Review on Internal Ballistics Research on Hybrid Rockets

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ABSTRACT

Some of major challenges in the internal ballistics research of hybrid rockets are outlined, especially when the heating rate dominates the fuel regression rate. Described are the effect of increasing Stanton number due to flow acceleration from turbulent boundary layer combustion flame, the effect of radiative heating from soot particles, and the effect of convection-radiation interaction. From the latter model, the positive minimum value of the relative time change rate of the radiation-to-convection ratio is determined as a condition for the oxidizer-to-fuel ratio to decrease with time.

1. Introduction

The performance of chemical rocket propulsion system is determined by its effective exhaust velocity, which is determined by the characteristic exhaust velocity and its efficiency determined by the combination of propellants, the applied environment such as ambient pressure, and nozzle characteristics and efficiency. The internal ballistics of a rocket is the aggregate of these, and researching it is nothing but researching the performance of the rocket propulsion system in detail.

A combination of propellants categorizes a chemical rocket into solid rockets, liquid rockets, and hybrid rockets. In this paper, we focus on the hybrid rocket and give a consideration on the evaluation of its internal ballistics. From the viewpoint of internal ballistic characterization, the hybrid rocket differs greatly from the remaining two in the sense that flow dynamics play a big role in the combustion efficiency of the propellant. In a hybrid rocket, where the amount of fuel gas generation is affected by heat feedback from the turbulent diffusion combustion flame in the boundary layer over fuel surface, it is required to construct an evaluation model by closely linking things, such as the heat transfer to fuel, the decomposition / evaporation of fuel, and the mixing and combustion of propellants, with phenomena from the minimum scale of turbulence to the macroscopic flow scale. Since it is extremely difficult to analyze everything comprehensively and directly, it has been practically divided into several elementary models. The internal ballistics of hybrid rockets and its modeling up to June 1971 was well summarized by Netzer [1].

In order to evaluate the internal ballistics of a hybrid rocket, it is extremely important to evaluate the fuel gasification phenomenon in consideration of the macroscopic flow phenomenon. The mass generation rate of fuel gas is equal to the product of the fuel regression rate and the density of solid fuel. When the fuel surface temperature is sufficiently high, the fuel mass generation rate is defined by the energy amount for the unit mass to gasify and the input energy from the outside. In this case, the phase change rate and the thermal decomposition rate are sufficiently high, so the rate is determined by the external heating rate. On the other hand, in the case of low temperature, such as ignition phase, the rate is defined by the temperature of the fuel material even if the heating rate is high. In this paper we will restrict ourselves to the former case and will deal with the

generation of fuel gas due to convective and radiative fuel heating. We will not deal with the evaluation of liquefied fuel entrainment, whose importance has been well known in recent years [2,3,4].

According to Netzer[1], the fuel regression rate due to convective / radiative heating can be reasonably analyzed by Eqs. (1)-(5) under several assumptions such as a) flat plate turbulent boundary layer analysis is applicable, b) regression rate of grain is controlled by heat transfer from a diffusion flame, c) flame zone is infinitesimally thin, d) oxidizer enters port as a uniform gaseous stream, e) no heat transfer into subsurface region of solid grain, f) Reynolds analogy is applicable between the flame and the wall in the presence of blowing, g) Le = Pr = 1, f) Pressure is constant.

$$\rho_f \dot{r} = \frac{\dot{Q}_c^*}{h_{v_{eff}}} \left(e^{-\dot{Q}_r/\dot{Q}_c^*} + \frac{\dot{Q}_r}{\dot{Q}_c^*} \right) \tag{1}$$

$$\dot{Q}_c^* = 0.036 \, h_{v_{eff}} \left(\frac{\bar{\rho}}{\rho_e} \right)^{0.6} \left(\frac{x}{\mu_e} \right)^{-0.2} G^{0.8} B^{0.23} \tag{2}$$

$$\dot{Q}_r = \sigma \, \epsilon_w \left(\epsilon_g \, T_r^4 - T_w^4 \right) \approx \sigma \, \epsilon_w \, \epsilon_g \, T_r^4 \tag{3}$$

$$G = \rho_e u_e = \left(\frac{\dot{m}_g}{A_p} \right) \tag{4}$$

$$\epsilon_g = 1 - e^{-\alpha N z} \tag{5}$$

Here, $\dot{Q}_c^*$ is the local convective heating rate without radiation, G the local mass flux defined by Eq. (4), and $\dot{Q}_r$ the radiative heating rate. In Eq. (3) ϵ_w is the emissivity of the fuel surface and ϵ_g is the emissivity of the gas, which is calculated by Eq. (5). The energy inflow from the outside is roughly divided into convective heat transfer and radiative heat transfer. However, the effect of mass addition (blowing B) from the fuel surface directly affects the amount of convective heat transfer, while it does not the radiative heating. Therefore, fuel generation augmented by an increase in radiative heat transfer enhances the blowing effect and as a result suppresses convective heat transfer. In Eq. (1), this effect is expressed by multiplying $e^{\dot{Q}_r/\dot{Q}_c^*}$ to the convective heating rate without radiation, then the fuel mass generation is not a linear sum of convective and radiative contributions [5]. In recent years, Eq, (1) has been further modified by Chiaverini et al [6] and Budzinski et al. [7].

