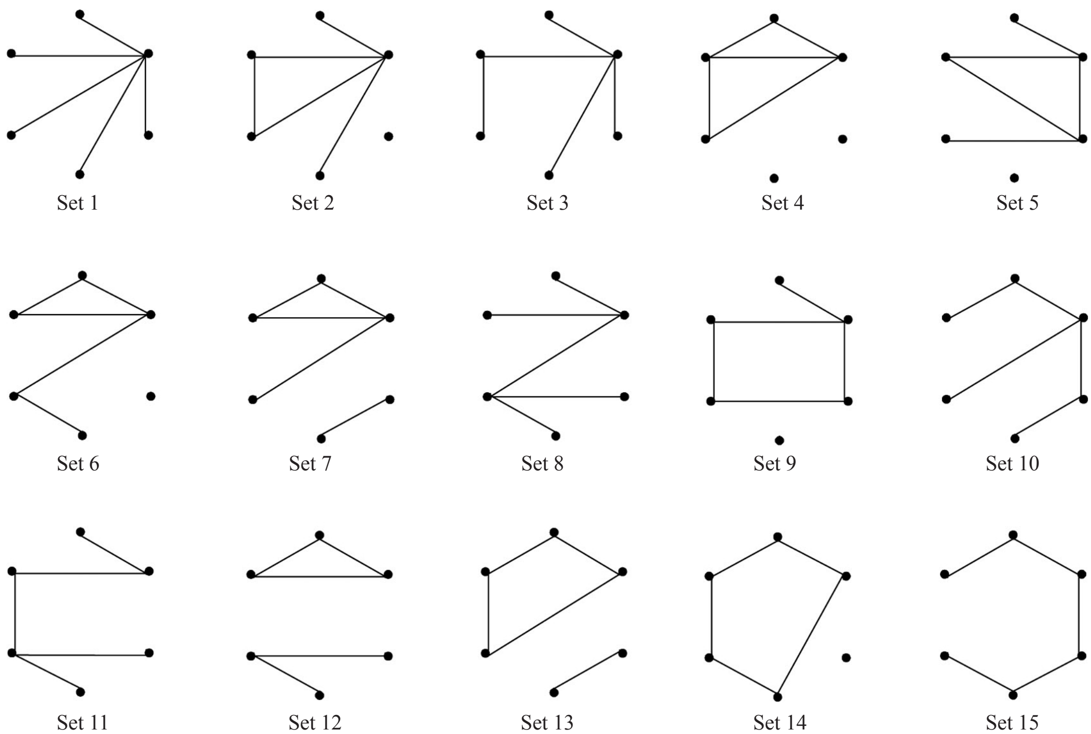


Figure 5. Representative example figures belonging to the fifteen isomorphic sets of (6, 5) figures. The code numbers (1–15) attached to the isomorphic sets do not imply any specific order.

There is abundant evidence that metric information such as location and orientation is crucial in the recognition of figures and objects. For example, the locations of component parts are crucial for recognition of an outlined object and the orientation of a rotated object is crucial for its recognition. This leads to the question of whether topological information also plays a significant role in figure recognition.

Because the term *topological hypothesis* could be interpreted too strongly, here I instead use the term *topological sensitivity hypothesis*. Without being constrained by Chen's assumption that topological structures can be directly perceived, I examined whether humans are sensitive to the topological structures of (6, n) figures when presented with a same/different discrimination task for a pair of figures.

1.2 *General methodology*

The present study consisted of a total of six experiments employing a same/different discrimination task. This task required participants to judge as quickly and as accurately as possible whether a simultaneously presented pair of (6, n) figures were the same or different. Two figures that were identical in shape but different in orientation were defined as different. Three types of pairs were prepared:

- An identical pair (Id pair) consisting of a pair of figures that were mutually identical both in shape and in orientation.
- A nonidentical and isomorphic pair (Iso pair) consisting of a pair of figures that were mutually different in shape or in orientation but whose members both belonged to the same isomorphic set.
- A nonidentical and nonisomorphic pair (Noniso pair) consisting of a pair of figures whose members belonged to different isomorphic sets and thus were different in shape.

To prevent participants from learning any set presentation patterns, the proportion of Id, Iso, and Noniso pairs was set to 2:1:1, except in experiment 4, where (6, 5) figures were used as stimuli. Because there are 105 combinations of the isomorphic sets for (6, 5) figures, the proportion of Id, Iso, and Noniso pairs was set to 5:1:4 to provide a more varied presentation of Noniso pairs to each participant within the limited number of trials. The presentation order of the pairs was randomized. This six-part study was approved by the Hakuoh University Department of Psychology, and all participants provided written informed consent prior to the start of the study.

1.2.1 *Participants.* A group of 10–16 male and female Japanese university students (age = 19–24 years) were recruited to participate in each experiment. All participants had normal or corrected-to-normal vision, and each person participated in only a single experiment.

1.2.2 *Stimuli.* For each experiment a stimulus consisting of two (6, n) figures of a specific pair type were simultaneously presented on a 17-inch LCD monitor (NEC AS171MC) controlled by a NEC MJ33AA-9 microcomputer. All stimuli were prepared as described below. The six vertices of the invisible regular hexagon, called the points, were stylized as small filled circles, and the points were presented as the stimulus presentations in experiments 1, 2, 4, 5, and 6. The presentation of the figures varied somewhat in experiment 3, where stimulus figures were presented without the points, to determine how the points might affect topological sensitivity.

1.2.3 *Generation of pairs.* Pairs were independently prepared for each participant. First, all of the (6, n) figures in the database satisfying both the number of line segments (n) and intersection state conditions were sorted into their respective isomorphic sets and subsequently pooled. The intersection condition concerned whether the figures that had intersecting line segments were included as stimuli. The presence of intersecting line segments inside of the regular hexagonal area was contingent upon the condition that the locations of the points were designated at the vertices of a regular hexagon, and thus the number of intersections was not a graph invariant. For example, if the number of lines was 3 and intersections were included, all 455 (6, 3) figures were sorted into five isomorphic sets of sizes 60, 20, 180, 180, and 15.

One figure was randomly selected from a specific isomorphic pool and was duplicated to constitute an Id pair. This selection and duplication process was sequentially applied to all isomorphic pools iteratively until the number of Id pairs reached a prespecified number (220 in experiments 1, 2, 3, 5, and 6; 250 in experiment 4). This prespecified number included the pairs used for both the practice and the test blocks.

