Proceedings of the SICE Annual Conference 2022
September 6-9, 2022, Kumamoto, Japan

Mathematical Representation of Decision-making Process in Forex Trading with Chaotic Behavior

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Abstract: The individual differences of the decision-making process in gold trading have already been represented reasonably by constructing a mathematical model. In forex trading, the representation of the process has not succussed yet, because forex trading is more complex than gold trading such as foreign exchange rate has chaotic behavior, whereas gold price does not have. In this research, the representation of the decision-making process in forex trading was tried by identifying a model, which has the proposal, integrator, and derivative elements, same as the representation in gold trading.

Each elemental coefficient can be regarded as the parameters reflecting the individual characteristics of the process. They were identified individually using the data of the foreign exchange rate and the trading amount of 18 persons, obtained in a created forex trading game. The identified parameters were compared with those in gold trading to examine the effect of the chaotic behavior in foreign exchange rate. In the result, they were almost converged respectively within 10 transactions, however their median values were distributed in wider range than that in gold trading. This convergence can be regarded as the response reflecting human's learning function in more complex forex trading, and such explanation is consistent with the results obtained from current psychological experiments. Thus, the decision-making process in forex trading could be basically represented by our model.

Keywords: decision-making process, mathematical model, forex trading, chaos.

1. BACKGROUND AND PURPOSE

Nowadays, the human decision-making process and psychological behavioral has being much researched mainly using psychological evaluation methods, such as magnitude estimation method [1][2]. However, these psychological evaluation methods have some problem because of their high cost and troublesome way to examination. In addition, especially in the situation with chaotic input, psychological approach is so difficult to identify the human decision-making process clearly, which are almost unknown yet.

Therefore, the decision-making process is being researched recently by using mathematical models of fuzzy inference [3] or structural equation modeling [4]. These models are also used to improve the education system for freshman or students, because they can be useful for understanding and examining the process more clearly, logically, and simply [3]. However, as most of the research using these models often requires the results of psychological evaluation tests for constructing the models, such as multiple measurements of emotion [5], there are not many chances where the models can be used.

In our laboratory, the decision-making process in gold trading was explained clearly using the original mathematical model using transfer function [6]. This model has a simple mathematical structure, and the individual characteristics of the decision-making process in gold trading can be analyzed by identifying coefficient parameters of the model using the data obtained not by psychological evaluation tests but by behavioral tests. Therefore, the presented model has potential to use easily for analyzing various decision-making process.

In these days, the decision-making process of human is known that related to the trend of subject of deciding [1] or gender [10]. Thus, decision-making process could be changed if subject of trading is changed. Forex trading is more complex than gold trading because foreign exchange rate could have chaotic behavior whereas gold price does not behave chaotically at all. However, the research about chaotic analysis focused on forex trading has not been done as possible as authors know. Thus, to compare the degree of chaos in input data, Lyapunov exponent which is the easiest and most popular way to represent chaotic degree was used.

In this research, the decision-making process in forex trading is represented by a mathematical model including proposal, integrator, and derivative elements, of which structure is the same as that for gold trading. And, for basic validation, the model parameters are identified and compared between in forex trading in gold trading to examine the effect of the input with chaotic behavior. In addition, the preliminary analysis of gender differences in decision-making process performed using the identified parameters.

† Mana Yabuki is the presenter of this paper.

2. CONSTRUCTION OF MODEL

In this research, the mathematical model of the decision-making process in forex trading was constructed with the model structure as shown in Fig. 1 [6] based on the hypothesis that the decision-making process consists of each response to the present, the accumulated, and the trend information.

The constructed model has the proportional, the integrator, and the derivative elements in parallel, which are corresponding to the responses to the present, the accumulated, and the trend information, respectively. And the constants *Kp*, *Ki*, and *Kd* are the coefficients of

the elements, whose differences can represent the individual differences in the decision-making process.

The input variable $u(t)$ in the model is the present forex price and the output variable $y(t)$ is the decided trading amount of forex.

