Two Accounts of Japanese Numerals

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Abstract

If we adopt a plural logical framework for a natural language semantics, a simple numeral like "three" can be regarded as a plural predicate that may be true of a number of individuals. But not all numerals are simple ones like "three". There are also complex numerals, and they should be given a compositional semantic account.

In this paper, two ways of giving a compositional account of Japanese numerals are examined. According to one, a numeral is a singular term that refers to a natural number, which is an abstract object, while according to the other it is a plural predicate which may be of any higher level. We argue that the latter account should be preferred because it does not induce an unrequired ontological commitment.

1 Introduction

It was once a common wisdom that number could not be a property of things like color and shape. The recent development of plural logic[1], however, has shown that there is no need to be afraid of going against this old wisdom. A number predication could be just an ordinary predication if predication is construed as essentially plural (or better, number-neutral). After all, such a construal is obligatory owing to the pervasiveness of plural phenomena in a natural language.

If we adopt a plural logical framework, then a simple numeral like "three" can be regarded as a plural predicate that may be true of a number of individuals. As it is well known, it can be even analyzed to be a logical predicate. But not all numerals are simple ones like "three"; there are complex numerals like "two hundred and fifty-three" as well. It is obvious that semantics of a numeral must be compositional so that it systematically explains how the semantic value of a complex numeral is determined by those of its components.

In this paper, we are going to examine two ways of giving such a compositional account of Japanese numerals. They differ on whether we commit to the existence of natural numbers in our metalanguage or not. On one hand,

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[1]See [McKay 2006], [Oliver and Smiley 2013], [Yi 2005] and [Yi 2006].

admitting natural numbers as individuals in our ontology makes the presentation of semantics of numeral NPs much simpler, but we may argue that such a commitment is undesirable and unnecessary. On the other, dispensing with natural numbers may be well motivated, but the resulting semantics appears to be a much more complicated affair. I will argue, however, that this appearance is a deceptive one, and that the second account is much better than it seems. One byproduct of the discussion is that it may lead to a recognition of the fact that higher-level plural predication is not so uncommon in a natural language as it is sometimes thought.

There are two reasons for taking up Japanese numerals besides the fact that Japanese is the language I understand best. First, in common with other classifier languages, Japanese is essentially a number-neutral language, and plural logic gives us a particularly good framework for its semantics; hence, a way should be found to give semantics of numerals in this framework. Secondly, Japanese has a very perspicuous system of numerals which easily yields to a compositional treatment; this can be seen from the fact that one introductory book on semantics presents an account of Japanese numerals as the first exercise in a formal semantics[2].

2 Syntax

Japanese numerals[3] are built from nine simple numerals

ichi,	*ni*,	*san*,	*shi*/*yon*,	*go*,	*roku*,	*shichi*/*nana*
one	two	three	four	five	six	seven
hachi,	*kyuu*,					
eight	nine					

and names for powers of ten

jyuu,	*hyaku*,	*sen*,	*man*,	*oku*,	...
ten	hundred	thousand	ten thousand	hundred million	
10^1	10^2	10^3	10^4	10^8	

As $10^0 = 1$, we count also *ichi* (one) as one of the names for powers of ten. We suppose that they are ordered in the order just shown starting from *ichi*. Namely,

$$ichi < jyuu < hyaku < \ldots < oku < \ldots$$

[2]See [Löbner 2002]. pp.211–214. In fact, it is a very limited account and deals with only the numerals for numbers less than 100. The semantic account of Japanese numerals given in §3 is an extension of this limited one.

[3]Japanese numerals are based on the Chinese ones, but there are two readings for most of them, one Chinese and the other native Japanese. In this paper, with some exceptions, a numeral will be given a Chinese reading. *Yon* for four and *nana* for seven are these exceptions.

We define a numeral of order δ by induction on $<$.

Definition: a numeral of order δ

1. A numeral of order *ichi* is one of the simple numerals *ichi*, *ni*, ..., *kyuu*. These are the only numerals of order *ichi*.
2. Let δ be one of the names for powers of ten. Suppose that numerals of order ϵ such that $\epsilon < \delta$ are defined. Let ν be a numeral of order ϵ such that $\epsilon < \delta$. Then,
   i. "$\nu\ \delta$" (concatenation of ν and δ) are numerals of order δ, and called "multiples of δ"; there are, however, two provisos:
      a. *Jyuu*, *hyaku* and *sen* are by themselves numerals of order *jyuu*, *hyaku* and *sen* respectively; in the case of *jyuu*, a concatenated form *ichi jyuu* is not allowed[4];
      b. for *hyaku* and *sen*, the preceding numeral ν must be a simple numeral (for *jyuu*, it is obvious that ν must be a simple numeral);
   ii. if δ' is a multiple of δ, then δ' itself and "$\delta'\ \nu$" are numerals of order δ;
   iii. only those which are known to be numerals of order δ from the above are numerals of order δ.

∎

Then, a numeral can be defined by the following[5].

Definition: a numeral

A Japanese expression α is a numeral $\leftrightarrow$ for some name δ for powers of ten, α is a numeral of order δ. ∎

You must have noticed that there are some irregularities in the simple construction of a Japanese numeral. It occurs in 2–i, namely, the exceptional clauses

[4] For numerals of order *man* and greater, a numeral may start like *ichi hyaku man* or *ichi sen man*, but numerals of order *hyaku* or *sen*, *ichi hyaku* and *ichi sen* are not common, though they are used sometimes.