To generate an Iso pair, two distinct figures were randomly selected from the same specific isomorphic pool. This process was applied to every isomorphic pool iteratively until a prespecified number was reached (110 in experiments 1, 2, 3, 5, and 6; 50 in experiment 4).

To generate a Noniso pair, one figure was randomly selected from one isomorphic pool, the other figure was randomly selected from another isomorphic pool, and the two figures were combined. This process continued in an iterative manner until the number of Noniso pairs reached a prespecified number (110 in experiments 1, 2, 3, 5, and 6; 200 in experiment 4).

The three types of pairs were separately accumulated to form three distinct sets of Id, Iso, and Noniso pairs. The three pair sets were then split into two subsets each. Three randomly chosen subsets of Id, Iso, and Noniso pairs were concatenated to form a block of pairs. Likewise, the remaining three subsets were concatenated to form the other block of pairs. In each block of pairs the order of pair presentation was randomized to prevent participants from learning any presentation patterns. The presentation of the figures on either the right or left side of the display was also randomized. Finally, the first 10 pairs in each block were assigned as practice pairs and the remaining pairs of the block as test pairs. Hence, the proportions of Id, Iso, and Noniso pairs in the test trials did not yield exactly 2:1:1 or 5:1:4.

1.2.4 *Procedures.* As previously noted, stimuli were presented on a microcomputer with a 17-inch LCD monitor. Participants made their responses by pushing a button (‘Enter’, ‘F6’, or ‘F5’, horizontally aligned from left to right) on a switch box. The response speed of the monitor was 5 ms, and the time was measured to the nearest 1 ms. Each participant sat directly in front of the monitor, with his or her head placed on a chin-rest 60 cm from the monitor screen.

At the start of each trial a ‘ready’ message appeared on the screen. When a participant pushed the Enter button, the message cleared and the blank screen remained for 2.5 s. Then, accompanied by an audible beep, a fixation cross appeared at the center of the display for 0.5 s. The fixation cross was then replaced by two stimulus figures constituting a pair. Participants were asked to judge whether a presented pair of figures were the same or different by pushing either the F5 or F6 button. Participants were instructed to use their index finger to push the F6 button and their middle finger to push the F5 button. The assignment of a different finger to each task was to prevent a confounding based on potential differences in finger response speed and same/different judgment speed. Before starting each block, participants were instructed as to which of the two buttons was designated as the ‘same’ button and which was designated as the ‘different’ button. Thus, assignments of the fingers to the respective judgments were counterbalanced. No information was given as to the nature of the types of the pairs. Stimulus figures remained onscreen until a participant responded. Emphasis was placed on both speed and accuracy.

A trial was designated as the sequence that started from a push of the Enter button to a push of either the F5 or F6 button. Trials were divided into two blocks according to the specific functions assigned to the F5 and F6 buttons, with the button functions alternating across the blocks. Each participant was given 10 practice trials before the test trials in each block to familiarize themselves with the task. The assignment of the button functions at the first block was randomized for each participant. In the practice trials participants received immediate feedback to help them become accustomed to the experimental procedures. No feedback was provided in the test trials. Response latency was defined as the time that elapsed between the presentation of a pair of stimulus figures on the LCD and the response of the participant.

2 Experiment 1

Although Chen (2005) claimed that topological perception occurs prior to the perception of local features, the distinction of isomorphic sets of (6, n) figures (when $n > 2$) cannot be made by a single graph invariant but only by a set of graph invariants. In this respect, topological sensitivity to stimuli, if present, should be based on comparisons of plural topological properties between the two figures of a given pair. As has been stated, topological properties are sufficient to discriminate two figures of a Noniso pair, but are incapable of discriminating two figures of an Iso pair. An Id pair is a special case of Iso pair in that the former not only preserves the topological structure (ie isomorphism) but also preserves the superficial properties (ie metric information) such as the distances (or lengths) and orientations defined by the respective plane of the geometrical origins of the two figures.

Because connotations of the term *property* are vague, the term *feature* will hereafter be used, and graph invariants will be called invariant features. According to the assumptions underlying feature comparisons, processing will terminate under the following conditions: when comparisons of the invariant and superficial features in a set have been completed between the two figures of an Id pair without encountering any difference in feature values; when encountering a difference in the comparisons of superficial feature values, but not in invariant feature values between the two figures of an Iso pair; and when encountering a difference either in invariant or superficial feature values between the two figures of a Noniso pair. Therefore, the critical comparison of the topological sensitivity hypothesis is between Iso and Noniso pairs. If individuals are sensitive to the topological structure of figures, they should detect a break of topological structure more quickly and with shorter judgment latencies in Noniso pairs than in Iso pairs.

2.1 *Methods*

2.1.1 *Participants.* Three male and seven female Japanese university students aged 20–22 years participated in this experiment.

2.1.2 *Stimuli.* The stimuli were a pair of simultaneously presented (6, 3) figures. Because an intersection of the line segments was not a graph invariant, experiment 1 allowed the possible inclusion of intersecting line segments in the stimulus figures.

The shortest line segment lengths were 3.8 cm, and the longest segments were 7.6 cm, with visual angles of 3.34 deg and 6.69 deg, respectively. Except for the points described below, the width and height of the area upon which each stimulus figure was projected were 6.6 cm and 7.6 cm, respectively. Points with diameters of 0.4 cm were also displayed on the LCD, and the locations of their centers were shifted 0.2 cm outward from the center of the invisible regular hexagon. Two figures of a stimulus were simultaneously presented at horizontally parallel positions, and the distance between the centers of the figures was 9.4 cm.

2.1.3 *Generation of pairs.* Because no control was exerted over the absence or presence of intersections during figure generation, the numbers of figures pooled in the isomorphic sets was identical to those described in the introduction. Examples of Id, Iso, and Noniso pairs are shown in figure 6.

2.1.4 *Procedures.* Each participant performed a total of 20 total practice trials and 420 test trials (10 practice trials and 210 test trials per block).