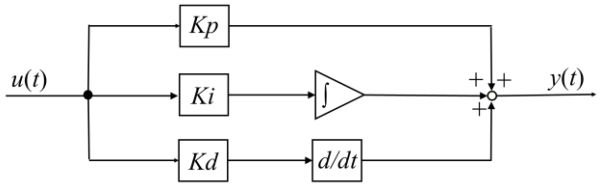

Fig. 1 Model structure of the decision-making process consisting of the proportional, the integrator, and the derivative elements.

The input-output relation of the model in Fig. 1 is shown by the equation (1) as continuous-time system, however, in this research, the discretization of this relation is required for the identification in the forex trading game as described later.

$$y(t) = Kd\frac{du(t)}{dt} + Kp\cdot u(t) + Ki\int_0^t u(\tau)d\tau \tag{1}$$

The equations (2) and (3) are those discretized the equation (1) by the backward difference method, respectively. Where T represents the sampling period.

$$y(i) = Kd\frac{u(i)-u(i-1)}{T} + Kp\cdot u(i) + Ki\sum_{j=0}^{i}\{u(j)\cdot T\} \tag{2}$$

$$y(i-1) = Kd\frac{u(i-1)-u(i-2)}{T} + Kp\cdot u(i-1) + Ki\sum_{j=0}^{i-1}\{u(j)\cdot T\} \tag{3}$$

Thus, the equation (4) is obtained by subtracting equation (3) from equation (2). Where 1 was assigned to T because play's number of times was used instead of time in the forex trading game.

$$\begin{aligned} y(i) = {} & y(i-1) + (Kp + Ki + Kd)\cdot u(i) \\ & -(Kp + 2Kd)\cdot u(i-1) + Kd\cdot u(i-2) \end{aligned} \tag{4}$$

3. DETERMINATION OF CHAOS

In this research, Lyapunov exponent was used to assess the strength of the chaos included in forex trading, because it is easiest and popular indicator of the strength of chaos.

Lyapunov exponent λ in an one-dimensional discrete-time dynamical system can generally be calculated using the equation (5), (6) and (7) [7]. Where, f represents a function or mapping which has the input variable x, and δ is infinitesimal changes of x.

$$\lambda = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n-1}\log_e\left|f'(x_i)\right| \tag{5}$$

$$f'(x_i) = \frac{f(x_i+\delta)-f(x_i)}{f(x_{i-1}+\delta)-f(x_{i-1})} \tag{6}$$

$$f'(x_1) = \frac{f(x_1+\delta)-f(x_1)}{\delta} \tag{7}$$

In this research, f in the above equations can be regarded as the social function which decides the value of forex from its trading amount in various places. In this case, its value and x correspond to $u(i)$ and $y(i)$ in the constructed model. However, x and δ in real data are unknown and cannot be assigned to these equations. Therefore, Lyapunov exponent in forex trading was calculated using the equation (8), In this equation, d correspond to the function value of f, that is the rate of forex.

$$\lambda = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n-2}\log_e\left|\frac{d_{i+2}-d_{i+1}}{d_{i+1}-d_i}\right| \tag{8}$$

4. SYSTEM IDENTIFICATION

Each coefficient of Kp, Ki, and Kd, shown in Fig. 1, were identified by the least-square method using the input and output data, which were obtained in a forex trading computer game created in our laboratory using C language. The data of foreign exchange rate used in the game were synthesized using the real data from October in 2020 to April in2021.

In the created game, ten sets are in one game, and the fictitious forex trading is weekly played for twelve weeks every set. In each trading, the players look the price of each financial product represented on the screen and decide and input the trading amount of forex after checking the forex price shown on the display. As the identification of the coefficients is performed every set, the ten identified results are obtained per one game by solving the simultaneous equation 7.

$$\frac{\partial J}{\partial \boldsymbol{a}} = 0 \tag{9}$$

Where J and $\boldsymbol{a}$ in the equation 9 are the variable which is the sum of the squares of the output errors and the vector of the coefficient parameters as shown by the equation (10) and (11).