[5] By this definition, expressions like the following are rightly judged not to be numerals.

(i) *jyuu* *shi* *go*
ten four five

(ii) *ni* *san* *jyuu*
two three ten

They do not signify single numbers; (i) means "about 14 or 15" and (ii) "from around 20 to around 30". Such expressions can be easily recognized because they contain a sequence of consecutive numerals like *shi go* or *ni san*, which never occurs in numerals.

about *jyuu*, *hyaku* and *sen*. Although there is nothing to prevent to form a numeral like

(a)	**jyuu*	*san*	*hyaku*,
	ten	three	hundred

(b)	**hyaku*	*ni*	*jyuu*	*sen*,
	hundred	two	ten	thousand

(c)	**ichi*	*jyuu*,
	one	ten

(a) and (b) are not allowed because of the second proviso to 2–i. It is similar to the fact that in English you should say "one thousand three hundred" instead of "thirteen hundred" but that you could say "one hundred twenty thousand". Considering the way a complex numeral is formed from the numerals of order *man*, there would not be anything wrong with (a) and (b). It might be even argued that the numerals of order *sen* could be all replaced by those of order *hyaku* if the expressions like (a) and (b) were allowed, and that the necessary names for powers of ten should run as follows:

ichi,	*jyuu*,	*hyaku*,	*man*,	*oku*.	...
one	ten	hundred	ten thousand	hundred million	
10^0	10^1	10^2	10^4	10^8	

However, there must be a good reason to have a proviso that forbids expressions like (a) and (b); it comes from some pragmatic consideration to lighten a load of calculating in the cases of "smaller large" numerals, and I expect similar phenomena to be found in many languages.

On the other hand, (c) is not allowed because of the first proviso to 2–i. In general, names for powers of ten cannot be a numeral by themselves; they must be preceded by some numeral of the smaller order. But *jyuu*, *hyaku*, *sen* are exceptions to this. This time again, the same sort of pragmatic considerations seem to be at work here. If there is no danger of ambiguity, it is better to have a more concise expression.

Now, it is a remarkable fact that a numeral of any order constructed according to this definition has no ambiguity and can be parsed in a unique way, even though any punctuation like parentheses are not used. In order to see how this is made possible, let us consider the following numeral as an example.

(d)	*yon*	*sen*	*go*	*hyaku*	*man*	*go*	*sen*
	four	thousand	five	hundred	ten thousand	five	thousand
	hachi	*jyuu*	*ni*				
	eight	ten	two				

The most important clue is that in any numeral there must be the highest one among the names for powers of ten occurring in it, and that its occurrence must be unique. It gives us the order of the numeral. Thus, (d) has the order *man*. At the occurrence of *man*, (d) can be divided into three parts, the preceding part (e), the *man* itself, and the succeeding part (f):

(e)	*yon*	*sen*	*go*	*hyaku*	
	four	thousand	five	hundred	
(f)	*go*	*sen*	*hachi*	*jyuu*	*ni*
	five	thousand	eight	ten	two

Concatenating (e) before *man* is the operation mentioned in 2–i of the definition, and concatenating (f) after the result of the former operation is that in 2–ii.

In general, a numeral of order δ is either of the form

$$\nu\,\delta\,\nu',$$

or, of the form

$$\nu\,\delta,$$

where ν and ν' are numerals of order $\epsilon < \delta$[6]. As an occurrence of concatenation before δ and that after it are different operations, let us distinguish them and write the above as

$$((\nu\,\delta) \oplus \nu'),$$

and

$$(\nu\,\delta).$$

Then, (e) and (f) can be represented as

(e) $((yon\ sen) \oplus (go\ hyaku))$

(f) $((go\ sen) \oplus ((hachi\ jyuu) \oplus ni))$

Thus, (d) itself is represented as

$$(((yon\ sen) \oplus (go\ hyaku))\ man) \oplus ((go\ sen) \oplus ((hachi\ jyuu) \oplus ni))$$

3 Semantics I: a numeral as a singular term

We may construe a numeral as a singular term which refers to a natural number. It is either a simple name like *san* which denotes the number three, or a description like *san jyuu go* that is constructed from three numerals and denotes the number thirty-five. As such a description may become indefinitely complex, its semantics should be given in a compositional way. It is not difficult to create a compositional account of them, based on the idea that each numeral refers to a particular natural number.

A semantic account of numerals consists of two parts corresponding to the syntactic construction of a numeral. First, there are semantic axioms for nine simple numerals and numerals for exponents of 10. Here are samples of them.

[6]When δ is one of *jyuu*, *hyaku*, and *sen*, a numeral may not have a part represented by ν.

Axiom I–1 (for *san*)

$$\mathrm{Val}(x, \text{“}\mathbf{san}\text{”}) \leftrightarrow x = 3. \qquad \blacksquare$$

Axiom I–2 (for *go*)

$$\mathrm{Val}(x, \text{“}\mathbf{go}\text{”}) \leftrightarrow x = 5. \qquad \blacksquare$$

Axiom I–3 (for *jyuu*)

$$\mathrm{Val}(x, \text{“}\mathbf{jyuu}\text{”}) \leftrightarrow x = 10. \qquad \blacksquare$$

As we are supposing that a numeral refers to a natural number which is an individual, the semantic value of a numeral must be always an individual. That is the reason why singular variables are used in the above. This use of individual variables for natural numbers testifies that this account commits to the existence of natural numbers as individuals.