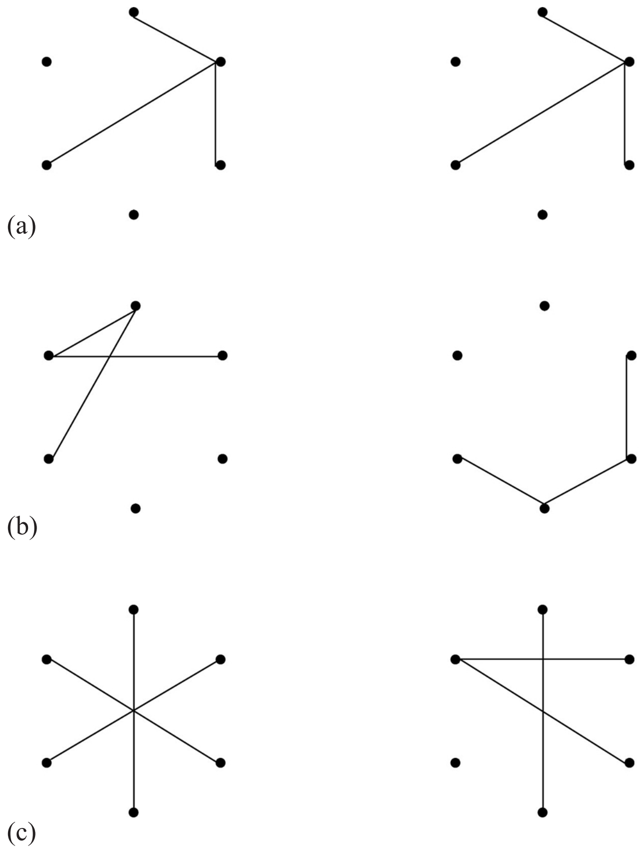


Figure 6. Examples of (a) Id, (b) Iso, and (c) Noniso pairs used in experiment 1.

2.2 *Results and discussion*

The rates of erroneous judgments for Id, Iso, and Noniso pairs were 3.5%, 4.7%, and 1.8%, respectively. Analysis of variance of the error rates revealed a significant main effect ($F_{2,27} = 5.4, p < 0.05$). A posteriori tests corrected for multiple comparisons indicated that the error rates were significantly different between only the Iso and Noniso pairs at 1% level.

As the critical test of the topological sensitivity hypothesis is the latency difference between Iso and Noniso pairs and as processing of judgments to Id and nonidentical pairs should not be unitary, I will henceforth present the latency results of only nonidentical pairs (ie Iso and Noniso pairs). The mean latencies of the correct responses were 656 ms (43 ms) for Iso pairs and 619 ms (46 ms) for Noniso pairs (standard errors are in parentheses). Analysis of variance showed a significant main effect ($F_{1,9} = 10.59, p < 0.01$) of nonidentical pair types (ie Iso and Noniso pairs).

Both the latency and error rate data suggest that discrimination was more efficient for Noniso pairs compared with the other pair types. As the trends of the latencies and of the error rates of the nonidentical pair types were parallel, no indication of a tradeoff between speed and accuracy was found. I obtained shorter latencies for Noniso pairs than for Iso pairs, supporting the topological sensitivity hypothesis.

3 Experiment 2

In experiment 1 the stimulus figures were classified according to graph-theory-defined isomorphic sets. In this way, two figures belonged to the same isomorphic set if they shared an identical correspondence of line segments connected by pairs of points. This was true regardless of the locations of the pairs. However, it has been previously asserted that the intersections of such line segments can be preattentively detected (eg Bergen and Julesz 1983; Wolfe and DiMase 2003). Although the state (ie presence versus absence) of such intersecting line segments is not a graph invariant, it is probable that any quick detections of an intersection would confound the data on the participant's sensitivity to topological differences. Therefore, experiment 2 was developed to further examine the topological sensitivity hypothesis using stimuli without line segment intersections. Because experiment 2 examined the two major characteristics of the present series of experiments (ie the number of lines was 3 and intersections were not included), it was taken as the gold standard experiment and the number of participants was increased.

3.1 *Method*

3.1.1 *Participants.* Four male and twelve female Japanese university students aged 20–24 years participated in this experiment.

3.1.2 *Stimuli.* Stimuli were composed of a pair of simultaneously presented (6, 3) figures. The sizes and the spatial arrangements of the stimuli were identical to those presented in experiment 1.

3.1.3 *Generation of pairs.* The method of generating pairs was identical to that used in experiment 1, with the exception that the figures with intersecting line segments were not included as stimulus figures. All (6, 3) figures were sorted according to the isomorphic sets, and figures with one or more intersections were discarded. Hence, of a total of 455 (6, 3) figures, the number of figures included in the present experiment was 60 in isomorphic set 1, 20 in set 2, 120 in set 3, 90 in set 4, and 5 in set 5, for a total of 295. Examples of Id, Iso, and Noniso pairs are shown in figure 7.

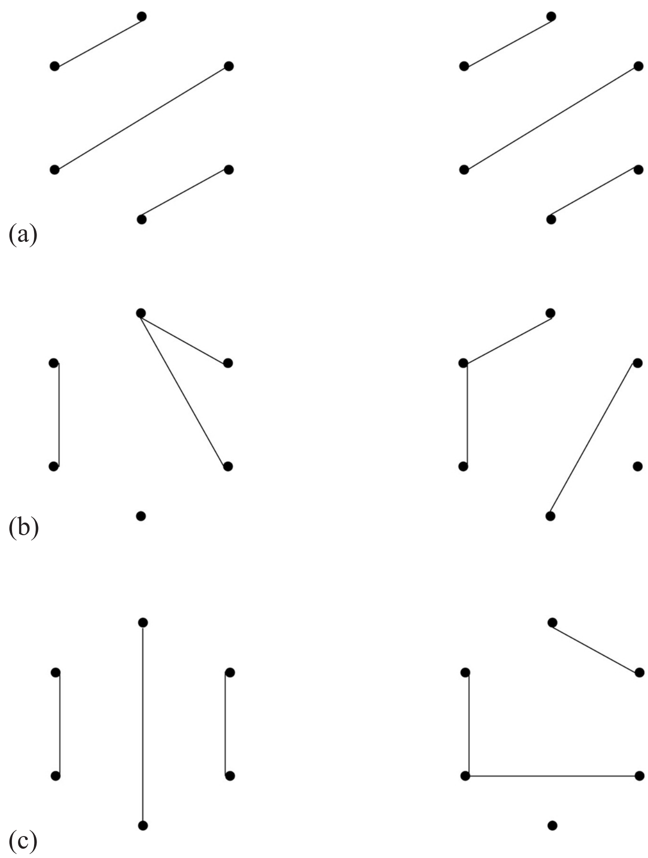


Figure 7. Examples of (a) Id, (b) Iso, and (c) Noniso pairs used in experiment 2.