2. Issues of convective heating rate evaluation

Equation (2) is Marxman's model and assumes that local heating due to the combustion flame in the boundary layer does not affect the velocity profile of the boundary layer [5, 8-10]. This assumption, however, was disproved by Jones [11], who conducted a turbulent boundary layer combustion experiment with mass addition and mainstream acceleration to find the maximum in the velocity profile near the flame, and the maximum can be larger than the mainstream as shown in Fig.1.
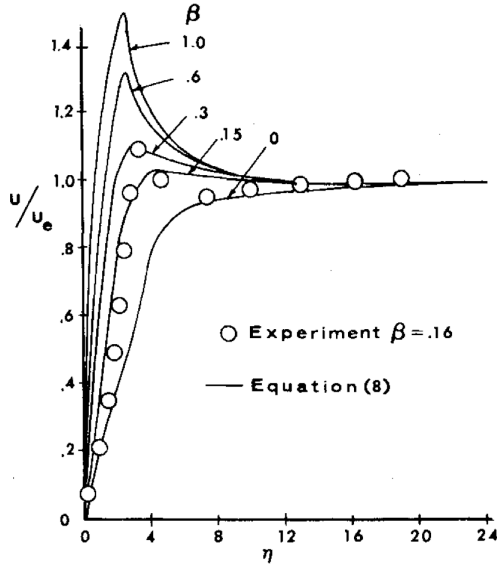

Fig. 1 Velocity profiles across a boundary layer in an accelerating flow with combustion [11].

This is also shown by CFD simulation results [12-14] of the internal flow of the hybrid rocket. These indicate that both local heating (flame) in the boundary layer and a negative pressure gradient in the axial direction of the combustion chamber yield a local acceleration region in the boundary layer, which results in convective heating rate increase. A simple model has already been proposed to modify the conventional convective heating evaluation considering this effect [15], but currently there are yet very few studies on this subject. Since this phenomenon is very complex in which a reactive turbulent boundary layer flow with surface mass addition, detailed research by experiments and CFD simulations is required to elucidate the phenomenon.

3. Issues of radiative heating rate evaluation

As of 1972, the radiative heating rate was expressed by Eq. (3). In this equation, ϵ_g is the emissivity of the gas and was calculated by Eq. (5). In Eq. (5), T_r is the effective radiation temperature, α an empirical parameter, and z radiation length. The parameter N denotes pressure for gas and n (number density of particles) for radiating solid particles existing in the gas. The radiative heating rate does not depend on the mass flux explicitly. However, a change in the mass flux affects the convective heating to change the fuel regression rate,

then to change O/F, which will change highly radiating particle density to change the radiative heating rate. For this reason, the evaluation of the effective radiation temperature and the emissivity of the gas becomes very important when using this model. However, in Ref. [1], description is limited about evaluation models for radiative heating from high-temperature gas molecules and soot particles.

In recent years, radiation from high-temperature gas molecules has been estimated by the Statistical Narrow Band Model (SNB) [16] in which radiation from high temperature gas such as CO, CO_2, H_2O can be evaluated up to 5000K. Leccese et al. [17], Migliorino et al. [18], and more recently, Naka et al [19, 20] have used the SNB model for internal ballistics simulations of paraffin wax fueled hybrid rockets.

Next, regarding radiation from high-temperature particles, as of 1972, particles were assumed to be particles produced by metal powder filled in fuel grain. However, in recent years, even in the case of metal-free fuels, it has been pointed out that soot particles, rather than hot gas molecules, are the main source of radiation in hybrid rockets [3]. In addition, Leccese et al. [17] conducted a paraffin wax hybrid rocket combustion experiment and compared it with the results of simulation of hot gas radiation only and reported that there was a large difference between the two radiative heat fluxes suggesting that soot contribution in total radiation is significant.

The mechanism of soot formation is not yet known in detail. In particular, there are still many unclear points about the mechanism of soot particles from molecules that are precursors of soot such as acetylene and PAHs [21]. Then, in our group, Naka et al. evaluated the internal ballistic characteristics in the hybrid rocket using the Global Soot Model [22-25], which is a semi-empirical model for the soot production rate, to show that the effect of radiation plays a very important role for conditions of the combustion experiments [26]. In Ref. [20], a three-dimensional radiative heat transfer analysis method is proposed and is now under preparation for submitting a journal article with the details. Furthermore, regarding the soot generation model, more detailed studies including evaluation of the surface area of soot are necessary in the future.