Next, there must be two axioms corresponding to two syntactic operations involved in constructing a complex numeral. Any complex numeral has its order numeral, and the two operations are either preposing a numeral to that order numeral or postposing a numeral to it. As you will see from the following two axioms, preposing semantically corresponds to multiplication, and postposing to addition.

Axiom I–4 (for preposing)

If δ is a name for a power of ten, ν is a numeral of order $\epsilon < \delta$, and "$\nu\ \delta$" is a numeral, then

$$\mathrm{Val}(x, \text{“}\nu\ \delta\text{”}) \leftrightarrow \exists x \exists y[\mathrm{Val}(y, \nu) \wedge \mathrm{Val}(z, \delta) \wedge x = y \times z].$$

$\blacksquare$

Axiom I–5 (for postposing)

Let δ be a name for a power of ten, δ' a multiple of δ, and ν a numeral of order $\epsilon < \delta$. Then

$$\mathrm{Val}(x, \text{“}\delta'\ \nu\text{”}) \leftrightarrow \exists y \exists z[\mathrm{Val}(y, \delta') \wedge \mathrm{Val}(z, \nu) \wedge x = y + z].$$

$\blacksquare$

By these axioms, the semantic value of a numeral *san jyuu go*, which can be parsed as "(*san jyuu*) *go*", is calculated in the following way.

1. Val(x, "**(san jyuu) go**")

2. $\exists y \exists z$[Val(y, "**san jyuu**") $\wedge$ Val(z, "**go**") $\wedge$ $x = y + z$] (1 and Axiom I–5)

3. $\exists y$[Val(y, "**san jyuu**") $\wedge$ $x = y + 5$] (2, Axiom I–2, and logic)

4. $\exists y[\exists v \exists w$[Val($v$, "**san**") $\wedge$ Val(w, "**jyuu**") $\wedge$ $y = v \times w] \wedge x = y + 5]$ (3, Axiom I–4, and logic)

5. $\exists y[y = 3 \times 10 \wedge x = y + 5]$ (4, Axiom I–1, Axiom I–3, and logic)

6. $x = 3 \times 10 + 5$ (5, logic)

From line 6, we can easily conclude that $x = 35$. It may be remarked that this inference is no longer within a semantic theory because it makes use of a mathematical fact which is not the matter of meaning. But it may not be straightforward to draw a boundary between the matter of meaning and that of facts. For one thing, this account of numerals uses the properties of operations like the product and the sum between natural numbers in order to explain their compositionality.

In Japanese, a numeral cannot directly modify a common noun as the ungrammaticality of (1) below shows; a numeral must be succeeded by a classifier as in (2) or (3)[7].

(1)	*San	kodomo	ga	waratta.		
	three	child(ren)	NOM	laughed		
(2)	San	nin	no	kodomo	ga	waratta.
	three	CL(for people)	GEN	child(ren)	NOM	laughed
	(Three children laughed.)					
(3)	San	nin	kodomo	ga	waratta.	
	three	CL(for people)	child(ren)	NOM	laughed	
	(Three children laughed.)					

In (2), a numeral *san* (three) forms a numeral noun phrase with a classifier *nin* and modifies a common noun *kodomo* (a child/children) by the intermediary of a case particle *no*. In (3), a numeral noun phrase *san nin* modifies *kodomo* adverbially. In both cases, the appearance of a classifier *nin* is obligatory[8].

Given the present assumption that a numeral is a singular term referring to a particular number, if we construe a numeral noun phrase like *san nin* as a plural predicate, the semantical function of a classifier should be that of turning a singular term to a predicate, Thus, we should set up an axiom like the following.

[7] Here is a list of abbreviations that will be used in the following. CL: classifier, COP: copula, GEN: genitive, IND: indeterminate, NOM: nominative, TOP: topic.

[8] There are two sorts of exceptions. First, there are idiomatic phrases like *san baka* (three fools) and *nana kusa* (seven herbs) which do not need a classifier. Secondly, a classifier *tsu* is used with native Japanese numerals like *hito tsu* (one), *futa tsu* (two), until *kokono tsu* (nine), but for a number bigger than nine, a Chinese-derived numeral is used without a classifier. In the latter case, it may be assumed that the classifier *tsu* is present without being pronounced.

Axiom I–6 (for classifier)

Let ν be a numeral and Cl a classifier. Then

$$\mathrm{Val}(X,\ \text{“}\nu\ \mathrm{Cl}\text{”}) \leftrightarrow \exists x[\mathrm{Val}(x,\ \nu) \wedge \text{the number of } X \text{ is } x].$$

∎

Note that, as its consequence, it does not make any semantical (i.e. truth conditional) difference which particular classifier is used. Hence, for example, the following holds.

$\mathrm{Val}(X,$ “**san nin**”$) \leftrightarrow \mathrm{Val}(X,$ “**san satsu**”$)$.

But isn't it obvious that a sentence like the following should not be admitted?

