Divisibility of Labor Supply and Involuntary Unemployment: A Perfect Competition Model

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Abstract

We show the existence of involuntary unemployment without assuming wage rigidity. We derive involuntary unemployment by considering utility maximization of consumers and profit maximization of firms in an overlapping generations model under perfect competition with decreasing or constant returns to scale technology. Indivisibility of labor supply may be a ground for the existence of involuntary unemployment. However, we show that under some conditions there exists involuntary unemployment even when labor supply is divisible.

Keywords: involuntary unemployment, perfect competition, divisible labor supply

JEL Classifications: E12, E24.

1. Introduction

According to Otaki (2009) the definition of involuntary unemployment consists of two elements.

1. The nominal wage rate is set above the nominal reservation wage rate.
2. The employment level and economic welfare never improve by lowering the nominal wage rate.

Otaki(2009) assumes that the wage rate is equal to the reservation wage rate at the equilibrium under indivisibility of labor supply. In such a situation, however, unemployment is not involuntary. He has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and R. M. Solow (1981). The arguments of this paper, however, do not depend on bargaining. Another reference about involuntary unemployment without wage rigidity is Umada (1997). He derived an upward-sloping labor demand curve from mark-up principle for firms under increasing returns to scale technology, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary

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unemployment without wage rigidity\(^1\). But his model of firms’ behavior is ad-hoc.

In this paper we consider utility maximization of consumers and profit maximization of firms in an overlapping generations model under perfect competition according to Otaki (2010, 2011, 2015) with decreasing or constant returns to scale technology, and show the existence of involuntary unemployment under divisibility of individual labor supply. Indivisibility of labor supply means that labor supply of each individual can be 1 or 0. On the other hand, if labor supply is divisible, it is a variable in \([0,1]\). As discussed by Otaki (2015) (Theorem 2.3) and Otaki (2012), if labor supply is infinitely divisible, there exists no unemployment. However, if labor supply by each individual is not so small, there may exist involuntary unemployment even when labor supply is divisible. In this paper the first element of Otaki’s two elements of involuntary unemployment should be

Labor supply of each individual is positive at the current real wage rate.

In some other papers we have shown the existence of involuntary unemployment under perfect or monopolistic competition when labor supply by individuals is indivisible\(^2\).

In the next section we analyze consumers’ utility maximization in an overlapping generations model with two periods. We consider labor supply by individuals as well as their consumptions. In Section 3 we consider profit maximization of firms under perfect competition. In Section 4 we show the existence of involuntary unemployment when labor supply is divisible.

Schultz (1992) showed that involuntary unemployment does not arise in an overlapping generations model. His arguments depend on the real balance effect caused by a fall in the nominal wage rate on consumption of the older generation consumers. In Appendix of this paper we will discuss this point.

2. Consumers

We consider a two-period (young and old) overlapping generations model under perfect competition according to Otaki (2010, 2011 and 2015). There is one factor of production, labor, and one good which is produced under perfect competition. There is a continuum of firms. The volume of firms is one. Consumers are born at continuous density \([0,1] \times [0,1]\) in each period. Each consumer supplies \(l\) units of labor when they are young (the first period), \(0 \leq l \leq 1\).

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\(^1\) Lavoie (2001) presented a similar analysis.

\(^2\) In Tanaka(2020a) we have shown the existence of involuntary unemployment in an overlapping generations model with indivisible labor supply. In Tanaka(2020b) we analyzed the balanced budget multiplier in a static model (not overlapping generations model) under monopolistic competition with indivisible labor supply and have shown the existence of involuntary unemployment.
We use the following notations.

\( c_i \): consumption of the good at period \( i, \ i = 1,2 \).

\( p_i \): price of the good at period \( i, \ i = 1,2 \).

\( W \): nominal wage rate.

\( \Pi \): profits of firms which are equally distributed to each consumer.

\( l \): labor supply of an individual.

\( L \): employment (the number of employed consumers (workers)).

