

Monetary economy and budget deficit in endogenous growth model

Yasuhito Tanaka 

Faculty of Economics, Doshisha University,
Kyoto, Japan

Correspondence

Yasuhito Tanaka, Faculty of Economics,
Doshisha University, Kamigyo-ku, Kyoto,
602-8580, Japan.
Email: ochibocho@gmail.com

Abstract

By an endogenous growth model with two-period overlapping generations structure and money holding of consumers, we examine the existence of budget deficit in an economy which endogenously grows by investments of firms. The main results are as follows. (1) Budget deficit is necessary for full employment under constant prices. (2) Inflation is induced if the actual budget deficit is greater than the value at which full employment is achieved under constant prices. (3) If the actual budget deficit is smaller than the value which is necessary and sufficient for full employment under constant prices, a recession occurs. Therefore, balanced budget cannot achieve full employment under constant prices. We do not assume that budget deficit must later be made up by budget surplus. This paper is an example of an analysis using a very simple model of the following statement by John Maynard Keynes.

“Unemployment develops, that is to say, because people want the moon;—men cannot be employed when the object of desire (i.e. money) is something which cannot be produced and the demand for which cannot be readily choked off. There is no remedy but to persuade the public that green cheese is practically the same thing and to have a green cheese factory (i.e. a central bank) under public control.”

In the appendix of this paper, we show that if money as well as goods are produced by capital and labor, budget deficit is not necessary for full employment under constant prices.

KEYWORDS

budget deficit, endogenous growth model, growing economy, money holding

JEL CLASSIFICATION

E12, E24

This is an open access article under the terms of the Creative Commons Attribution License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

© 2023 The Authors. *International Studies of Economics* published by John Wiley & Sons Australia, Ltd on behalf of Shanghai University of Finance and Economics.

1 | INTRODUCTION

The so-called endogenous growth theory, originated by Arrow (1962), Sidrauski (1967), Uzawa (1965) and developed by Romer (1986), Lucas (1988) and others, is based on the idea that the rate of economic growth is not given exogenously, but that the economy grows through human investment including education, physical investment and research and development. For example, the “Uzawa–Lucas model” (Lucas, 1988; Uzawa, 1965) is a model that explains long-term economic growth as consequence of human capital accumulation. Among others the AK-type endogenous growth model is one of the most simple and representative model. In this paper by an AK-type endogenous growth model with two-period overlapping generations structure and money holding of consumers, we examine the existence of budget deficit (fiscal deficit) in an economy which endogenously grows by investments of firms (specifically about a model we refer to Grossman and Yanagawa (1993) and, in particular, Maebayashi and Tanaka (2021), but our model is a simplified version adding money holding of consumers).

Since the purpose of the paper is to prove that budget deficit is necessary or inevitable in a growing economy when people hold money, or people derive utility not only from consumption but also from money holdings, we use the standard and most simple endogenous growth model, the AK-type model in which the production function has a constant returns to scale property, and investment in capital is characterized as increasing labor productivity (labor-augmenting productivity).

Various analyses have been conducted by Jumpei Tanaka on the significance of budget deficit and government debt in the economy and the intergenerational burden. Please see, Tanaka (2010, 2011a, 2011b, 2013).¹ Jumpei Tanaka's analysis focuses on the difference in economic welfare across generations with or without government debts, but his main model does not include economic growth and he assumes that all government debts are redeemed by taxes. Our interest in this paper lies elsewhere. Our interest is to prove that budget deficit is necessary and inevitable in a growing economy where consumers hold money. In our model, people save primarily through capital, but unlike previous traditional models, we consider that consumers will seek to hold money other than capital for reasons such as securing liquidity.