$$J = \sum_{i=2}^{N}\left\{\begin{aligned} & y(i)-y(i-1)-(Kp+Ki+Kd)\cdot u(i) \\ & +(Kp+2Kd)\cdot u(i-1)-Kd\cdot u(i-2) \end{aligned}\right\}^2 \tag{10}$$

$$\boldsymbol{a} = \begin{bmatrix} Kp & Ki & Kd \end{bmatrix}^T \tag{11}$$

The created games of forex trading and gold trading were played by the volunteers aged 19 to 26 (male: 11, female: 7), who were ethically approved as the experimental subjects in Tokai University, under the condition which game is played was chosen at random. The players could decide the amount of trading and input their decision to the computer, however were not

informed which game they play, and how many weeks they play in one set, before starting the game. Also, the trading amount was not limited in the game. Before and after playing game, the players answered some questions about the multiple mood scale in order to validate the reasonability of the obtained input-output data. This multiple mood scale is one of method for psychologically measuring human's emotional condition.

The coefficients of *Kp*, *Ki*, and *Kd* in the decision-makings not only in forex trading but also in gold trading were identified by the method as described previously, for comparing both decision-making process. In addition, the data of gold price used in the game were synthesized using the real data from January in 2018 to April in 2021.

5. RESULTS

The variations of the foreign exchange rate and the gold price in the created trading games were shown in Fig.2. The foreign exchange rate fluctuated more heavily than the gold price. This characteristic in the variation of the rate and price is similar to that in the actual trading of forex and gold.

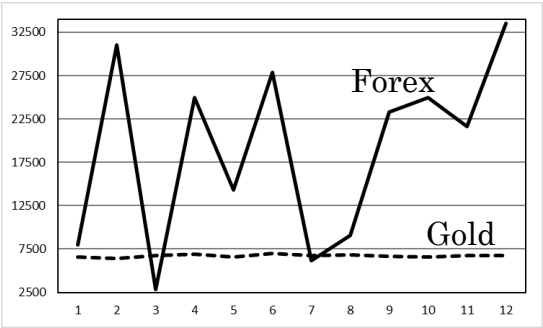

Fig.2 variation of the rate in each trading game

Each Lyapunov exponent of the variations of foreign exchange rate and gold price in the created game and the real market were shown by the box plots in Fig. 3. The length of boxes and the line in the boxes show the quartile deviation and the median value of Lyapunov exponents calculated by the equation (8). The variations of Lyapunov exponent in the games was lower than those in real market, and the Lyapunov exponent in forex trading game was higher than that in gold trading game.

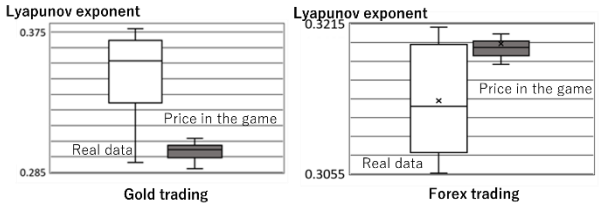

Fig. 3 Box plots of Lyapunov exponent in gold trading and forex trading.

In this research, as the trading amount was not limited, the distribution of *Kp* value was especially widespread. For that reason, *Ki* and *Kd* were standardized by the method of dividing by the absolute value of *Kp*. The distributions of one player's *Ki*/*Kp* averaged per set were illustrate by an example in the box plots of Fig. 4, and had wider quartile deviation in forex trading game. Most players had the similar trend except several players.

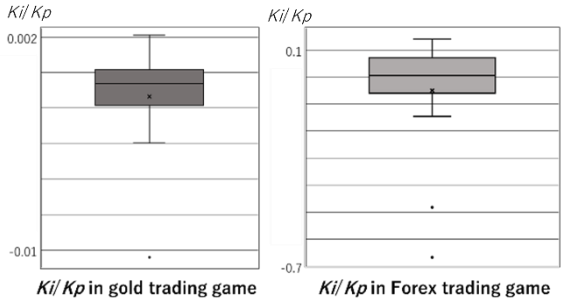

Fig. 4 Box plots of the distribution of one player's *Ki*/*Kp* averaged per set.

Table 1 indicates the descriptive statistic values about the distribution of *Ki*/*Kp* averaged per set in all players. The *Ki*/*Kp* in forex trading game distributed in around 25 times wider quartile deviation than those in gold trading game, also in this table.

