Theoretical analysis of ion concentration changes in astrocyte caused by Na^+,K^+-ATPase and NKCC1

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***Abstract*— The amounts of water flow into astrocyte are controlled by some transporters on cell membrane. In this research, the mathematical model representing the ion concentration changes caused by Na,K-ATPase and Na-K-2Cl Cotransporter 1 was constructed and analyzed. In the result, the intracellular and extracellular ion concentrations reasonably changed within the physiological range and those values converged all under the constant conditions. Thus, this model can be expected to simulate the water flow into astrocyte by its extension considering other concerned transporters.**

I. INTRODUCTION

Cytotoxic cerebral edema is mainly caused by the excess water flow into astrocytes due to the dysfunction of ion transporters including Na,K-ATPase and Na-K-2Cl Cotransporter 1 (NKCC1) [1]. The construction of the mathematical model representing the changes of ion concentrations caused by the function of ion transporters must enables to analyze the change of the amount of intracellular water and to simulate the cytotoxic cerebral edema.

In this research, the mathematical model representing the functions of Na,K-ATPase and NKCC1 were constructed and analyzed.

II. CONSTRUCTION OF MODEL

The transportation of Na^+ and K^+ by Na,K-ATPase can be numerically analyzed using the mathematical model representing its conformation change [2].

The transportation rates of Na^+, K^+, and Cl^- by NKCC1 can be calculated by Nernst-Plank equation deformed as follow. Where, the below equation is the example of calculating the transportation rates of Na+, and D, c, Z, F, R, T, and Em are diffusion constant, ion concentration, valence of ion, faraday constant, gas constant, absolute temperature, and membrane potential. The membrane potential can be estimated as its optimal value by steepest descent method.

$$J_{ca}[\mathrm{mol/s}] = -\mathrm{D}\{(c_i - c_o) + \frac{\mathrm{ZF}}{\mathrm{RT}} dE_m\} \quad (1)$$

The amount that added the amount of ion transport by other transporters to the flux by Na,K-ATPase and NKCC1 are the change rate of the intracellular ion concentration. In this way, the simultaneous differential equations can be formulated, which were analyzed by Runge-Kutta method.

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III. RESULTS AND DISCUSSION

The initial value of each ion's intracellular and extracellular concentration was respectively set at physiologically lower value and normal value, for analyzing its transient phenomenon. The analysis results were shown in Fig 1.

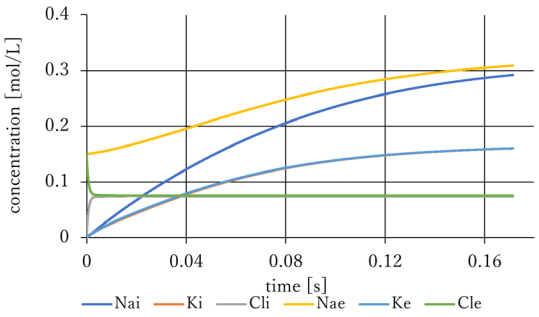

Figure 1. Change of each ion's intracellular and extracellular concentration.

The intracellular and extracellular concentrations of Na^+, K^+, and Cl^- converged all within 0.18 [s]. The final value of Na^+'s intracellular concentration (Nai) was lower than its extracellular concentration (Nae), which was physiologically natural result.

As the present model was constructed based on physical and physiological laws, there should be no essential problems in its structure. And the ion concentrations could be calculated using this model without divergence. However, the model parameters are not always set all to reasonable values, because most of them are unknown yet. Therefore, the investigation or estimation of each parameter's reasonable value is next challenge. In addition, the extension of this model by the addition the function of remaining transporters is also necessary for final validation the model.

IV. CONCLUSION

In this research, the mathematical model representing the functions of Na,K-ATPase and NKCC1 were constructed and the change of each ion's intracellular and extracellular concentration was partially analyzed by using the constructed model.

REFERENCES

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