(4)	*San	satsu	no	kodomo	ga	waratta.
	three	CL (books)	GEN	child(ren)	NOM	laughed

I have argued elsewhere[9] that a sentence like (4) is incorrect because it has a conventional implicature that is false. If we are concerned only with truth conditions, (2) and (4) have the same truth condition, but (4) should be avoided because of the falsity of its conventional implicature.

4 Semantics II: a numeral as a plural predicate

The preceding semantics of a numeral is based on the assumption that our ontology in the metalanguage includes natural numbers among individuals and that numerals are singular terms that denote them. Then, for example, the occurrence of a noun phrase *san nin no kodomo* (three children) is that of a predicate that applies to a number of children if and only if they have a certain relation to an abstract object, namely, the number three. Could we believe, however, that we are talking of some abstract object when we wish to say just that three children laughed?[10]

Given our construal of a numeral NP as a plural predicate, we could do better; we can dispense with an ontology of natural numbers, if we do not regard a numeral as a singular term but as an adjectival expression; *san* (three) is not a name for an abstract object three, but a predicate which is true of three things. Now a classifier is no longer an expression which turns a singular term into a predicate; it does not change an adjectival status of a numeral, but it signals that the preceding numeral will be applied to a count domain which is given by the succeeding common noun.

[9] [Iida 2015] §6.3.

[10] The reason why this does not sound outrageous at all to most linguists working in formal semantics is that they are working in the framework of generalized quantifier theory, which has a built-in commitment to sets as mathematical objects; after all, natural numbers are special sorts of sets, and in such a framework, a classifier is a function from one kind of sets to another kind of sets.

Although there are common nouns like *ringo* (apple(s)) the occurrence of which might be either count or mass, there are also intrinsically count nouns like *kodomo* (child(ren)) and *isu* (chair(s)). For each of intrinsically count nouns, there exists a particular classifier that goes with it, like *nin* for *kodomo* and *kyaku* for *isu*. The presence of a classifier like *nin* and *kyaku* signals that some count noun for which the classifier is suitable will follow.

Thus, for a simple numeral like *san* (three), we set the following as an axiom

Axiom II–1 (for *san*)

$$\mathrm{Val}(X,\ \text{“}\textbf{san}\text{”}) \leftrightarrow X \text{ are three.} \qquad \blacksquare$$

We have similar axioms for any simple numeral and also any name for powers of ten, in particular the previous Axioms 2 and 3 should be modified in a similar way.

Axiom II–2 (for *go*)

$$\mathrm{Val}(X,\ \text{“}\textbf{go}\text{”}) \leftrightarrow X \text{ are five.} \qquad \blacksquare$$

Axiom II–3 (for *jyuu*)

$$\mathrm{Val}(X,\ \text{“}\textbf{jyuu}\text{”}) \leftrightarrow X \text{ are ten.} \qquad \blacksquare$$

Another axiom says that a classifier Cl has no truth-conditional content.

Axiom II–6 (for a classifier)

If ν is a numeral and Cl is a classifier, then

$$\mathrm{Val}(X,\ \text{“}\nu \text{ Cl”}) \leftrightarrow \mathrm{Val}(X,\ \nu). \qquad \blacksquare$$

It is well known that a plural predicate like “X are three” can be defined by using only logical concepts in the following way[11].

$$X \text{ are three} \leftrightarrow \exists x \exists y \exists z[x\eta X \wedge y\eta X \wedge z\eta X \wedge x \neq y \wedge y \neq z \wedge x \neq z \wedge \forall w[w\eta X \rightarrow [w = x \vee w = y \vee w = z]]].$$

Should we incorporate this into our semantics of *san*? I believe that the answer is “No”. For, giving a semantic account of an expression is one thing, and giving a conceptual analysis is another, although it is sometimes difficult to distinguish the two. Thus, axioms for simple numerals like the above are enough for our purpose.

The challenge here is to give a compositional account of a complex numeral like *san jyuu go* (thirty five). It is easy to see how we should proceed to define “X are n” in the same style as above for bigger and bigger n, although it

[11]It is assumed that the *among* relation $X\eta Y$ is logical relation in plural logic. The individualhood and identity are both defined by η. See [McKay 2006].

will be extremely tedious to write down the actual definitions. But it does not give us a compositional account of Japanese numerals (or, for that matter, English numerals either). Such an account should explain how each occurrence of a simple numeral like *san*, *jyuu*, and *go* contributes to the meaning of the complex numeral *san jyuu go*.

There exists an account of the semantics of complex numerals without presupposing that each numeral refers to an abstract number[12]. It is given, however, in a singularist framework, that is, a numeral is essentially predicated to a single individual ("a plural object") which can be divided into as many parts as the numeral indicates[13]. Thus, such an account entails the existence of a new kind of objects, namely, plural objects.

We can give a compositional account of unmodified numerals without presupposing the ontology of numbers or introducing that of plural objects, if we are allowed to assume that a predication may be plurally plural, plurally plurally plural, and so on; such higher-level plural predication is sometimes called "superplural" predication[14]. If we allow higher-level plural predication, then there must also be higher-level *among* relations; as number predicates can be defined by *among* relation, a numeral could be used also for higher-level plural predication.

Japanese is one of the languages which provide grounds for a belief in the existence of superplural predication in a natural language. Japanese has several classifiers that are used for pairs of particular sorts.