Table 1 Descriptive statistics values of the distribution of averaged *Ki*/*Kp* in all players.

	Gold Trading Game	Forex Trading Game
Maximum Value	0.0116	0.0890
75 Percentile	0.0013	0.0622
Median	-0.035	0.0100
25 Percentile	-0.0034	-0.0545
Minimum Value	-0.0709	-0.6669

Similarly, the distributions of one player's *Kd*/*Kp* averaged per set were illustrate by an example in the box plots of Fig. 5, and the descriptive statistic values about the distribution of *Kd*/*Kp* in all players were indicated in Table 2. The quartile deviation were more obviously wider in forex trading game than in gold trading game in both an intrapersonal comparison illustrated by the figure and an intragroup comparison shown by the table. Also, in gold trading game, *Ki*/*Kp* had wider quartile deviations than *Kd*/*Kp* in both the intrapersonal distribution and the intragroup distribution, however such significant difference was not observed in forex trading game.

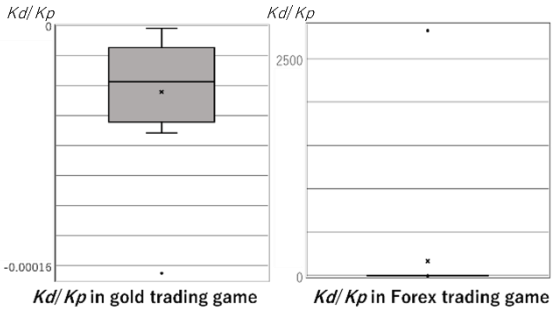

Fig. 5 Box plots of the distribution of one player's *Kd*/*Kp* averaged per set.

Table 2 Descriptive statistics values of the distribution of averaged *Kd*/*Kp* in all players.

	Gold Trading Game	Forex Trading Game
Maximum Value	0.0003	2821.9926
75 Percentile	0.000005	-0.0037
Median	-0.000002	-0.0110
25 Percentile	-0.000001	−0.0804
Minimum Value	-0.0014	-2.8361

Because integration and differentiation are completely opposite mathematical operations, it is reasonable to illustrate individual characteristics in the decision-making process as vectors on Cartesian coordinates, where the horizontal and vertical axes represent the ratio of *Kp* to the absolute value of *Kp* and the difference between *Ki*/*Kp* and *Kd*/*Kp*, respectively. In this paper, the characteristics of the player numbered 3 in forex trading and gold trading were shown in Figure 6 as the example of the display by the vectors, where the number 1 to 10 in the figure are represented the number of the set of game.

In the case of the player numbered 3, the recall ratio of each set of forex trading game was 20%, whereas 90% in gold trading game. This difference indicates that the decision-making process in forex trading game was less stable than gold trading game in this player, who answered after the game that the forex trading game was more difficult to decide the rule to trade during the game. The result that the difference of recall ratios is consistent to difficulty to decide the rule were observed in other players.

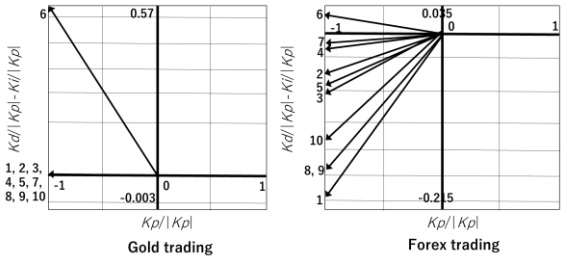

Fig. 6 Characteristics of the decision-making process of the player numbered 3, identified in gold trading game and forex trading.

In the case of the player numbered 4, the forex trading was more difficult to decide the rule to trade as is with the case the player of number 3, which was reflected in the characteristics of the decision-making in both games shown in Fig. 7.

The common point in above 2 examples is that the declination angle of the vectors shown in the Fig. 6 and 7 did not converge in forex trading whereas those converged in gold trading.

On the other hand, the player numbered 5 answered that the forex trading was rather easier than gold trading. Vectors' declination angle of this player did not converge in both games as shown in Fig. 8, and the recall ratio were 70% in gold trading game and 30% in forex trading game.