4. A Study on the Coupling Effect of Convective Heating and Radiative Heating

The experiment targeted in the analysis by Naka et al. [20] was a combustion experiment conducted by Messineo et al. [26] using paraffin wax fuel and a relatively low flow rate of LOx. From the experimental and calculated results presented in Ref. [20], the time history of O/F (Fig. 2) and the spatial-averaged heating rate (Fig. 3) are cited here for further discussion.

What is interesting here is that, as far as the experiments of Messineo et al. [26] are concerned, the O/F decreases with time as the combustion progresses, and this trend is confirmed both by a reconstruction of experimental data and by the numerical analysis as shown

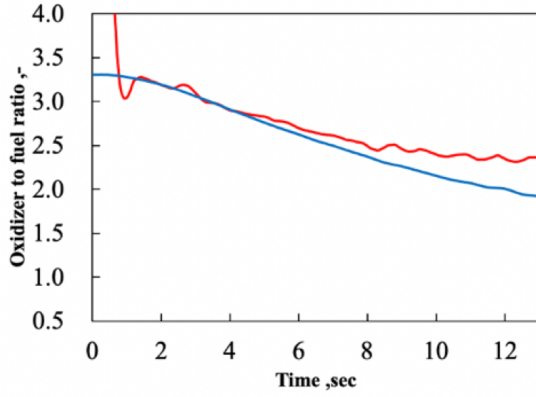in Fig.2. Also, in this experiment, since the oxidizer mass

Fig. 2 Numerical result and the reconstruction result of the time history of the oxidizer-to-fuel ratio. (Blue: numerical simulation, Red: reconstruction from experiment) [20].
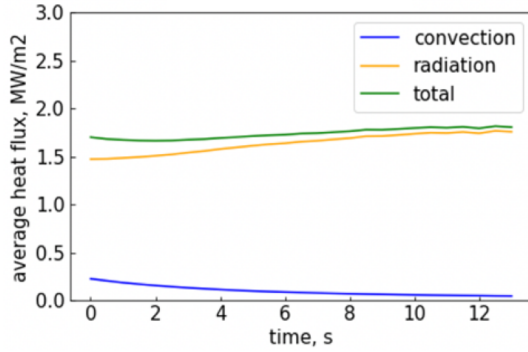

Fig. 3 Numerical result of the history of spatial-averaged heat flux over fuel surface (blue: convection, yellow: radiation, green: total) [20].

flux is relatively low, the radiation heating is dominant, and the convective heating rate is decreasing with time while the radiative heating increases as shown in Fig.3.

There are at least two ways to explain the decay of the O/F happened in the experiment. One is to consider the form of fuel regression rate as $\dot{r} = aG^n$ and explain it with $n < 0.5$. That is, in this case, the O/F is expressed as

$$O/F \propto D^{2n-1}, \tag{6}$$

where D is the port diameter. So, if $n < 0.5$, then O/F decreases with time. It is true that in traditional practice this is often an interpretation of the phenomena. But are the findings and prospects from this approach good?

Here, we consider another method. It is a method explained by the increase of the ratio of radiative heating to convective heating over time. It is a background philosophy that the fuel regression rate (or heating rate) is well-described by the power of the mass flux when the

convective heat transfer is physically dominant. If there is other physical processes' dominance, it would be more predictable to evaluate the fuel regression rate by other indicators. A case where radiation is dominant is such a situation. It should be noted that the effect of the injector, which is often referred to an n-index modifier, need not be considered here. This is because the numerical calculation assumes that the oxidizer enters in parallel into the chamber, and it is shown that the O/F decreases over time as is in the reconstruction of experimental data.

Now, writing only the result, when the fuel regression rate is expressed by Eq. (1) considering the coupling of radiation and convection, the O/F can be written as follows.

$$O/F \propto \frac{D^{0.6}}{f(\xi)} \tag{7}$$

$$f(\xi) \equiv e^{-\xi} + \xi \tag{8}$$

$$\xi \equiv \frac{\dot{Q}_r}{\dot{Q}_c^*} \tag{9}$$

In this case, in order for the O/F to decrease over time, that is, $\frac{dO/F}{dt} < 0$,

$$\frac{d\ln f}{dt} > 0.6 \frac{d\ln D}{dt} \tag{10}$$

must be satisfied. For the fuel regression rate $\dot{r}$, there is a relationship of $\frac{dD}{dt} = 2\dot{r}$, so Eq. (10) is expressed as,

$$\frac{d\ln f}{dt} > \frac{1.2\,\dot{r}}{D}. \tag{11}$$

Rewriting the left side of Eq. (11) using the ratio of radiative heating to effective convective heating,

$$\eta \equiv \frac{\dot{Q}_r}{\dot{Q}_c} = \xi e^{\xi}, \tag{12}$$

one obtains,

$$\frac{d\ln f}{dt} = \frac{-1+e^{\xi}}{1+e^{\xi}\xi} \frac{d\xi}{dt} = \frac{1-e^{-\xi}}{(1+e^{\xi}\xi)(1+\xi)} \frac{d\eta}{dt}. \tag{13}$$