\( L_f \): population of labor or employment at the full-employment state.

\( y(Ll) \): labor productivity, which is decreasing or constant with respect to "employment \times labor supply \((Ll)\)", \( y(Ll) \geq 1, \ y' \leq 0 \).

We drop the time argument of \( c_1 \) and \( c_2 \), which should be written as \( c_{1t} \) and \( c_{2t+1} \), to simplify notation.

We call \( Ll \) "net employment". We define the elasticity of the labor productivity with respect to \( Ll \) as follows,

\[
\zeta = \frac{y'}{y(Ll)}.
\] (1)

We assume that \(-1 < \zeta \leq 0\), and \( \zeta \) is constant. Decreasing (constant) returns to scale means \( \zeta < 0 \) (\( \zeta = 0 \)). The output is \( Lly(Ll) \). \( \zeta > -1 \) or \( 1 + \zeta > 0 \) means that the output is increasing with respect to \( Ll \). If the good is produced under constant returns to scale technology, \( \Pi = 0 \).

The utility of a consumer of one generation over two periods is

\[
U(c_1, c_2) = u(c_1, c_2) - G(l).
\] (2)

We assume that \( u(c_1, c_2) \) is homogeneous of degree one (linearly homogeneous). \( G(l) \) is a function of disutility of labor which is continuous, strictly increasing, differentiable and strictly convex, thus \( G' > 0, \ G'' > 0 \).

The budget constraint for an employed individual is

\[
p_1c_1 + p_2c_2 = Wl + \Pi.
\] (3)

\( p_2 \) is the expectation of the price at period 2. In our model the good is produced by only labor, and there is no capital. We assume that the interest rate is zero. The Lagrange function is

\[
\mathcal{L} = u(c_1, c_2) - G(l) - \lambda[p_1c_1 + p_2c_2 - (Wl + \Pi)].
\]

\( \lambda \) is the Lagrange multiplier. The first order conditions are

\[
\frac{\partial u}{\partial c_1} - \lambda p_1 = 0, \ \frac{\partial u}{\partial c_2} - \lambda p_2 = 0.
\] (4)
They are rewritten as
\[ \frac{\partial u}{\partial c_1} c_1 = \lambda p_1 c_1, \quad \frac{\partial u}{\partial c_2} c_2 = \lambda p_2 c_2. \] (5)

Since \( u(c_1, c_2) \) is homogeneous of degree one,
\[ \frac{\partial u}{\partial c_1} c_1 + \frac{\partial u}{\partial c_2} c_2 = u(c_1, c_2) = \lambda (p_1 c_1 + p_2 c_2) = \lambda (Wl + \Pi), \]
Also, since \( u(c_1, c_2) \) is homogeneous of degree one, \( \lambda \) is a function of \( p_1 \) and \( p_2 \), and \( \frac{1}{\lambda} \) is homogeneous of degree one with respect to \( (p_1, p_2) \) because proportional increases in \( p_1 \) and \( p_2 \) reduce \( c_1 \) and \( c_2 \) at the same rate given \( Wl + \Pi \). Then, we obtain the following indirect utility function.
\[ V = \frac{1}{\varphi(p_1, p_2)} (Wl + \Pi) - G(l). \] (6)

\( \varphi(p_1, p_2) \) is a function of \( p_1 \) and \( p_2 \). It is positive, increasing with respect to \( p_1 \) and \( p_2 \), and homogeneous of degree one with respect to \( (p_1, p_2) \). Therefore, the indirect utility function \( V \) is decreasing with respect to \( p_1 \) and \( p_2 \), and homogeneous of degree zero with respect to \( (p_1, p_2, Wl + \Pi) \). Maximization of \( V \) with respect to \( l \) implies
\[ W = \varphi(p_1, p_2) G'(l). \] (7)

Let \( \rho = \frac{p_2}{p_1} \). From (7)
\[ \omega = \frac{W}{p_1} = \varphi(1, \rho) G'(l). \] (8)

\( \omega \) is the real wage rate. If the value of \( \rho \) is given, \( l \) is obtained from (8) as a function of \( \omega \). Since \( G'' > 0 \), labor supply \( l \) is increasing with respect to the real wage rate \( \omega \).