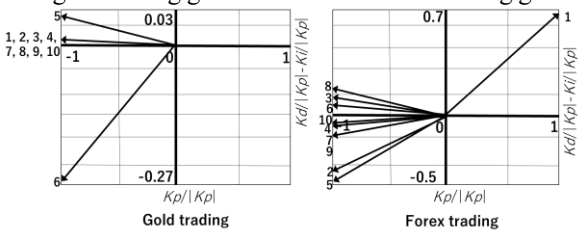

Fig. 7 Characteristics of the decision-making process of the player numbered 4, identified in gold trading game and forex trading game.

This result was not consistent to the difficulty this player felt, and rather was not conflicting to the fact that forex trading is more complex and difficult than gold trading.

The average recall ratios of all players were 31.7% in gold trading game and 25.6% in forex trading. And the players who could explain the decision-making process of one's own in trading were twelve in forex trading game and sixteen in gold trading.

Therefore, the difference of not only the convergent degree of model parameters' vectors but also the recall ratios can indicate the difficulty of trading. That is these results indicate that forex trading is more difficult than gold trading, and this is not conflict with previous knowledge.

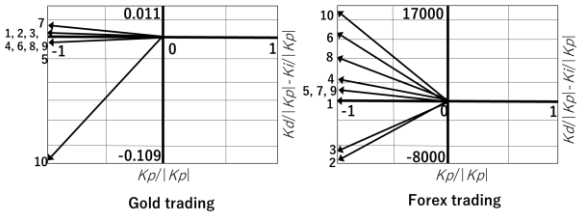

Fig. 8 Characteristics of the decision-making process of the player numbered 5, identified in gold trading game and forex trading game.

The gender difference of *Ki*/*Kp* and *Kd*/*Kp* in each game were shown by the box plots in Fig. 9 and 10. In gold trading, the median values of *Ki*/*Kp* and *Kd*/*Kp* of the male players were -0.00096 and -0.00045, and those of female players were -0.00038 and -0.00017. On the other hand, in forex trading, those of male were 0.05630 and -0.00827, whereas of female were -0.03041 and -0.01897.

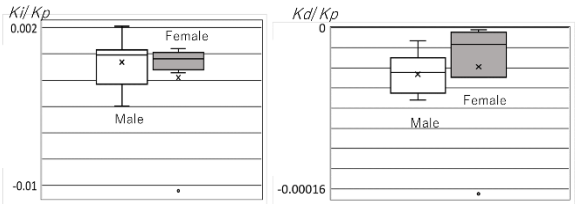

Fig. 9 Box plot of gender differences of *Ki*/*Kp* and *Kd*/*Kp* in gold trading game.

In gold trading game, *Ki*/*Kp*'s distribution of the male

players was more widespread and *Kd*/*Kp*‘s distribution was less widespread than those of female players.

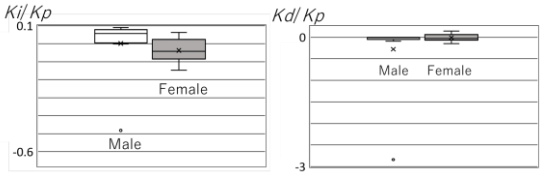

Fig. 10 Box plot of gender differences of *Ki*/*Kp* and *Kd*/*Kp* in forex trading game.

However, in forex trading game, the distributions of both parameters were more widespread in female than in male in reverse. The distribution of *Ki*/*Kp* and *Kd*/*Kp* were both not different significantly between male and female.

6. DISCUSSION

This research’s purpose focused to represent psychological function, decision-making, mathematically, and the psychological method such as tests and/or experiments was not used. However, for validating the constructed model, the calculated results in the model have to be checked from the results of psychological experiments in the future. In this paper, instead of such validation, the characteristics of the decision-making process in forex trading were compared with those in gold trading. The reasons why this comparison is effective for validating the model are as following.

First, both forex trading and gold trading have quantified input and output variables, such as foreign exchange rate and trading amount. A mathematical model such as our model requires such quantified input and output variables for calculation. However, it has less chance to obtain the quantified data available for the model in daily living. Therefore, the experiment of financial trading is usable for obtaining the input and output data, as this research.