(g)	ni	soku	no	kutsu
	two	CL(for footwear)	GEN	pair(s) of shoes

(two pairs of shoes)

(h)	ichi	zen	no	hashi
	one	CL(for pair(s) of chopsticks)	GEN	pair(s) of chopsticks

(a pair of chopsticks)

Most importantly, there is a classifier *kumi* which is used for pluralities in general.

(i)	san	kumi	no	oyako
	three	CL(for pluralities)	GEN	parent-child

(three groups of parent(s) and child(ren))

(j)	go	kumi	no	tsukue	to	isu
	five	CL(for pluralities)	GEN	desk(s)	and	chair(s)

(five sets of desks and chairs)

[12] [Ionin and Matushansky 2006].

[13] Ionin and Matushansky claim that a numeral could not be construed as a predicate if we wish to give a compositional account of complex numbers. But this is no longer true if we move from a singularist conception of plurality to plural logical framework, as will be shown in the following.

[14] See [Oliver and Smiley 2013], p.275ff. There exists now an excellent monograph [Wagner 2015] on superplural logic.

In (i) the numeral NP *san kumi* is applied to *oyako* which may denote a number of parent(s) (*oya*)–child(ren) (*ko*) combinations. Semantic values of *oyako* may be more than one parent-child combination, which consists of a number of people who stand in the relation of a parent and a child. Hence, we are dealing with a plurally plural domain.

In (j) what are counted are not individual desks and chairs but combinations of desk(s) and chair(s). It must be noted that the particle *to* is used for forming a plural expression like English "and" in "John and Mary". It is even more obvious than (i) that a numeral *go* is plurally predicated to pluralities that consist of a desk or desks and a chair or chairs.

With nouns like *oyako* and NPs like *tsukue to isu*, we can easily find examples of plurally plural quantification.

(5)

Dai-bubun	no	oyako	wa	naka
large part	GEN	parent-child	TOP	relationship
ga	yoi.			
NOM	good			

(Most of the parent-child are on good terms.)

(6)

Dono	tsukue	to	isu	mo	tagai-ni
IND	desk(s)	and	chair(s)	∀	each other
pittari	da				
fit right	COP				

(Every desk-chair combination is perfectly matched.)

These should be compared to the following.

(7)

Dai-bubun	no	kodomo	ga	waratta.
large part	GEN	child(ren)	NOM	laughed

(Most of the children laughed.)

(8)

Dono	tsukue	mo	atarashii.
IND	desk(s)	∀	new

(Every desk is new.)

Cases of quantification in Japanese can be classified into two varieties; on one hand, there are cases of quantification by means of a quantity noun like *dai-bubun*, and on the other, there are those by means of an indeterminate phrase like *dono*. (5) and (7) are cases of the former, and (6) and (8) the latter.

In (5), quantification is over a domain consisting of a number of parent-child combinations, while it is over a domain of individual children in (7). Similarly, in (6) it is over a number of desk-chair combinations, in contrast to (8) in which the quantificational domain consists of individual desks. Moreover, the predicates *naka ga yoi* (be on good terms) in (5) and *tagai-ni pittari da* (perfectly matched each other) in (6) are only applicable to a number of individuals, never to a single individual like *waratta* (laughed) in (7) or *atarashii* (is new) in (8).

I suppose that these examples show that there are plurally plural predication and quantification in Japanese.

As several authors have argued[15], superplural logic is a natural extension of plural logic. We could even argue that, as there are good reasons for moving from singular to plural logic, the same consideration forces us to move from plural to superplural logic. Examples of Japanese give us another reason to do the same.

Let us suppose that our metalanguage has not only singular variables

$$x, y, z, \ldots$$

and plural variables

$$X, Y, Z, \ldots$$

but also higher-level plural variables[16]

$$X^2, Y^2, Z^2, \ldots, X^3, Y^3, Z^3, \ldots, X^n, Y^n, Z^n, \ldots,$$

and we may write instead of "$x, y, z, \ldots$"

$$X^0, Y^0, Z^0, \ldots,$$

and instead of "$X, Y, Z, \ldots$"

$$X^1, Y^1, Z^1, \ldots$$

We also suppose that *among* relation η could be of any level n ($n \geq 1$). Thus, we write like this:

$$X^2 \eta^2 Y^2.$$

Let us consider the following sentence in which a complex numeral *san jyuu* occurs.

(9)	San	jyuu	nin	no	kodomo	ga	atsumatta.
	three	ten	CL(for person)	GEN	child(ren)	NOM	assembled

(Thirty children came together.)

The thirty children who are said to have come together may be divided into three groups of ten children each. In other words, we may construe

(k)	san	jyuu	nin	no	kodomo
	three	ten	CL(for person)	GEN	child(ren)

(thirty children)

as

[15] See [Oliver and Smiley 2013] and [Wagner 2015].

[16] The use of Arabic numerals for superscripts does not commit us to the existence of natural numbers. It may be replaced by any system of symbols which has the same order structure.

(l) san kumi no jyuu nin no kodomo
three CL(for pluralities) GEN ten CL(for person) GEN child(ren)
(three groups of ten children each)

(l) is just like (i) except that it has *jyuu nin no kodomo* (ten children) instead of *oyako* (parent-child). As *jyuu nin no kodomo* denotes a number of people, namely, ten children, (l) is a plurally plural expression.