For an unemployed individual the budget constraint is \( p_1 c_1 + p_2 c_2 = \Pi \), and his indirect utility function is
\[ \frac{1}{\varphi(p_1, p_2)} \Pi. \] (9)

Let
\[ \alpha = \frac{p_1 c_1}{Wl + \Pi}, \quad 1 - \alpha = \frac{p_2 c_2}{Wl + \Pi} \] (10)

We have \( 0 < \alpha < 1 \). Demand for the good of each employed consumer of younger generation is
\[ c_1 = \frac{\alpha (Wl + \Pi)}{p_1}. \] (11)

\( ^3 \lambda \) is decreasing with respect to \( p_1 \) and \( p_2 \).
His demand in the second period is
\[ c_2 = \frac{(1 - \alpha)(WL + \Pi)}{p_2}. \quad (12) \]

For an unemployed consumer \( c_1 = \frac{\alpha\Pi}{p_1} \), \( c_2 = \frac{(1 - \alpha)\Pi}{p_2} \). Let \( \bar{c}_2 \) be demand of an older generation consumer who works in the previous period, \( \bar{l}, \bar{W}, \bar{\Pi} \) and \( \bar{\alpha} \) be his labor supply, the nominal wage rate, the profit and the value of \( \alpha \) when he is young. Then
\[ \bar{c}_2 = \frac{(1 - \bar{\alpha})(\bar{W}\bar{l} + \bar{\Pi})}{p_1}. \quad (13) \]

\((1 - \bar{\alpha})(\bar{W}\bar{l} + \bar{\Pi})\) is his savings carried over from his first period. For an older generation consumer who is unemployed in the previous period \( \bar{c}_2 = \frac{(1 - \bar{\alpha})\bar{\Pi}}{p_1} \). Denote the total savings of the older generation consumers by \( M \). Then, their demand for the good is
\[ \frac{M}{p_1}. \quad (14) \]

The total demand for the good is
\[ c = \frac{Y}{p_1}. \quad (15) \]

\( Y \) is the effective demand defined by
\[ Y = \alpha(WLl + Lf\Pi) + G + M. \quad (16) \]

\( G \) is the government expenditure. The government expenditure as well as consumptions of younger and older generations constitutes the national income (about this demand function please see Otaki (2007), Otaki (2009), Otaki (2015)).

3. Firms

In this section we consider firms’ profit maximization behavior. Let \( x \) and \( z \) be the output and the net employment (employment \( \times \) labor supply) of a firm. We have \( x = y(z)z \) and
\[ \zeta = \frac{y'(z)}{y(z)z}. \quad (17) \]

Thus,
\[ \frac{dz}{dx} = \frac{1}{y(z) + y'z} = \frac{1}{(1 + \zeta)y(z)}. \quad (18) \]
The profit of a firm is
\[ \pi = p_1 x - \frac{x}{y(z)} W. \] (19)

\( p_1 \) is given for each firm. The condition for profit maximization under perfect competition is
\[ p_1 - y(z) - xy' \frac{dz}{dx} W = p_1 - \frac{1 - y'z}{y(z)} W = p_1 - \frac{1}{(1 + \zeta) y(z)} W = 0. \] (20)

Therefore \( p_1 = \frac{1}{(1+\zeta) y(z)} \). This means the marginal cost pricing because the marginal cost is \( \frac{1}{(1+\zeta) y(z)} \). Since at the equilibrium \( x = c \) and \( z = LL \), we obtain
\[ p_1 = \frac{1}{(1 + \zeta) y(LL)} W. \] (21)

With decreasing (constant) returns to scale \(-1 < \zeta < 0 \) (\( \zeta = 0 \)).