3.1.4 *Procedures.* All procedures were identical to those used in experiment 1.

3.2 *Results and discussion*

The percentage errors for Id, Iso, and Noniso pair discriminations were 4.1%, 5.6%, and 2.4%, respectively. Analysis of variance on the error rates did not reveal a significant main effect ($F_{2,45} = 1.82$, $p > 0.05$).

The mean latencies of the correct responses were 701 ms (50 ms) for Iso pairs and 654 ms (42 ms) for Noniso pairs (standard errors are in parentheses). Analysis of variance showed a significant main effect ($F_{1,15} = 19.3$, $p < 0.001$) of nonidentical pair types.

There was no indication of a tradeoff between speed and accuracy. The pattern of latencies supports the topological sensitivity hypothesis. Although the respective latencies of the pair types increased from those of experiment 1 by 35–71 ms, the absence of intersecting lines in the stimulus figures did not significantly affect the pattern of latencies across the three types of pairs.

4 Experiment 3

Even if the shape of the two figures was identical, the figures were defined as being different when they were specified by different point pairs. As such, the two figures were different in their orientations. That is, the points, which were emphasized by the enlarged filled circles, provided definitive information as to the orientation differences of (6, n) figures with the same shape. However, the presentation of the points in itself was not a necessary condition for examination of the topological sensitivity hypothesis.

It is possible that the presence of these emphasized points could distract or confuse the invariant feature-based perceptions of participants. In this respect, the purpose of experiment 3, using (6, 3) figures as stimuli, was to replicate experiment 2 without presenting the points.

4.1 *Method*

4.1.1 *Participants.* Three male and seven female Japanese university students aged 20–22 years participated in this experiment.

4.1.2 *Stimuli.* The stimuli used in this experiment were a pair of simultaneously presented (6, 3) figures. The sizes and the spatial arrangements of the stimuli were identical to those presented in experiment 1, with no points displayed.

4.1.3 *Generation of pairs.* The method used to generate pairs was identical to that used in experiment 2. Figures with one or more line segment intersections were not included as stimulus figures. Examples of Id, Iso, and Noniso pairs used in this experiment are shown in figure 8.

4.1.4 *Procedures.* All procedures were identical to those used in experiment 1.

4.2 *Results and discussion*

The rates of erroneous judgments for Id, Iso, and Noniso pairs were 5.1%, 7.0%, and 2.7%, respectively. Analysis of variance on the error rates revealed no significant main effect ($F_{2,27} = 1.76$, $p > 0.05$).

The mean latencies of the correct responses were 674 ms (49 ms) for Iso pairs and 616 ms (47 ms) for Noniso pairs (standard errors are in parentheses). Analysis of variance showed a significant main effect ($F_{1,9} = 94$, $p < 0.001$) of nonidentical pair types.

In comparison with the results of experiment 2 (the standard experiment), the latencies of the respective pair types of this experiment were 40–70 ms shorter, which suggests that the removal of the points made their judgments easier. Nevertheless, the results again showed shorter latencies for Noniso pairs than for Iso pairs, further supporting the topological sensitivity hypothesis even without the emphasized points.

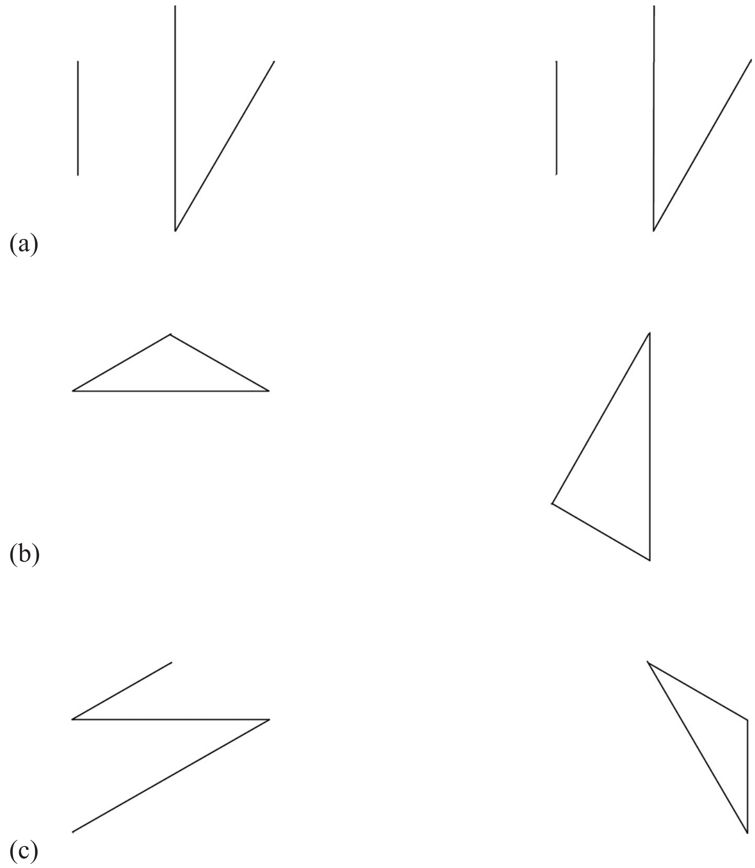


Figure 8. Examples of (a) Id, (b) Iso, and (c) Noniso pairs used in experiment 3.