For (l) to denote the same children as (k) does, three groups of children mentioned in (l) should not overlap each other. In other words, any two of them should not have a common member. A number of groups that satisfy such a condition are said to be "pairwise disjoint".

We may speak of pluralities of pluralities and name this second-level plurality by "A^2", indicating by the index 2 that we are talking about plurally plural things. In our example, "A^2" refers plurally to three groups of children. We say that A^2 consists of pairwise disjoint pluralities when

$$\forall X \forall Y[[X\eta^2 A^2 \wedge Y\eta^2 A^2 \wedge X \not\equiv Y] \rightarrow \neg\exists Z[Z\eta X \wedge Z\eta Y]]].$$

Let us abbreviate this as

$$\mathrm{PD}(A^2)$$

Another concept we need is the sum of pluralities. For, to say that three groups of ten children each amount to thirty children altogether is to say that the result of assembling three groups are the same as the children denoted by (k), provided that there is no overlap between the three groups of the children. Let A^2 be the same as before. Then, we define the sum of A^2, ΣA^2, all individuals that are among some pluralities among A^2, namely,

$$x\eta\Sigma A^2 \leftrightarrow \exists X[X\eta^2 A^2 \wedge x\eta X].$$

These concepts should be generalized to any higher-level plurality.

Let X^{n+1} be a plurality of level $n+1$. X^{n+1} consists of pluralities of level n. What we need for our purpose is not the pairwise disjointedness of these pluralities of level n but a condition which assures us that they have no individuals in common. For example, consider

(m) ni jyuu man nin no kodomo
two ten ten thousand CL(for person) GEN child(ren)
(twenty thousand children)

For this, we have to consider a third-level plurality C^3 which consists of two second-level pluralities C_1^2 and C_2^2. Each of them, in turn, consists of ten first-level pluralities containing ten thousand children each: say, $C_{10}^1, C_{11}^1, \ldots, C_{19}^1$ for C_1^2, and $C_{20}^1, C_{21}^1, \ldots, C_{29}^1$ for C_2^2. It is necessary for C^3 to consist of exactly twenty thousand children such that i there is no overlap between pluralities among C_1^2,

ii that there is no overlap between pluralities among C_2^2, and

iii that no plurality among C_1^2 overlaps any plurality among C_2^2.

The first two require that each of C_1^2 and C_2^2 be pairwise disjoint. The last one requires that these two must be pluralities that come from two groups of individuals that have no individual in common.

By "$\mathsf{ind}(X^{n+1})$" let us denote a first-level plurality that consists of all the individuals from which any of the pluralities of any level of less than $n+1$ that appear in the formation of X^{n+1}. Then the condition (iii) says that $\mathsf{ind}(C_1^2)$ and $\mathsf{ind}(C_2^2)$ have no individual in common. Let us say that they are totally disjoint.

How can we get the first-level plurality $\mathsf{ind}(X^{n+1})$ from the $n+1$-level plurality X^{n+1}? First, we have to generalize the concept of the sum of pluralities to higher-level plurality. It can be done in this way.

Definition: the sum of pluralities

Let A^{n+1} be a plurality of level $n+1$ ($n \geq 1$). Then, the sum of A^{n+1}, that is, ΣA^{n+1}, are those X^{n-1} such that

$$\exists X^n[X^n \eta^{n+1} A^{n+1} \wedge X^{n-1} \eta^n X^n].$$

∎

Then, $\mathsf{ind}(X^{n+1})$ can be obtained by the iterated applications of the sum operation to X^{n+1}.

Definition: the individual basis of a plurality

Let A^{n+1} be a plurality of level n ($n \geq 1$). Then, the individual basis of A^n is

$$\underbrace{\Sigma\,\Sigma\,\ldots\,\Sigma}_{n \text{ times}} A^{n+1}.$$

∎

Now we can define a form of disjointedness we need.

Definition: pairwise totally disjoint plurality

Let A^{n+1} be plurality of level $n+1$ ($n \geq 1$). Then, A^{n+1} consist of pairwise totally disjoint pluralities of level n, or, PTD(A^{n+1}), if and only if

$$\forall X^n \forall Y^n [[X^n \eta^{n+1} A^{n+1} \wedge Y^n \eta^{n+1} A^{n+1} \wedge X^n \not\equiv Y^n] \rightarrow$$
$$\neg \exists x [x \eta \, \mathsf{ind}(X^n) \wedge x \eta \, \mathsf{ind}(Y^n)]].$$

∎

After this preparation, we can state an axiom that corresponds to Axiom I–4.

Axiom II–4 (for preposing)

If δ is a name for a power of ten, ν is a numeral of order $\epsilon < \delta$, and "$\nu\ \delta$" is a numeral, then

$$\mathrm{Val}(X^n, \text{“}\nu\ \delta\text{”}) \leftrightarrow \exists X^{n+1}[\mathrm{Val}(X^{n+1}, \nu) \wedge \mathrm{PTD}(X^{n+1})$$
$$\wedge \forall Y^n [Y^n \eta^{n+1} X^{n+1} \rightarrow \mathrm{Val}(Y^n, \delta)] \wedge X^n \equiv \Sigma X^{n+1}].$$

∎

Although this looks rather formidable mainly because of many superscripts, what it says is not difficult to understand.