4. Main Results

4.1. Involuntary Unemployment

From (21) the real wage rate is
\[ \omega = \frac{W}{p_1} = (1 + \zeta) y(LL). \] (22)

Under decreasing (constant) returns to scale, since \( \zeta \) is constant, \( \omega \) is decreasing (constant) with respect to \( LL \). From (8) and (22) we get
\[(1 + \zeta) y(LL) = \varphi(1, \rho) G'(l) \] (23)

From (23) labor supply of an individual \( l \) is obtained as a function of \( L \). Denote it by \( l(L) \). Since \( G'' > 0 \) (convex disutility of labor) and \( y' \leq 0 \) (decreasing or constant returns to scale), we have
\[ \varphi(1, \rho) G''(l) - (1 + \zeta) y'L > 0. \] (24)

This guarantees that \( l(L) \) is decreasing and \( LL(L) \) is strictly increasing with respect to \( L \) because
\[ \frac{dl(L)}{dL} = \frac{(1 + \zeta) y'l(L)}{\varphi(1, \rho) G''(l) - (1 + \zeta) y'L} \leq 0, \] (25)

and
\[ \frac{dLL(L)}{dL} = l(L) + L \frac{dl(L)}{dL} = \frac{\varphi(1, \rho) G''(l) l(L)}{\varphi(1, \rho) G''(l) - (1 + \zeta) y'L} > 0. \] (26)
Then, the real wage rate $\omega$ is decreasing with respect to $L$ because $y' \leq 0$.

Alternatively, from (23) $l$ is obtained as a function of $L_l$. Denote it by $l(L_l)$. Then,

$$\frac{dl(L_l)}{d(L_l)} = \frac{(1 + \zeta)y'}{\varphi(1, \rho)G''} \leq 0. \quad (27)$$

The (nominal) aggregate supply of the good is equal to

$$WL_l + \Pi = p_1 L_l y(L_l). \quad (28)$$

$L_l$ is an abbreviation of $L_l(L)$ or $L_l(L_l)$. The (nominal) aggregate demand is

$$\alpha(WL_l + \Pi) + G + M = \alpha p_1 L_l y(L_l) + G + M. \quad (29)$$

Since they are equal,

$$p_1 L_l y(L_l) = \alpha p_1 L_l y(L_l) + G + M, \quad \text{or} \quad p_1 L_l y(L_l) = \frac{G + M}{1 - \alpha}. \quad (30)$$

In real terms$^4$

$$L_l y(L_l) = \frac{1}{1-\alpha}(g + m), \quad \text{or} \quad L_l = \frac{1}{(1-\alpha)y(L_l)}(g + m), \quad (31)$$

where

$$g = \frac{G}{p_1}, \quad m = \frac{M}{p_1}.$$

(31) means that the net employment (employment $\times$ labor supply) $L_l$ is determined by $g + m$. $L_l y(L_l)$ is strictly increasing with respect to $L_l$ because

$$\frac{d(L_l y(L_l))}{d(L_l)} = y(L_l) + L_l y' = y(L_l) \left(1 + \frac{L_l y'}{y(L_l)}\right) = y(L_l)(1 + \zeta) > 0. \quad (32)$$

Therefore, there exists the unique value of $L_l$ which satisfies (31) given $g + m$. It is strictly increasing with respect to $g + m$. From (23) we obtain the value of $l(L_l)$, and the value of $L$ is determined by $L = \frac{L_l}{l(L_l)}$. $L_l$ can not be larger than $L_l(L_l)$. However, it may be strictly smaller than $L_l(L_l)$. Then, there exists involuntary unemployment, that is, $L < L_l$ because $L_l$ is strictly increasing with respect to $L$.