In the next section, we will present our model and prove that a budget deficit is necessary to achieve full employment under constant goods prices. The reasons are as follows. In a growing economy, the increase in savings translates into an increase in investment in capital and money holdings, but the increase in money holdings reduces investment demand, resulting in insufficient demand relative to productive capacity and necessitating a budget deficit. In our model, money is supplied by the payment of wages, which consumers use to pay taxes and to pay for consumption and capital investment when they are young. What remains after that is the amount of money held. In old age, people consume from invested capital, its earnings (interest), and money carried over. Section 3 briefly discusses recession and inflation. If the actual budget deficit is greater than what is necessary and sufficient for full employment under constant prices, inflation occurs. This result appears to be an assertion that fiscal spending or budget deficits are the cause of inflation, but it also holds that inflation is due to monetary factors, since the budget deficit increases as people hold more money. On the other hand, if the actual budget deficit is less than what is necessary and sufficient for full employment under constant prices, recession occurs. Therefore, balanced budget cannot achieve full employment under constant prices. We do not assume that budget deficit must later be made up by budget surplus.

¹In a previous study about the existence of involuntary unemployment, we used a similar but different overlapping generations model according to Otaki (2007, 2009, 2015), and in Tanaka (2022), similar issues have been analyzed by a traditional overlapping generations model with exogenous economic growth according to Diamond (1965).

In Section 4, we will very briefly analyze the monetary policy by issuance of government bonds. Issuance of government bonds in a period decreases the wage rate in the next period by decreasing the investment in capital.

This paper is an example of an analysis using a very simple model of the following statement by John Maynard Keynes.

Unemployment develops, that is to say, because people want the moon;—men cannot be employed when the object of desire (i.e. money) is something which cannot be produced and the demand for which cannot be readily choked off. There is no remedy but to persuade the public that green cheese is practically the same thing and to have a green cheese factory (i.e. a central bank) under public control. (Keynes, 1936, chap. 17)

In the appendix we show that if money as well as goods are produced by capital and labor, budget deficit is not necessary for full employment under constant prices. However, if money circulates with a value greater than the cost of production, a budget deficit will be necessary. The difference between the value of money and its production cost is the so-called *seigniorage*. A moderate seigniorage to economic growth without inflation is necessary unless the production of money is quite costly.

Some authors studied the problem of public debt using an overlapping generations model. For example, Yakita (2008) and Arai (2011) studied sustainability of public debt, and Tamai and Kamiguchi (2019) analyzed the welfare effects of deficit-financed public investment in an overlapping generations model. However, these authors, and most of the others, do not take into account money holdings by people. Of course, neither Diamond (1965) nor Maebayashi and Tanaka (2021) take into account people's money holdings. Critical to the conclusions of this paper is that we are considering a growing economy and that we are taking into account people's money holdings. Indeed, without money holdings, it would not be possible to say that budget deficits are essential. It is also crucial, as noted immediately above, that the money be unproduced.

We also do not consider the existence of government bonds, except in Section 4. Since government bonds do not have the liquidity of money, people do not derive utility from the holding of government bonds itself. Therefore, if government bonds do not generate interest similar to the return that capital generates, they will not be held. We assume in Section 4 that government bonds are held as a substitute for capital.

2 | BUDGET DEFICIT IN MONETARY ECONOMY

We introduce money demand or money holding of consumers into a simple endogenous growth model with overlapping generations structure. Mainly, we refer to Maebayashi and Tanaka (2021) about formulation of the overlapping generations endogenous growth model. Our model is its simplified version adding money holding of consumers.

Consumers live over two periods, younger (working) period and older (retired) period. Population of consumers is constant.

The utility of a younger consumer in Period 1 is represented by

$$u_1 = \left(c_1^y\right)^\beta \left(c_2^o\right)^\gamma \left(\frac{m_1}{p_1}\right)^{1-\beta-\gamma}, \quad 0 < \beta < 1, 0 < \gamma < 1, 0 < \beta + \gamma < 1.$$

c_1^y is his consumption in Period 1, c_2^o is his consumption in Period 2. In Period 2, he belongs to the older generation. m_1 is his money holding or demand for money in Period 1. Consumers

derive utility from holding money as well as from consumption in youth and old age. The budget constraint is