By performing a little calculation using the definitions of f and η, one can finally obtain the following relationship,

$$\frac{d\eta}{dt} > \frac{1.2(1+\xi e^{\xi})(1+\xi)}{1-e^{-\xi}} \frac{\dot{r}}{D} \equiv \dot{\eta}_{Bound}. \tag{14}$$

Or, rewriting Eq. (14) for the relative rate of change of η, we get

$$\frac{d\ln\eta}{dt} > \frac{1.2(1+\xi e^{\xi})(1+\xi)}{\xi(e^{\xi}-1)} \frac{\dot{r}}{D} \equiv e_{\dot{\eta}_{Bound}}. \tag{15}$$

This equation means that the relative time change rate of the radiation-convection heating rate ratio must not fall below the lower limit $e_{\dot{\eta}_{Bound}}$ in order for the O/F to decrease over time.

Keeping in mind that ξ and η have a one-to-one relationship with $\eta = \xi e^{\xi}$, by using Eq. (15), the contour lines of $e_{\dot{\eta}_{Bound}}$ are drawn on a plane $(\eta, \dot{r}/D)$ as shown in Fig. 4.

In the experiment by Messineo et al., $\dot{r}/D$ is about 0.02 [1/s] and η is about 10, and Fig. 4 shows that
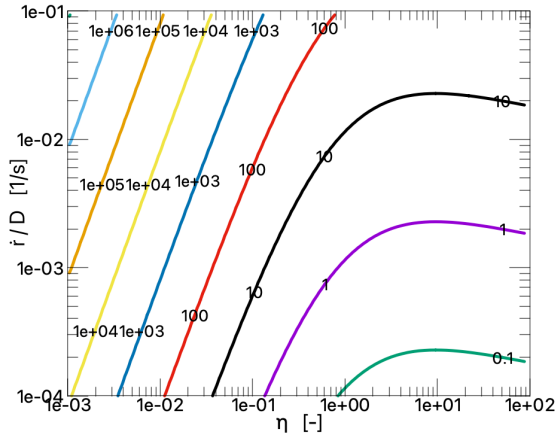
Fig. 4 Contours of the lowest values of the relative temporal change [%/s] of the radiative-to-convective ratio, $e_{\dot{\eta}_{Bound}}$, projected on $(\eta, \dot{r}/D)$ plane.

$e_{\dot{\eta}_{Bound}} \approx 10[\%/s]$ in that case. In the 10 seconds of burning period, η became 5 times (500%) easily, which corresponds to an average rate of 50 [%/s]. That is, it is confirmed that the conditions for reducing O/F were sufficiently satisfied.

From Fig.4, we can see various other things. For example, when radiative heating is sufficiently smaller than convective heating, the lower limit of the relative rate of change of η is extremely high for any port diameter, which is extremely difficult to satisfy. That is, the O/F does not decrease over time.

Even if the fuel regression rate is the same, if the port diameter is relatively large, the lower limit of the relative change rate becomes lower for the same radiative to convective ratio, resulting in a tendency of temporal decrease in O/F.

Further, from Eq. (15), it is true that lower limit of the relative time change rate is always positive. Therefore, when the radiative heating rate decreases with time relative to the convection heating rate, the O/F does not decrease over time.

As mentioned above, such an explanation may be possible as a reason why the O/F is decreasing with time. In this way, by correctly bringing radiative heating into the evaluation of internal ballistics of hybrid rocket, one can reach a viewpoint different from the conventional one.

5. Concluding Remarks

Some major challenges of conventional models for the internal ballistics of hybrid rockets are outlined, especially when the heating rate dominates the fuel regression rate. Those are 1) the treatment of the effect of increasing Stanton number due to the acceleration effect of the turbulent boundary layer combustion with mass addition, 2) the detailing of the soot generation model, and 3) the interference between the convection and the radiation. Future improvement of these models is desired.

As a result of examining the conditions for the O/F to decrease with time using a fuel regression rate model with convection-radiation interaction, a positive value of the lower limit of the relative time change rate of the radiation to convection heating has been obtained as a function of the ratio of the regression rate to the port diameter and the radiation-to-convection heating rate ratio. Some new findings could be obtained in this way, and all of these are problems in which flow dynamics, chemical reaction, and radiation are combined, so many efforts for detailed research of both numerical analysis and experiments should be encouraged.

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