(31) means

$$L_l y(L_l) = \frac{G + M}{(1 - \alpha)p_1}. \quad (33)$$

This is the aggregate demand function given $G + M$. $L_l y(L_l)$ is constant given $g + m$. It is increasing with respect to $G + M$ given $p_1$, and decreasing with respect to $p_1$ given $G + M$. Since $L_l y(L_l)$ is increasing with respect to $L_l$, (33) means that the net employment $L_l$ is increasing with respect to $G + M$ given $p_1$ and decreasing with respect to $p_1$ given $G + M$.

$^4 \frac{1}{1-\alpha}$ is a multiplier.
4.2. Summary of Discussions

The real aggregate demand and net employment $Ll$ are determined by the real value of $g + m$. Labor supply of each individual is determined by $Ll$ according to (23), and the employment $L$ is determined by

$$L = \frac{Ll}{l(Ll)} \tag{34}$$

The employment may be smaller than the population of labor, then there exists involuntary unemployment. The real wage rate is determined by $Ll$ according to (22). There exists no mechanism to reduce involuntary unemployment unless $g + m$ is increased.

If

$$(1 + \zeta)y(Ll) > \varphi(1, \rho)G'(l) \quad \text{for any} \quad 0 < l < 1, \text{given} \quad L,$$

individuals choose $l = 1$, and then the labor supply is indivisible.

On the other hand, if

$$\lim_{Ll \to 0} (1 + \zeta)y(Ll) < \varphi(1, \rho)G'(0), \tag{35}$$

individuals choose $l = 0$. However, if $G'(0)$ is sufficiently small, $l > 0$.

Involuntary unemployment occurs when aggregate demand for the good is insufficient. Firms determine the number of employed workers needed to meet the demand for the good. If demand for the good is insufficient, the number of employed workers may be smaller than the population. Then, there exists involuntary unemployment. Under decreasing returns to scale the real wage rate is decreasing with respect to the output or employment. It may affect the individual labor supply. However, labor demand or employment (the number of employed workers) is not determined by the real wage rate. Under constant returns to scale the real wage rate is constant. Labor demand is determined by the aggregate demand for the good.

4.3. Comment on the Nominal Wage Rate

The reduction of the nominal wage rate induces a proportionate reduction of the price even when there exists involuntary unemployment, and it does not rescue involuntary unemployment (please see Chapter 2 of Otaki (2016)).

In the model of this section no mechanism determines the nominal wage rate. When the nominal value of $G + M$ increases, the nominal aggregate demand and supply increase. If the nominal wage rate rises, the price also rises. If the rate of an increase in the nominal wage rate is smaller than the rate of an increase in $G + M$, the real aggregate supply and the employment increase. Partition of the effects by an increase in $G + M$ into a rise in the nominal wage rate

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5 However, there is room for improvement of employment by the real balance effect if the nominal value of consumption by the older generation consumers is maintained. About this point please see Appendix.
(and the price) and an increase in the employment may be determined by bargaining between labor and firm\(^6\).

4.4. Full-employment Case

If \( L = L_f \), full-employment is realized. Then, (31) is written as

\[
L_f l(l_f) y(L_f l(L_f)) = \frac{1}{1 - \alpha}(g + m).
\]

(36)

\( l(l_f) \) is obtained from

\[
(1 + \zeta) y(L_f l) = \varphi(1, \rho) G'(l).
\]

(37)

\( L_f l(L_f) > Ll(L) \) for any \( L < L_f \) because \( Ll(L) \) is strictly increasing with respect to \( L \). Since \( L_f l(L_f) \) is constant, (36) is an identity not an equation. On the other hand, (31) is an equation not an identity. (36) should be written as

\[
\frac{1}{1 - \alpha}(g + m) \equiv L_f l(l_f) y(L_f l(L_f)).
\]

(37)

This defines the value of \( g + m \) which realizes the full-employment state.