$$p_2 c_2^o = (1 + r_1) \left[(1 - \tau) w_1 l_1 - p_1 c_1^y - m_1 \right] + m_1. \quad (1)$$

w_1 is the wage rate, τ is the tax rate ($0 < \tau < 1$), and r_1 is the interest rate or the rate of return on capital in Period 1. The tax is paid by employed consumers. p_1 and p_2 are the price levels in Periods 1 and 2. l_1 is an indicator of whether the consumer is employed or not in Period 1, and takes the value of 1 if the consumer is employed and 0 if he is not employed. The (nominal) investment in capital is

$$(1 - \tau) w_1 l_1 - p_1 c_1^y - m_1. \quad (2)$$

It generates income at the rate of r_1 . For simplicity we omit depreciation of capital. The consumption in Period 2 equals (1). (1) is rewritten as

$$p_1 c_1^y + \frac{1}{1 + r_1} p_2 c_2^o + \frac{r_1}{1 + r_1} m_1 = (1 - \tau) w_1 l_1.$$

The Lagrange function for an employed consumer is

$$\mathcal{L} = (c_1^y)^\beta (c_2^o)^\gamma \left(\frac{m_1}{p_1} \right)^{1-\beta-\gamma} - \lambda \left[p_1 c_1^y + \frac{1}{1 + r_1} p_2 c_2^o + \frac{r_1}{1 + r_1} m_1 - (1 - \tau) w_1 l_1 \right].$$

The conditions for utility maximization are

$$\begin{aligned} \beta (c_1^y)^{\beta-1} (c_2^o)^\gamma \left(\frac{m_1}{p_1} \right)^{1-\beta-\gamma} &= \lambda p_1, \\ \gamma (c_1^y)^\beta (c_2^o)^{\gamma-1} \left(\frac{m_1}{p_1} \right)^{1-\beta-\gamma} &= \lambda \frac{1}{1 + r_1} p_2, \end{aligned}$$

and

$$(1 - \beta - \gamma) (c_1^y)^\beta (c_2^o)^\gamma \left(\frac{m_1}{p_1} \right)^{-\beta-\gamma} = \lambda \frac{r_1}{1 + r_1} p_1.$$

Or,

$$\begin{aligned} \beta (c_1^y)^{\beta-1} (c_2^o)^\gamma \left(\frac{m_1}{p_1} \right)^{1-\beta-\gamma} &= \lambda p_1 c_1^y, \\ \gamma (c_1^y)^\beta (c_2^o)^{\gamma-1} \left(\frac{m_1}{p_1} \right)^{1-\beta-\gamma} &= \lambda \frac{1}{1 + r_1} p_2 c_2^o, \\ (1 - \beta - \gamma) (c_1^y)^\beta (c_2^o)^\gamma \left(\frac{m_1}{p_1} \right)^{1-\beta-\gamma} &= \lambda \frac{r_1}{1 + r_1} m_1. \end{aligned}$$

Then, for employed consumers, we get

$$\begin{aligned} p_1 c_1^y &= \beta (1 - \tau) w_1, \\ p_2 c_2^o &= \gamma (1 + r_1) (1 - \tau) w_1, \end{aligned}$$

and

$$m_1 = \frac{1 + r_1}{r_1} (1 - \beta - \gamma)(1 - \tau)w_1. \quad (3)$$

For unemployed consumers

$$c_1^y = c_2^o = m_1 = 0.$$

However, unemployment insurance could also be incorporated into the model without changing the results. For simplicity, we exclude them. Let L_1 and L_f be the employment and the population of consumers (or labor supply) in Period 1. L_f is constant.