5 Experiment 4

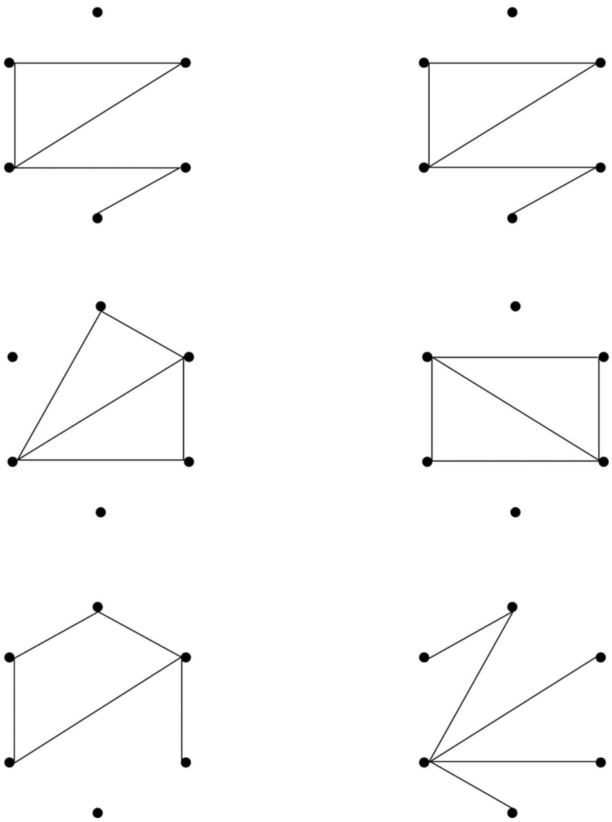
The stimulus figures used in the previous three experiments were (6, 3) figures. However, it may be worthwhile to determine whether topological sensitivity still persists even when figural complexity increases. Thus, experiment 4 employed (6, 5) figures as the stimuli. Because there were 15 isomorphic sets for (6, 5) figures in contrast to 5 sets for (6, 3) figures, the number of participants and the number of pairs for each were increased in this experiment.

5.1 *Method*

5.1.1 *Participants.* Six male and ten female Japanese university students aged 19–24 years participated in this experiment.

5.1.2 *Stimuli.* The stimuli were a pair of simultaneously presented (6, 5) figures. The sizes and the spatial arrangements of the stimuli were identical to those presented in experiment 1. The points were displayed.

5.1.3 *Generation of pairs.* The method used to generate the pairs was identical to that used in experiment 1, with the following exceptions: figures with one or more intersecting line segments were not included in the stimulus figures. Hence, out of a total of 3003 (6, 5) figures, the number of figures satisfying the intersection condition was 6 in isomorphic set 1, 90 in set 2, 48 in set 3, 30 in set 4, 120 in set 5, 120 in set 6, 48 in set 7, 27 in set 8, 60 in set 9, 72 in set 10, 72 in set 11, 18 in set 12, 6 in set 13, 6 in set 14, and 48 in set 15. Examples of Id, Iso, and Noniso pairs used in this experiment are shown in figure 9.



(a)

(b)

(c)

Figure 9. Examples of (a) Id, (b) Iso, and (c) Noniso pairs used in experiment 4.

5.1.4 *Procedures.* All procedures were identical to those used in experiment 1, except that the participants completed 10 practice and 240 test trials per block (20 practice and 480 test trials total), and the total numbers of Id, Iso, and Noniso pairs were 250, 50, and 200, respectively.

5.2 *Results and discussion*

The percentage errors for Id, Iso, and Noniso pair discriminations were 2.9%, 7.3%, and 2.7%, respectively. Analysis of variance on the error rates revealed a significant main effect ($F_{2,45} = 5.4$, $p < 0.01$). A posteriori tests corrected for multiple comparisons indicated that the error rates were significantly different between Id and Iso as well as Iso and Noniso pairs at the 5% level.

The mean latencies of the correct responses were 711 ms (25 ms) for Iso pairs and 662 ms (22 ms) for Noniso pairs (standard errors are in parentheses). Analysis of variance showed a significant main effect ($F_{1,15} = 28$, $p < 0.001$) of nonidentical pair types.

Sensitivity to topological differences was again confirmed by the shorter latencies in Noniso pairs than in Iso pairs. The latency difference for each pair type between experiment 2 and this experiment was small (ie less than 10 ms). Hence, it appears that participants' ability to discriminate figures was not overly influenced by the complexity of the present type of stimulus figures.

6 Experiment 5

In the previous experiments the stimulus figures were defined by six points located at the vertices of the invisible regular hexagon. However, according to the definition of isomorphism, there is no constraint on the locations of the points between which each line segment is drawn.

Because topological structures are preserved under continuous deformations on a given figure, experiment 5 was designed to examine whether the topological sensitivity hypothesis accounted for the data, even when the stimulus figures were deformed—namely, when the 1 : 1 aspect ratio of the stimulus figures was changed to a 1 : 2 ratio.

6.1 *Method*

6.1.1 *Participants.* Six male and four female Japanese university students aged 19–22 years participated in this experiment.

6.1.2 *Stimuli.* The stimuli were a pair of simultaneously presented (6, 3) figures with lengths ranging from 1.9 to 6.8 cm and visual angles of 1.81 deg to 6.49 deg, respectively. Except for the area of the points, the width and height of the area upon which each figure was projected were 6.6 cm and 3.8 cm, respectively. The point sizes and display were identical to those used in the previous experiments.

6.1.3 *Generation of pairs.* The method used to generate the pairs was identical to that used in experiment 2. Figures with one or more intersecting line segments were not included in the stimulus figures. Examples of Id, Iso, and Noniso pairs are shown in figure 10.

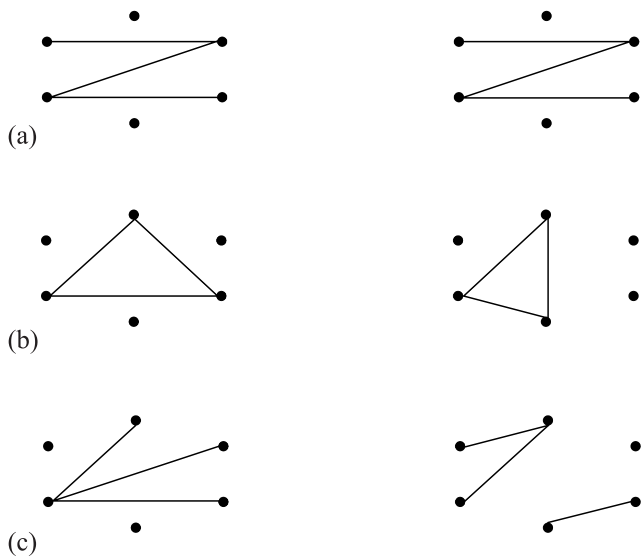


Figure 10. Examples of (a) Id, (b) Iso, and (c) Noniso pairs used in experiment 5.

6.1.4 *Procedures.* All procedures were identical to those used in experiment 1.

6.2 *Results and discussion*

The rates of erroneous judgments for Id, Iso, and Noniso pairs were 3.0%, 5.3%, and 2.3%, respectively. Analysis of variance on the error rates revealed no significant main effect ($F_{2,27} = 2.8$, $p > 0.05$).