From (37) we have

\[
p_1 = \frac{1}{(1 - \alpha)L_f l(l_f) y(L_f l(L_f))}(G + M),
\]

(38)

where

\[
g = \frac{G}{p_1}, \quad m = \frac{M}{p_1}.
\]

Therefore, the price level \( p_1 \) is determined by \( G + M \), which is the sum of nominal values of the government expenditure and consumption by the older generation. Also the nominal wage rate is determined by

\[
W = (1 + \zeta) y(L_f l(L_f)) p_1.
\]

(39)

4.5. Government Expenditure to Reduce Unemployment

Now we assume that the government collects a lump-sum tax \( T \) from the younger generation consumers. The aggregate demand and the aggregate supply are

\[
\alpha(WlL + \Pi - T) + G + M = \alpha(p_1 Ll y(Ll) - T) + G + M = p_1 Ll y(Ll).
\]

(40)

If \( Ll \) is a steady state value with constant price and output, the savings of the younger generation must be equal to \( M \). Thus,

\[
(1 - \alpha)(p_1 Ll y(Ll) - T) = G - T + M = M.
\]

(41)

\(^6\) As mentioned in Introduction, Otaki (2009) has shown the existence of involuntary unemployment using efficient wage bargaining. The arguments of this paper, however, do not depend on bargaining.
This means $G - T = 0$. Now we suppose that full-employment is realized by the government expenditure $G'$ with the tax $T'$ under constant price. Then, we have

$$(1 - \alpha)(p_1 L_f l y(L_f l) - T') = G' - T' + M. \quad (42)$$

Assume $p_1 L_f l y(L_f l) - T' > p_1 L l y(Ll) - T$, that is, the realization of full employment will increase consumers' disposable income. Then, from (41) and (42) we get $G' - T' > 0$. Therefore, budget deficits are necessary to move from the presence of involuntary unemployment to full-employment.

**A Simple Example**

Assume $M = 0$ and $T = 0$ in (40) with $L = L_f$. Then,

$$\alpha p_1 L_f l y(L_f l) + G = p_1 L_f l y(L_f l). \quad (43)$$

This means

$$G = (1 - \alpha)P_1 L_f l y(L_f l). \quad (44)$$

This is the government expenditure necessary to achieve full employment when the savings of the older generation is zero, and it is equal to the savings of the younger generation. Let denote it by $M'$. Let $G'$ and $T'$ be the government expenditure and the tax in the next period. The following relation holds under constant price.

$$p_1 L_f l y(L_f l) = \alpha[p_1 L_f l y(L_f l) - T'] + G' + M'. \quad (45)$$

To maintain full-employment with $T' = 0$, the savings of the younger generation must be $M'$. Therefore, we need

$$(1 - \alpha)P_1 L l y(Ll) = G' + M' = M'. \quad (46)$$

This means

$$G' = 0. \quad (47)$$

**4.6. Graphical Representation**

The output of the good $L l y(Ll)$ is an increasing function of the net employment $L l$ as shown in (32). (21) means that the price of the good is an increasing function of $L l$ given $W$ due to decreasing returns to scale $y'(Ll) \leq 0$. Therefore, the price of the good is an increasing function of the output $L l y(Ll)$ given $W$. Let denote this relation by

$$p_1 = \Psi(L l y(Ll)), \quad \text{with} \quad \Psi' \geq 0. \quad (48)$$
Figure 1 presents a graphical representation of arguments. The line (48) shows Eq. (48). If the production process is constant returns to scale, the price is constant, and the line (48) is horizontal. The curve (33) shows Eq. (33). (33) expresses that the aggregate demand for the good is decreasing with respect to $p_1$ given $G + M$ which is the sum of the nominal values of the government expenditure and the savings of the older generation consumers. The intersection point represents the equilibrium. The equilibrium value of $L_I y(L_I)$ may be smaller than $L_f y(L_f I)$. The thick curve shows Eq. (33) when the government expenditure $G$ increases. An increase in $G$ increases the equilibrium value of $L_I y(L_I)$.