Let K_1 and K_2 be the capital in Period 1 and that in Period 2. The real value of capital in Period 2 is derived from (2) as follows.

$$\begin{aligned} K_2 &= \left[\frac{(1 - \tau)w_1 - m_1}{p_1} - c_1^y \right] L_1 = \left[\frac{1 - \beta}{p_1} (1 - \tau)w_1 - \frac{m_1}{p_1} \right] L_1 \\ &= \left[1 - \beta - \frac{1 + r_1}{r_1} (1 - \beta - \gamma) \right] \frac{(1 - \tau)w_1}{p_1} L_1. \end{aligned} \quad (4)$$

This means

$$p_1 K_2 = \left[1 - \beta - \frac{1 + r_1}{r_1} (1 - \beta - \gamma) \right] (1 - \tau)w_1 L_1.$$

From this we verify the following relation.

$$\begin{aligned} p_2 c_2^o L_1 &= (1 + r_1)p_1 K_2 + m_1 L_1 \\ &= (1 + r_1) \left[1 - \beta - \frac{1 + r_1}{r_1} (1 - \beta - \gamma) \right] (1 - \tau)w_1 L_1 + \frac{1 + r_1}{r_1} (1 - \beta - \gamma)(1 - \tau)w_1 L_1 \\ &= (1 + r_1)[1 - \beta - (1 - \beta - \gamma)](1 - \tau)w_1 L_1 = \gamma(1 + r_1)(1 - \tau)w_1 L_1. \end{aligned}$$

From (4)

$$\frac{m_1}{p_1} L_1 = \frac{(1 - \tau)w_1}{p_1} L_1 - c_1^y L_1 - K_2.$$

This equation implies that money holding is equal to wages paid minus taxes, investment in capital, and consumption, as described in the introduction. The capital in Period 1 is

$$K_1 = \frac{1}{p_0} \left[(1 - \tau)w_0 - p_0 c_0^y - m_0 \right] L_0.$$

p_0 and w_0 are the price level and the wage rate in Period 0, which is the previous period of Period 1. L_0 is the employment in Period 0. From this

$$p_1 c_1^o L_0 = (1 + r_0)p_0 K_1 + m_0 L_0 = (1 + r_0)\gamma(1 - \tau)w_0 L_0. \quad (5)$$

We assume full employment and constant prices until Period 0. Thus,

$$L_0 = L_f.$$

In Period 1 the production function of each firm is

$$Y_1 = K_1^\alpha (\theta \bar{K}_1 L_1)^{1-\alpha}, \quad 0 < \alpha < 1.$$

Y_1 , K_1 , and L_1 are the output, capital input, and labor input. θ is a positive constant. $\theta \bar{K}_1$ is the labor-augmenting productivity. \bar{K}_1 is the average capital per population, that is, $\bar{K}_1 = \frac{K_1}{L_f}$. However, K_1 in this formulation is the capital over all the economy, and \bar{K}_1 is given for the firms. The number of firms is normalized to one. The profit of a firm is

$$\pi_1 = p_1 K_1^\alpha (\theta \bar{K}_1 L_1)^{1-\alpha} - r_1 p_0 K_1 - w_1 L_1.$$

$p_0 K_1$ is the nominal amount of the capital at the time when the investment is made, Period 0. The first order conditions for profit maximization are

$$p_1 \alpha K_1^{\alpha-1} (\theta \bar{K}_1 L_1)^{1-\alpha} = p_0 r_1,$$

and

$$p_1 \theta \bar{K}_1 (1 - \alpha) K_1^\alpha (\theta \bar{K}_1 L_1)^{-\alpha} = w_1.$$

In the equilibrium $\bar{K}_1 = \frac{K_1}{L_f}$. For simplicity we assume $L_f = 1$, and $0 < L_1 \leq 1$. Thus, $\bar{K}_1 = K_1$, and

$$p_0 r_1 = p_1 \alpha K_1^{\alpha-1} (\theta K_1 L_1)^{1-\alpha} = p_1 \alpha (\theta L_1)^{1-\alpha},$$

or

$$r_1 = \frac{p_1}{p_0} \alpha (\theta L_1)^{1-\alpha}, \quad (6)$$

and

$$w_1 = p_1 \theta K_1 (1 - \alpha) K_1^\alpha (\theta K_1 L_1)^{-\alpha} = p_1 (1 - \alpha) \theta^{1-\alpha} L_1^{-\alpha} K_1. \quad (7)$$