The mean latencies of the correct responses were 725 ms (27 ms) for Iso pairs and 661 ms (27 ms) for Noniso pairs (standard errors are in parentheses). Analysis of variance showed a significant main effect ($F_{1,9} = 152$, $p < 0.001$) of nonidentical pair types.

Even when the shapes of the stimulus figures were deformed, participants' judgments of Noniso pairs were faster than those of Iso pairs, replicating the results of the previous experiments using the stimulus figures with the regular aspect ratio of 1 : 1. This result provides strong support for the topological sensitivity hypothesis.

7 Experiment 6

All five experiments indicated that humans are sensitive to topological differences of a given pair of figures when making a same/different judgment. As has been stated previously, the topological sensitivity hypothesis does not claim that invariant features alone are responsible for the recognition of figures. The crucial role of superficial features in figure recognition is obvious, given the fact that Id pairs were correctly judged as being the same and Iso pairs were correctly judged as being different in most trials. It is therefore important to ask whether superficial features (ie metric measures) alone can explain the results, particularly the performance differences between Iso and Noniso pairs.

There are two types of superficial features: (a) those dependent on invariant features, and (b) those not dependent on invariant features. Metric measures such as the distance between two endpoints in a figure or the orientation formed by two isolated points in a figure are examples of type-a features. The orientation of a figure, the total length of line segments in a figure, the centroid location of a figure, and the location of an intersection of line segments are examples of type b. With respect to the type-a superficial features, the metric information of the feature makes sense only after specifying an invariant feature, which the superficial feature depends on, and thus metric information of this type should be taken as derived from the relevant invariant feature.

Yet, at the same time, the values of invariant features may vary in Noniso pairs depending on the isomorphic sets selected. If the value of an invariant feature of one figure is different from the value of the same invariant feature of the other figure, there is no correspondence of the superficial feature values between the two figures. For example, if the number of endpoints in one figure is 2 and the number of endpoints in the other figure is 0, we cannot define the angular disparity between the respective locations of the endpoints of the two figures. Such noncorrespondence of invariant feature values occurs frequently in Noniso pairs. In this respect, type-a superficial features would not provide any persistent effects on participants' decisions with respect to Noniso pairs.

On the other hand, if type-b superficial features are not controlled for while generating stimulus figures, the effect of metric information could be confounded with the effect of topological differences and thus could invalidate claims of topological sensitivity. As the participants understood the irrelevancy of the absolute location information (eg the centroid of a figure, the location of an intersection) to the present discrimination task, the critical information for discriminations concerns relational measures like distances (or lengths) and orientations.

Concerning the factor of total line lengths of respective figures, I compared the average line length differences between Iso pairs and Noniso pairs for a virtual participant by simulating the same pair-generation procedure of (6, 3) figures with the condition that the figures having intersecting line segments were excluded. The length of each side of an invisible regular hexagon was taken as a unit of length. In three simulations, the average length difference for Iso pairs was always smaller than that for Noniso pairs ($t_{218} = 3.77$, $t_{218} = 2.28$, $t_{218} = 2.61$, all ps < 0.05, two-tailed). It could thus be inferred that the total line length factor was confounded with the topological difference factor for the stimuli used in the preceding experiments.

With regard to the orientation differences between Iso and Noniso pairs, the results of simulations were more ambiguous. The points along each line segment of a figure were sampled with an equal interval. The slope of the principal component of the points sampled from a (6, 3) figure without intersections was computed and expressed as one of the six orientations (figure 11). Discrepancies in the orientations of the two figures of a pair were defined as the smaller of the clockwise or counterclockwise angle formed by the two

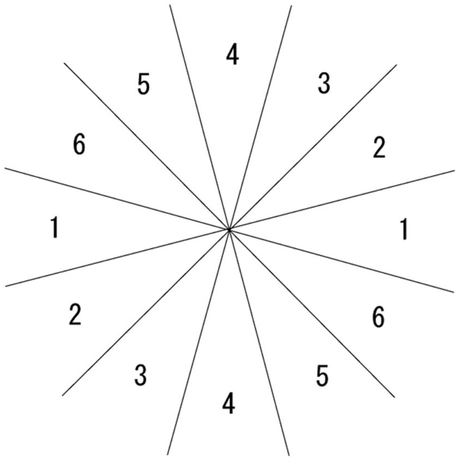


Figure 11. Representative orientations. Numbered sectors represent orientations with a 30° step counterclockwise from the horizontal sector.

orientations, and were classified into one of the four classes representing the discrepancies 0°, ±30°, ±60°, and ±90°, respectively. The discrepancies obtained by simulated trials that had fallen into the respective classes were tallied and their probabilities were calculated. Finally, the simulated probability distributions of Iso and of Noniso pairs were compared. In three simulations the distributions differed significantly for one case ($\chi^2_2 = 16.9, p < 0.01$). This was not the case for the other two simulations ($\chi^2_2 = 1.65$ and 1.41, ps > 0.05).

Taking the results of these simulations into consideration, I stress that experiment 6 was intended to examine whether the topological sensitivity was still present even when the total line lengths were equal between a given pair of figures. However, any figures in isomorphic set 2 without intersections had different total line lengths than those of any figures without intersections belonging to isomorphic set 5. As the number of the figures without intersections belonging to isomorphic set 2 is 20 and the number of belonging to isomorphic set 5 is 5, I excluded the figures of set 5 from the stimuli. The exclusion of the figures of set 5 from the stimuli also reduced the orientational differences between Iso and Noniso pairs. With regard to figure orientations, there was no significant difference between the three newly attempted simulations ($\chi^2_2 = 3.46$, 1.82, and 0.09, all ps > 0.05). The results of these simulations suggest that the orientation factor was controlled in the present experiment.

7.1 *Method*

7.1.1 *Participants.* Three male and seven female Japanese university students aged 19–22 years participated in this experiment.

7.1.2 *Stimuli.* Stimuli were composed of a pair of simultaneously presented (6, 3) figures. The sizes and the spatial arrangements of the stimuli were identical to those presented in experiment 1. There was no difference in the total line lengths between the two figures of any pairs.