Let us consider a case in which $n = 1$. Further suppose that Japanese numerals ν and δ translate respectively to N and D in English. Then what Axiom II–4 tells us is that a complex numeral "$\nu\ \delta$" is true of X if and only if X can be divided into pairwise totally disjoint N groups each of which consists of D individuals. You can see from this axiom how the semantic values of a complex numeral "$\nu\ \delta$" are determined by those of ν and δ.

In order to see what happens when $n \neq 1$, take *ni jyuu man* (literally, "two ten ten-thousand", meaning two hundred thousand) as an example. Note that *ni jyuu man* can be parsed as "(*ni jyuu*) *man*".

$$\mathrm{Val}(X, \text{“}\textbf{(ni jyuu) man}\text{”}) \leftrightarrow$$

$$\exists X^2[\mathrm{Val}(X^2, \text{“}\textbf{ni jyuu}\text{”}) \wedge \mathrm{PTD}(X^2) \wedge \forall Y [Y \eta^2 X^2$$
$$\rightarrow \mathrm{Val}(Y, \text{“}\textbf{man}\text{”})] \wedge X \equiv \Sigma X^2]. \quad \text{(by Axiom II–4)}$$

Consider the first conjunct within "$\exists X^2$", namely,

$$\mathrm{Val}(X^2, \text{“}\textbf{ni jyuu}\text{”})$$

By Axiom II–4 again, this is equivalent to

$$\exists X^3[\mathrm{Val}(X^3, \text{“}\textbf{ni}\text{”}) \wedge \mathrm{PTD}(X^3) \wedge \forall Y [Y \eta^3 X^3 \rightarrow \mathrm{Val}(Y, \text{“}\textbf{jyuu}\text{”})]$$
$$\wedge X^2 \equiv \Sigma X^3]$$

If you put this back, then you will see that a complex numeral *ni jyuu man* is true of X when X can be divided into pairwise totally disjoint groups X^2 which satisfy the following two conditions[17]:

(i) X^2 consists of groups of one hundred thousand individuals each.

(ii) X^2 can be divided into two disjoint groups so that they form two supergroups each of which comprises ten groups.

After all this, it must be easy to see what an axiom corresponding to Axiom I–5 should be.

Suppose that there are thirty-five children. Then, we can divide them into thirty children and five children which do not overlap each other. If we denote the former by "A" and the latter by "B", then the sum of them must denote the original thirty-five children. As we have already the general concepts of pairwise total disjointedness and the sum of pluralities, we need nothing new for our task.

Suppose that a superplurality that consists of two pluralities X and Y is denoted by "$[X, Y]$". We write "PTD(X, Y)" instead of "PTD$([X, Y])$". Similarly, we write "$\Sigma(X, Y)$" instead of "$\Sigma[X, Y]$". Then, our axiom runs as follows.

Axiom II–5 (for postposing)

Let δ be a name for a power of ten, δ' a multiple of δ, and ν a numeral of order $\epsilon < \delta$. Then, for any plurality X^n of level n ($n \geq 1$),

$$\begin{aligned}&\mathrm{Val}(X^n, \text{“}\delta'\nu\text{”}) \leftrightarrow \exists Y^n \exists Z^n [\mathrm{Val}(Y^n, \delta') \wedge \mathrm{Val}(Z^n, \nu) \wedge \mathrm{PTD}(Y^n, Z^n)\\ &\quad \wedge\ X^n \equiv \Sigma(Y^n, Z^n)].\end{aligned}$$

∎

If you compare this with the previous Axiom I–5, you will notice that they are very similar, although the present version makes no mention of natural numbers. Instead of the addition between numbers, it has the sum operation between totally disjoint pluralities.

Finally, in order to see how the two axioms for preposing and postposing work together, let us calculate the semantic values of *san jyuu go* again, but according to the new axioms this time.

1. Val(X, "(**san jyuu**) **go**").

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[17] Of course, we need the following two axioms:

Axiom (for *ni*)

Val(X, "**ni**") $\leftrightarrow$ X are two.

Axiom (for *man*)

Val(X, "**man**") $\leftrightarrow$ X are ten thousand.

2. $\exists Y \exists Z[\mathrm{Val}(Y, \text{“}\mathbf{san\ jyuu}\text{”}) \wedge \mathrm{Val}(Z, \text{“}\mathbf{go}\text{”}) \wedge \mathrm{PTD}(Y, Z) \wedge X \equiv \Sigma(Y, Z)]$.
   (1 and Axiom II–5)

3. $\exists Y \exists Z[\mathrm{Val}(Y, \text{“}\mathbf{san\ jyuu}\text{”}) \wedge Z \text{ are five} \wedge \mathrm{PTD}(Y, Z) \wedge X \equiv \Sigma(Y, Z)]$.
   (2 and Axiom II–2)

4. $\exists Y \exists Z[\exists Y^2[\mathrm{Val}(Y^2, \text{“}\mathbf{san}\text{”}) \wedge \mathrm{PTD}(Y^2) \wedge \forall W[W\eta^2 Y^2 \rightarrow$
   $\mathrm{Val}(W, \text{“}\mathbf{jyuu}\text{”})] \wedge Y \equiv \Sigma Y^2] \wedge Z \text{ are five} \wedge \mathrm{PTD}(Y, Z) \wedge$
   $X \equiv \Sigma(Y, Z)]$. (1 and Axiom II–5)

5. $\exists Y \exists Z[\exists Y^2[Y^2 \text{ are three} \wedge \mathrm{PTD}(Y^2) \wedge \forall W[W\eta^2 Y^2 \rightarrow$
   $W \text{ are ten}] \wedge Y \equiv \Sigma Y^2] \wedge Z \text{ are five} \wedge \mathrm{PTD}(Y, Z) \wedge$
   $X \equiv \Sigma(Y, Z)]$. (4 and Axioms II–1, II–3)

This may look too complicated, but in reality it is not more complex than what the first account delivered. For, in that account, we could use our knowledge about arithmetic operations like multiplication and addition, which would look very complex indeed if its content is presented in full.