The wage rate is proportional to the capital input. From them, we obtain

$$\begin{aligned} p_0 r_1 K_1 &= p_1 \alpha (\theta L_1)^{1-\alpha} K_1, \\ w_1 L_1 &= p_1 (1 - \alpha) (\theta L_1)^{1-\alpha} K_1, \\ p_0 r_1 K_1 + w_1 L_1 &= p_1 (\theta L_1)^{1-\alpha} K_1, \end{aligned} \quad (8)$$

and

$$p_1 Y_1 = p_1 K_1^\alpha (\theta K_1 L_1)^{1-\alpha} = p_1 (\theta L_1)^{1-\alpha} K_1 = p_0 r_1 K_1 + w_1 L_1.$$

The same procedure is used to obtain the values of the variables in Period 0. Since we assume constant prices and full employment until Period 0,

$$r_0 = \alpha\theta^{1-\alpha}, w_0 = p_0(1 - \alpha)\theta^{1-\alpha}K_0.$$

From (4), (6), and (8),

$$\frac{K_2}{K_1} = \left[1 - \beta - \frac{1 + r_1}{r_1}(1 - \beta - \gamma) \right] (1 - \tau)(1 - \alpha)(\theta L_1)^{1-\alpha}, \quad (9)$$

with

$$r_1 = \frac{p_1}{p_0} \alpha (\theta L_1)^{1-\alpha}.$$

Under constant prices and full employment,

$$\frac{K_2}{K_1} = \left[1 - \beta - \frac{1 + r_1}{r_1}(1 - \beta - \gamma) \right] (1 - \tau)(1 - \alpha)\theta^{1-\alpha}$$

with

$$r_1 = \alpha\theta^{1-\alpha} = r_0.$$

$\frac{K_2}{K_1}$ is the gross growth rate, and the growth rate is $\frac{K_2}{K_1} - 1$. When $\gamma \rightarrow 1 - \beta$ ($1 - \beta - \gamma \rightarrow 0$),

$$\frac{K_2}{K_1} \rightarrow (1 - \beta)(1 - \tau)(1 - \alpha)(\theta L_1)^{1-\alpha}.$$

This is the gross growth rate when consumers hold no money. We should assume that it is larger than one, that is, the growth rate is positive. (9) is increasing in β and γ . Thus, the growth rate when consumers hold money is smaller than that when consumers hold no money. However, we assume that the gross growth rate is larger than one, and the growth rate is positive even when consumers hold money. Therefore,

$$\frac{K_2}{K_1} > 1, \quad \frac{K_2}{K_1} - 1 > 0. \quad (10)$$

$\frac{K_2}{K_1}$ is the gross growth rate from Period 1 to Period 2. The gross growth rate from Period 0 to Period 1 is $\frac{K_1}{K_0}$. Under constant prices and full employment they are equal.

From (7) and $L_0 = 1$,

$$\frac{w_1}{w_0} = \frac{p_1}{p_0} (L_1)^{-\alpha} \frac{K_1}{K_0}.$$

From (3)

$$\frac{m_1}{m_0} = \frac{\frac{1+r_1}{r_1} \frac{w_1}{w_0}}{\frac{1+r_0}{r_0} \frac{w_0}{w_0}} = \frac{1+r_1}{1+r_0} \left(\frac{p_0}{p_1} \right) (L_1)^{\alpha-1} \frac{w_1}{w_0} = \frac{1+r_1}{1+r_0} (L_1)^{-1} \frac{K_1}{K_0}. \quad (11)$$

Let G_1 be the fiscal spending in Period 1. The market equilibrium condition is

$$p_1 c_1^y L_1 + p_1 c_1^o + p_1 (K_2 - K_1) + G_1 = p_1 Y_1 = p_0 r_1 K_1 + w_1 L_1.$$