7.1.3 *Generation of pairs.* The method of generating pairs was identical to that used in experiment 2, with the exception that if a randomly selected pair of figures happened to have different total line lengths, the pair was discarded and the pair-selection procedure was reiterated. All (6, 3) figures except for those belonging to isomorphic set 5 were sorted according to their isomorphic sets, and figures with one or more intersections were discarded. Hence, of a total of 455 (6, 3) figures, the number of figures included in the present experiment was 60 in isomorphic set 1, 20 in set 2, 120 in set 3, and 90 in set 4, for a total of 290.

However, equating the lengths of the total line segments of the two figures constrained the variation of their shapes especially for Iso pairs because the sizes of the isomorphic sets of (6, 3) figures are small. As a result, many selected Iso pairs of figures were mutually reflected or identical except for their rotations in this experiment. Examples of Id, Iso, and Noniso pairs are shown in figure 12.

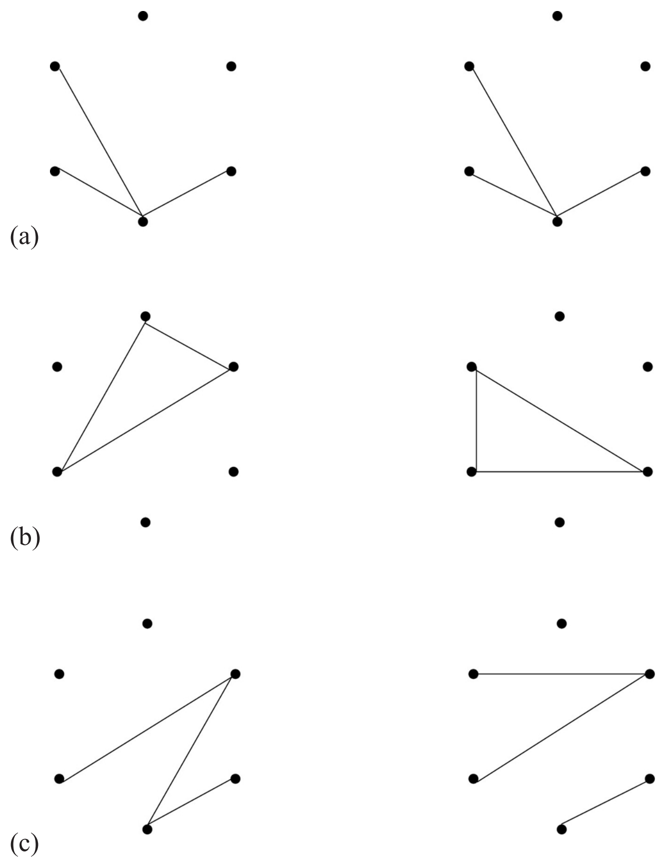


Figure 12. Examples of (a) Id, (b) Iso, and (c) Noniso pairs used in experiment 6.

7.1.4 *Procedures.* All procedures were identical to those used in experiment 1.

7.2 *Results and discussion*

The percentage errors for Id, Iso, and Noniso pair discriminations were 3.7%, 11.4%, and 2.4%, respectively. Analysis of variance on the error rates revealed a significant main effect ($F_{2,27} = 14.8$, $p < 0.01$). A posteriori tests corrected for multiple comparisons indicated that the error rates were significantly different between Id and Iso and Iso and Noniso pairs at the 1% level.

The mean latencies of the correct responses were 707 ms (58 ms) for Iso pairs and 655 ms (49 ms) for Noniso pairs (standard errors are in parentheses). Analysis of variance showed a significant main effect ($F_{1,9} = 22.5$, $p < 0.01$) of nonidentical pair types.

Even when the total line lengths between the two figures of any pair were equal, the latencies were significantly shorter for Noniso pairs than for Iso pairs, indicating topological sensitivity. It was also assumed that the effects of orientation differences were not significantly confounded by the effects of topological differences in the present experiment. As the total line lengths and orientations of the figures were the two major invariant feature-independent superficial features, it seems obvious that the persistently observed superior discrimination of Noniso pairs then Iso pairs could not be attributed to differences in the metric information between the two types. These results are consistent with Bedford's (2001) claim that figures having topological differences should be more easily discriminated than figures having different metric measures.

Looking at the error rates, it is noteworthy that the rate for Iso pairs was clearly higher than the rates of the other experiments, whereas the rates for Id and Noniso pairs were roughly comparable with those of the previous experiments. It is probable that the inclusion of many reflected pairs of figures in the Iso pairs could increase the difficulty for judging Iso pairs as different correctly. This inference will be further argued in the general discussion.

8 General discussion

In my examination of the topological sensitivity hypothesis of visual perception, systematic control was exerted on the selection of (6, *n*) figures as stimuli, which were classified according to graph-theory-defined isomorphic sets. The figures belonging to the same isomorphic set had the same topological structure, whereas the figures belonging to different sets had different topological structures.

A series of experiments investigated the roles of various properties of the stimuli such as (i) intersecting line segments, (ii) display of the points, (iii) figural complexity, (iv) the aspect ratios of the figures, and (v) metric information such as the total line lengths of the figures. Despite absolute differences in these factors, for a given condition the latencies were always higher for Iso pairs than for Noniso pairs. This finding constitutes clear support of the topological sensitivity hypothesis. Of particular importance were the findings of experiment 5, in which topological sensitivity was retained even when the shapes of the stimulus figures were deformed as predicted from the nature of topology. Likewise, the results of experiment 6 indicated that metric information was irrelevant to the consistent shorter latencies for Noniso pairs than for Iso pairs.

These findings lead to the question of what the constituents of topological sensitivity are. Although Chen considered that topological information is definable by global properties such as closures (eg Chen 1982; Han et al 1999), a local invariant feature alone is capable of classifying the topological structure of (6, *n*) figures if the figures are simple. For example, the number of endpoints, the number of central points, and the maximum degree points are locally specifiable invariant features that can by themselves classify all 105 (6, 2) figures into two isomorphic sets. However, a global invariant feature such as the number of cycles cannot classify (6, 2) figures because two straight line segments are insufficient to form any closed shapes. In fact, local versus global differences may not provide a sufficient basis to make a case for topological perception. In addition, the isomorphism of any two (6, *n*) figures (when $n > 2$) can be determined not by the value of a single invariant feature but by the values of multiple invariant features.