Thus, let me explain what the last line says. It gives us the condition for some things X to have the number property expressed by a numeral *san jyuu go*. The line 5 says that for X to have that property there must be some things Y and Z that satisfy the following three conditions.

5a. $\exists Y^2[Y^2 \text{ are three} \wedge \mathrm{PTD}(Y^2) \wedge \forall W[W\eta^2 Y^2 \rightarrow$
    $W \text{ are ten}] \wedge Y \equiv \Sigma Y^2]$

5b. $Z \text{ are five} \wedge \mathrm{PTD}(Y, Z)$

5c. $X \equiv \Sigma(Y, Z)$

First, 5a says that Y can be divided to form three collections of ten individuals each, which do not have any member in common. The use of a phrase like “collections” is not intended to imply that there are some special sort of plural entities. I use a second-level plural variable “Y^2” in order to talk plurally of plurally given individuals, that is, thirty individuals divided into three groups. As plural predication and quantification do not introduce any new sorts of entities, higher-level ones do not either.

Next, 5b says that there exist another five individuals which are different from any that is found in the three collections of ten individuals which are said to exist in 5a.

Finally, 5c says that the semantic values of *san jyuu go* are those that are among some ten individuals among the three collections of them or another five individuals.

If we use a numeral *san jyuu go* with some common noun, say *kodomo*, we will have a noun phrase

(n)	san	jyuu	go	nin	no	kodomo
	three	ten	five	CL(for person)	GEN	child(ren)

(thirty five children)

If we use a classifier *kumi* for plural pluralities, in our case, *jyuu nin no kodomo* (ten children), then the semantic values of (n) can be given by

(o)	san	kumi	no	jyuu	nin
	three	CL(for pluralities)	GEN	ten	CL(for person)
	no	kodomo	to	go	nin
	GEN	child(ren)	and	five	CL(for person)
	no	kodomo			
	GEN	child(ren)			

This can be roughly translated as

three collections of ten children and five children

San kumi no (three collections of) expresses second-level plural quantification in 5a, and *to* (and) corresponds to the sum operation in 5c. That there is no commitment to anything other than individuals could be confirmed, if you rewrite 5a by "analyzing" "Y^2 are three" and "W are ten" so that they are expressed entirely in logical terms, although it will be an extremely tedious task and you will need more than thirty different individual variables.

5 Conclusion

Our starting hypothesis was that a number predication could be just an ordinary predication. Obviously a numeral plays an essential role in any number predication, and a numeral may become as complex as one likes. Hence, our task has been to supply a compositional account of complex numerals. We have considered two such accounts for Japanese numerals.

According to the first account, a numeral is a singular term that refers to a natural number, and a classifier turns a singular term into a plural predicate. In contrast, in the second account, a numeral is by itself a plural predicate, and a classifier has no truth-conditional content.

The first account presupposes that there are natural numbers on which the operations of multiplication and addition are defined. The second account does not presuppose the existence of natural numbers; instead of multiplication and addition between natural numbers, it utilizes higher-level plural predication and the concept of sum of pluralities.

According to the first account, only first-level plural predication is enough for a number predication. However, higher-level plurality is necessary for an overall semantics of Japanese after all, as we saw above that it has plurally plural quantification like (5) and (6). Hence, when higher-level plurality is available, it seems to be reasonable to utilize it and dispense with an ontology of abstract objects.

It might be objected that according to the second account a numeral like *san* (three) or *san jyuu go* (three-ten-five, i.e., thirty-five) will not be single predicates but a plurality of predicates of different levels. For example, consider the following.

(p)	san	jyuu	go	man	san	jyuu	go	nin
	three	ten	five	ten thousand	three	ten	five	CL

(three hundred fifty thousand and thirty-five persons)

The first occurrence of a complex numeral *san jyuu go* is that of a second-level plural predicate, whereas its second occurrence is that of a first-level one. Moreover, the first occurrence of a simple numeral *san* is that of a third-level plural predicate, whereas its second occurrence is that of a second-level one.

This only shows that, however, numeral expressions belong to the logical part of a language just as expressions for quantification do. Remember our example of plurally plural quantification.

(6)	Dono	tsukue	to	isu	mo	tagai-ni
	IND	desk(s)	and	chair(s)	$\forall$	each other
	pittari	da.				
	fit right	COP				

(Every desk-chair combination is perfectly matched.)

Here an expression *dono ... mo* expresses second-level plural quantification; it is systematically ambiguous in that it may be of any level. It is not surprising that a numeral expression has a similar ambiguity; it can be analyzed in terms of "among" relation, which is different relation on a different level.

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