The left-hand side is the total demand, and the right-hand side is the total supply. $p_1 (K_2 - K_1)$ represents the cost required so as to increase the real value of the capital from K_1 to K_2 . Since

$$p_1 c_1^y = \beta(1 - \tau)w_1, p_1 c_1^o = (1 + r_0)p_0 K_1 + m_0,$$

we have

$$\beta(1 - \tau)w_1 L_1 + (1 + r_0)p_0 K_1 + m_0 + p_1 (K_2 - K_1) + G_1 = p_0 r_1 K_1 + w_1 L_1. \quad (12)$$

From this

$$\beta(1 - \tau)w_1 L_1 + (1 + r_0)p_0 K_1 + m_0 + p_1 (K_2 - K_1) + G_1 - \tau w_1 L_1 = p_0 r_1 K_1 + w_1 L_1 - \tau w_1 L_1.$$

Therefore,

$$(1 + r_0)p_0 K_1 + m_0 + p_1 (K_2 - K_1) + G_1 - \tau w_1 L_1 = p_0 r_1 K_1 + (1 - \beta)(1 - \tau)w_1 L_1.$$

Since, from (4)

$$p_1 K_2 = (1 - \beta)(1 - \tau)w_1 L_1 - m_1 L_1,$$

or

$$(1 - \beta)(1 - \tau)w_1 L_1 = p_1 K_2 + m_1 L_1,$$

we obtain

$$(1 + r_0)p_0 K_1 + m_0 + p_1 (K_2 - K_1) + G_1 - \tau w_1 L_1 = p_0 r_1 K_1 + p_1 K_2 + m_1 L_1. \quad (13)$$

Since

$$r_0 = \alpha \theta^{1-\alpha}, r_1 = \frac{p_1}{p_0} \alpha (\theta L_1)^{1-\alpha},$$

we have

$$\begin{aligned} G_1 - \tau w_1 L_1 &= m_1 L_1 - m_0 + p_0 (r_1 - r_0) K_1 + (p_1 - p_0) K_1 \\ &= m_1 L_1 - m_0 + \alpha [p_1 (\theta L_1)^{1-\alpha} - p_0 \theta^{1-\alpha}] K_1 + (p_1 - p_0) K_1. \end{aligned} \quad (14)$$

Suppose constant prices and full employment. Then,

$$p_1 = p_0, L_1 = L_f = 1, r_1 = r_0, \frac{K_1}{K_0} = \frac{K_2}{K_1}.$$

Therefore, from (11) and (14) we get

$$\begin{aligned}
 G_1 - \tau w_1 &= m_1 - m_0 = \left(\frac{K_1}{K_0} - 1 \right) m_0 = \left(1 - \frac{1}{\frac{K_1}{K_0}} \right) m_1 \\
 &= \left(1 - \frac{1}{\frac{K_1}{K_0}} \right) \frac{1 + r_1}{r_1} (1 - \beta - \gamma) (1 - \tau) w_1 \\
 &= \left(1 - \frac{1}{\frac{K_1}{K_0}} \right) \frac{1 + r_1}{r_1} (1 - \beta - \gamma) \frac{p_1 c_1^y}{\beta}.
 \end{aligned} \tag{15}$$

By (10), so long as $1 - \beta - \gamma > 0$, this is positive. Thus, we have shown the following result.

Proposition 1. *If the economy endogenously grows at a positive rate, and the consumers derive positive utility from holding money, we need positive budget deficit to maintain full employment under constant prices.*

3 | INFLATION AND RECESSION

Equation (15) in the previous section means that we need the budget deficit described in (15), or we need the fiscal spending,

$$G_1 = \left(1 - \frac{1}{\frac{K_2}{K_1}} \right) m_1 + \tau w_1,$$

to maintain full employment under constant prices.

3.1 | Inflation

Suppose full employment only. Thus, $L_1 = 1$. Then, (14) is reduced to

$$G_1 - \tau w_1 = m_1 - m_0 + \alpha \theta^{1-\alpha} (p_1 - p_0) K_1 + (p_1 - p_0) K_1. \tag{16}$$

By (6) and (11), $\frac{m_1}{m_0}$ is increasing in p_1 given p_0 . Therefore, if the actual budget deficit is larger than (16) with $p_1 = p_0$, which is the threshold value of the budget deficit, p_1 should increase. Then, inflation is triggered.