The error rates were persistently higher in Iso pairs than in Noniso pairs, suggesting that Iso pairs were more prone to be judged identical than Noniso pairs. These results could bolster the interpretation that comparisons of invariant feature values dominate over those of superficial feature values when participants make same/different discriminations of the present figure types. However, most Id pairs were still correctly discriminated from Iso pairs, which would indicate that superficial features were also used for their identification.

Although not of critical comparisons in this study, I applied a posteriori tests corrected for multiple comparisons on the latencies seen in the discrimination of the three pair types. For all experiments the results showed that latencies were the shortest for Noniso pairs but were not significantly different between Iso and Id pairs. The pattern of latencies was also consistent with the assumption that sequential, self-terminating invariant features comparisons should be conducted between the two figures of a pair.

The question of whether a set of invariant and superficial features is fixed or variable cannot be fully addressed, although several potential answers to this question can be inferred. In cases where a specific invariant feature does not exist in the figure (ie the value of that invariant feature is 0), it would be difficult to make a comparison between the figure having a value of 0 and a figure with a value other than 0. For example, by looking at figure 4, one sees that there is no endpoint in the figures of set 2; no isolated point in the figures of set 5, and no cycle in the figures of sets 1, 3, 4, and 5. In addition, the values of superficial features cannot be defined if the relevant invariant features do not exist. If participants had made use of a fixed set of a fairly limited number of invariant and superficial features, erroneous decision rates should have been fairly high. However, the obtained error rates for Id and Iso pairs were not extremely high (2.9%–11.4%). To avoid highly erroneous decisions, the size of the fixed set of features must be sufficiently large.

A cognitive system furnished with the computation capabilities of a large number of invariant and superficial features, many of which will not be used depending on what is presented, seems inefficient. Therefore, a variable set assumption, in which invariant and superficial features are variably activated from a repertoire of features according to the figures presented, seems more viable than a fixed-set assumption.

Although not reaching the same assertion as Chen, the present topological sensitivity hypothesis does not assume that the sensitivity is derived from the direct perception of the topological structure of a given figure. The results of the present experiments help to clarify the nature of topological sensitivity. A model based on sequential, self-terminating comparisons of invariant features and invariant feature-dependent superficial features should consistently and comprehensively explain why the latencies to Noniso pairs were smaller than those to Iso and Id pairs.

With respect to the role of figures' superficial features, if their comparisons were executed in a purely sequential, self-terminating manner, the latencies should be longer for Id pairs than Iso pairs. However, no significant difference between the two types in fact resulted. There may be two possible causes for this lack of difference: (a) the small size of a superficial feature set, and (b) the nature of superficial feature comparisons. Under the sequential comparisons of a small number of superficial features [ie possibility (a)], at the time the comparisons are exhausted without encountering any value difference between the figures of a pair, the pair would be determined to be the same. If the size of the set is fairly small, the chance of not encountering a value difference could be high, and the error rates for Iso pairs should accordingly be extremely high. In this respect, the obtained error rates for Iso pairs (ie 4.7%–11.4%) were seemingly too low.

The other possibility (b) assumes that the processing of superficial features can be easily distracted rather than self-terminated in the sequence of comparisons. If this happens, the latencies would protract and the error rates would be high for Iso pairs because of their complex patterns of superficial value differences, whereas the error rates for Id pairs would not be high because there is no difference in superficial values.

Taking the level of the percentage errors into consideration, an account based on the easily distracted nature of superficial feature comparisons would be more consistent with the patterns of the experimental results described here, and more consistent with the already stated assumption that superficial feature comparisons should be subordinate to invariant feature comparisons.

Let me conclude the discussion with the implications of the present study to some areas of visual shape perception. When people are asked to judge whether disoriented pairs of figures are the same or different regardless of their orientations, they would first extract features in the respective figures and compare the states of features, and if they fail to find a difference in the states, then mental rotation could be induced (eg Corballis 1988; Takano 1989). Between rotated and mirror-reflected pairs of figures, there is no difference in the states or values of the extracted features. Therefore, discrimination is more difficult when pairs consist of mutually reflected figures than when they consist of irrelevant figures. As an example of this, compare the performance difference in the distinctions of disoriented *p*s from disoriented *q*s as reported by Corballis and McLaren (1984) with the distinctions of disoriented letters from disoriented digits as reported by Corballis and Nagourney (1978). Sometimes the discrimination of identical pairs from mutually reflected pairs is almost impossible. For example, in same/different decisions of pairs of disoriented figures, I (Kanbe 2002) reported that the percentage errors to axisymmetric pairs were 53.9% for (6, 3) figures and 56.7% for (6, 5) figures and compared those with identical pairs for (6, 3) and (6, 5) figures, which were 4.8% and 7.9%, respectively. Here, axisymmetric pairs of two figures were mutually symmetrical about one of the six axes of 0°, 30°, 60°, 90°, 120°, or 150° counterclockwise from the right horizontal line. Many reflected pairs were included in Iso pairs in experiment 6. Although the definition of identity in the present experiments did not allow rotation, unlike in Kanbe (2002), the difficulty in discriminating reflected (ie axisymmetric) pairs from identical pairs may still be related to the high error rate for Iso pairs. As the permutations of the locations of corresponding invariant feature values between the two figures should systematically occur about an axis of symmetry in a reflected pair, such complex patterns of locational shifts and shifting directions could perturb processing. In this respect, the high error rate for the judgments of Iso pairs in experiment 6 is congruent with possibility (b), which assumes an easily distracted nature of superficial feature comparisons.

In the present study stimulus figures were randomly selected from the pools of candidate figures to form the three types of pairs. However, the sizes of such pools (or the sizes of subsets of figures) satisfying prescribed conditions vary from condition to condition. For example, the sizes of isomorphic sets vary from 15 to 180 for (6, 3) figures and from 6 to 360 for (6, 5) figures. As was already described, Wagemans et al (1994) reported that the sizes of sets of figures defined by affine transformations affected their discriminability. On the other hand, Garner (1974) defined a subset of five-dot patterns in which any member can transform itself into other members by 90° rotations and reflections around the horizontal, vertical, right diagonal, and left diagonal axes. By this rule, subsets with sizes of 1, 4, and 8 were generated. Garner compared the sizes of the subsets and rated the goodness of patterns belonging to the respective subsets. The results showed that the smaller the sizes of subsets, the greater the goodness ratings of patterns. If we accept the assumption that good figures are easily recognizable, it would be worth examining in future research how the sizes of the topologically equivalent sets of figures can affect their discriminability.

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