The next reason is that forex trading is more complex than gold trading because of its chaotic behavior. This feature is usable for verifying the model performance when the model parameters are identified as the indicators that forex trading is more difficult. Therefore, in this paper, the fact that forex trading has chaos was verified by calculating Lyapunov exponents, and the standardized parameters *Ki*/*Kp* and *Kd*/*Kp* were identified as they indicated that forex trading was more difficult than gold trading.

Psychological experiments are necessary for more precise validation of the constructed model in the future, however at least the model has be partially validated at the point of the success to represent the difference of the difficulty between forex trading and gold trading.

In this research, the vector representing the characteristics of decision-making was invented and the convergence of vector's declination angle was analyzed. This invention is consider to enable the analysis of individual learning function. The reason is that the angle must converge more rapidly when a players learn the adequate rule of trading more during the game. Thus, the result that the recall ratio in forex trading was lower than that in gold trading game can show that most players could not learned the adequate rule of forex trading during the game. Considering the foreign exchange rate is more chaotic than gold price as shown in Fig. 2 and 3, the result concerned with the convergence of vector's declination angle indicates that learning the rule in forex trading is more difficult than in gold trading due to its chaos. This is consistent with the natural fact that the decision-making with chaotic input is more difficult than with non-chaotic input.

After the game, some players answered the difficulty of deciding the trading amount in forex trading of which the rate behaved chaotically. However, the result of the recall ratio was not always consistent with player’s answers. In this research, the players of the created games of forex and gold trading were only eighteen people. Thus, as the result of the relation between the answers after the game and the feeling during the game is limited, more experiments using trading game are required to discuss it more statistically, including the relation between the multiple mood scale and Lyapunov exponent.

As shown in Fig. 10, the absolute values of *Ki*/*Kp*‘s median value was larger in the female players than in the male players, in forex trading. Also, *Ki*/*Kp*‘s median value in the female players was almost equal or slightly less, however *Ki*/*Kp*‘s quartile deviation was obviously narrower in gold trading, as shown by Fig. 9. The gender difference such as these results must be as the check point for model validation.

In recent years, the gender difference of decision-making process was examined using psychological experimental method such as Iowa gambling task [10], and the fact that female consider past information more than male is revealed from such experiments. Thus, the analysis of gender difference using our model can be usable for model validation by comparing with such experimental knowledge. However, as the female players were only seven in this research especially, the additional experiment has to be performed particularly in female volunteers to verify gender difference more precisely and discuss its usability for model validation.

The Lyapunov exponent of the input data in the trading game is different to that of real data. Thus, the created games cannot simulate the real trading but can be useful to obtain the input and output data used for identifying the coefficient parameters. Especially, as this method of obtaining necessary data has the merit that the experimental conditions are easier under the control by experimenters, it is expectable for applying to basic psychological experiments and tests, which are often affected by various known and unknown factors.

Additionally, Lyapunov exponent is the easiest indicator for analyzing chaotic behavior. However, as it can be impossible to make a complete judgment with Lyapunov exponent alone [7][8]. Therefore, the analysis of chaotic behavior in input data using other indicator is one of the next challenges in this research.

From the above discussion, the constructed simple model consisting of only 3 elements is potentially useful for representing and analyzing the decision-making

process, regardless of that psychological process is complex with many unknown factors.

7. CONCLUSION

The simple model which consists of the proportional, the integrator, and the derivative elements was constructed, and its parameters were identified using the data obtained by the games of forex trading and gold trading. And the identified parameters were compared between in forex trading and in gold trading, and between of male and of female for basically validation of the constructed model. The conclusion in this step is that our model is potentially useful for representing and analyzing the decision-making process in forex trading with chaotic behavior.

The next challenge is the mathematical representation of the learning process for adequate trading rule, and analysis of the individual difference in learning process. For that, the additional element representing learning function is being constructed.

APPENDIX

This research was under the approval by the ethical committee for human experimental subjects at Tokai University, especially in accordance with Japanese research guidelines on Novel Coronavirus (COVID19).

ACNOWLEDGEMENT

We appreciate to all experimental volunteers, who were students in school of engineering and graduate school of engineering at Tokai university.

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