![Graphical representation of arguments](image)

Figure 1: Equilibrium with Involuntary Unemployment

## 5. Concluding Remark

In this paper we have examined the existence of involuntary unemployment using a simple perfect competition model with divisible individual labor supply under decreasing or constant returns to scale technology. It seems to be possible to extend the analyses in this paper to monopolistic competition with increasing or constant returns to scale technology without changing main conclusions.

### Appendix

As mentioned in Introduction Schultz (1992) showed the impossibility of involuntary unemployment in an overlapping generations model. His arguments depend on the real balance effect caused by a fall in the nominal wage rate on consumption of the older generation consumers. It is stated in Page 69 of Schultz(1992) that:

In our model the presence of an old generation makes the real balance effect so strong that
involuntary unemployment is excluded.

As noted in footnote 5, the real balance effect caused by falling nominal wage rate and prices may reduce unemployment. We consider a three generations overlapping generations model with pay-as-you-go pension to explore the possibility of avoiding the real balance effect. Let us consider the following economy.

1. Each consumer lives three periods, Period 0 (childhood period), Period 1 (younger period) and Period 2 (older period). There are three generations, childhood, younger and older generations. In Period 0 the consumers consume $D$ units of the good by borrowing money from consumers in the previous generation. $D$ is constant. They must repay the debts in the next period when they are young.

2. In Period 1 the consumers are employed by firms or unemployed. If they are unemployed, they can not repay the debts. Therefore, each unemployed consumer receives unemployment benefit which is equal to his debt in the childhood period. The unemployment benefits are covered by taxes on employed younger consumers. Also employed younger consumers pay taxes for pay-as-you-go pensions for older generation consumers.

3. In Period 2 consumption of the consumers who retire is financed by their savings and pensions.

4. Since consumption in Period 0 of each consumer is constant, he determines his consumptions in Period 1 and Period 2 at the beginning of Period 1 given a state where he is employed or unemployed.

We use the following notations.

$D$: consumption of an individual in the childhood period. It is constant.
$\bar{D}$: consumption of a next generation consumer in the childhood period.
$R$: unemployment benefit for an unemployed individual. $R = D$.
$\Theta$: tax payment by an employed individual for the unemployment benefit.
$Q$: pay-as-you-go pension for an individual of the older generation.
$\bar{Q}$: pay-as-you-go pension for an individual of the younger generation when he retires.
$\Psi$: tax payment by an employed individual for the pay-as-you-go pension for the older generation consumers.

The following relationships hold.

$$L\Theta = (L_f - L)D, \quad L\Psi = L_f Q.$$  \hfill (A.1)
The budget constraint of an employed consumer is
\[ p_1 c_1 + p_2 c_2 = WI + \Pi - D - \Theta - \Psi + \hat{Q}, \] (A.2)
and that of an unemployed consumer is
\[ p_1 c_1 + p_2 c_2 = \Pi + \hat{Q}. \] (A.3)

The analysis that follows is similar. The (nominal) effective demand is
\[ Y = \alpha(WLI + L_f \Pi - L(D + \Theta) - L\Psi + L_f \hat{Q}) + G + M + \bar{D} \]
\[ = \alpha(WLI + L_f \Pi - L_f D - L_f Q + L_f \hat{Q}) + G + M + \bar{D}. \] (A.4)

\( M \) is the total consumption of the older generation consumers which are financed by their savings and pay-as-you go pensions. Their net savings is
\[ M - L_f Q. \] (A.5)

The Effects of Falling in the Nominal Wage Rate

If the nominal wage rate falls and the price of the good proportionately falls, we can assume that the real values of the following variables are maintained.
\[ \Pi, Q, \hat{Q}, G \text{ and } \bar{D}. \]

On the other hand, the nominal values of \( D \) and \( M - L_f Q \) are not changed. Therefore, whether a fall in the nominal wage rate increases or decreases the effective demand depends on whether
\[ M - \alpha L_f D - L_f Q. \] (A.6)

is positive or negative. If it is negative, a fall in the nominal wage rate decreases the effective demand and increases involuntary unemployment.

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