Summarizing the results,

Proposition 2. *If the actual budget deficit is larger than the value which is necessary and sufficient for full employment under constant prices, an inflation is triggered.*

3.2 | Recession

Assume constant prices, that is, $p_1 = p_0$. Then, (14) is reduced to

$$G_1 - \tau w_1 L_1 = m_1 L_1 - m_0 + \alpha p_1 [(\theta L_1)^{1-\alpha} - \theta^{1-\alpha}] K_1. \quad (17)$$

From (11),

$$\frac{m_1 L_1}{m_0} = \frac{1 + r_1}{1 + r_0} \frac{K_1}{K_0}.$$

Since r_1 is increasing in L_1 (see 6), this is increasing in L_1 . If the actual budget deficit is smaller than (17) with $L_1 = 1$, which is also the threshold value of the budget deficit, L_1 should be smaller than $L_f = 1$, and recession occurs.

Summarizing the results,

Proposition 3. *If the actual budget deficit is smaller than the value which is necessary and sufficient for full employment under constant prices, a recession occurs.*

3.3 | Recovery from recession

We consider Period 2 when recession occurred in Period 1. Suppose that we recover full employment in Period 2 under constant prices. Taking into account that $L_1 < 1$, (14) in Period 2 becomes

$$G_2 - \tau w_2 = m_2 - m_1 L_1 + p_1 \alpha [\theta^{1-\alpha} - (\theta L_1)^{1-\alpha}] K_2. \quad (18)$$

Under full employment and constant prices, (15) is rewritten for Period 2 as follows.

$$G_2 - \tau w_2 = m_2 - m_1. \quad (19)$$

Comparing (18) with (19), if $L_1 < 1$ (recession in Period 1), (18) is larger than (19). Therefore, we get the following result.

Proposition 4. *If a recession occurs in a period, we need a larger budget deficit in the next period to recover full employment than that when full employment is continuously maintained.*

3.4 | Some discussion

Equation (14) expresses the most general relation. It does not assume constant price nor full employment in Period 1, although constant price and full employment are assumed until Period 0. The claim of Proposition 1 is that to maintain full employment without price change in Period 1, the budget must be in deficit. The reasons are as follows. In a growing economy, the increase in savings translates into an increase in investment in capital and money holdings, but the increase in money holdings reduces investment demand, resulting in insufficient demand relative to productive capacity and necessitating a budget deficit.

On the other hand, Proposition 2 appears to be an assertion that fiscal spending or budget deficits are the cause of inflation, but it also holds that inflation is due to monetary factors, since the budget deficit increases as people hold more money, $m_1 - m_0$ (see 16).

Proposition 3 shows that a recession occurs when the budget deficit is insufficient under constant prices. Although recessions are thought to be accompanied by involuntary unemployment, this paper does not address the question of why involuntary unemployment occurs. However, if the budget deficit is insufficient and prices remain unchanged, involuntary unemployment must occur.

The threshold value of the budget deficit for inflation and recession is obtained from (15) as follows:

$$(m_1 - m_0)|_{p_1=p_1, L_1=1} = \left(\frac{K_1}{K_0} - 1 \right) m_0.$$

$\frac{K_1}{K_0} - 1$ is the growth rate. K_1 has been determined by the savings in Period 0. Therefore, the threshold value has been determined in Period 0.

Proposition 4 reveals that overcoming the recession would require an even larger budget deficit than if full employment were maintained. The budget deficit for this purpose should not be covered by later taxes. To do so would plunge the economy back into recession.

4 | MONETARY POLICY BY GOVERNMENT BONDS

We have analyzed only fiscal policy by fiscal spending and taxes. Suppose that the government issues government bonds at the same rate of return as capital. Let b_1 be the government bonds held by each consumer in Period 1. Then, from (2) and (4) the investment in capital is

$$\left[\frac{(1 - \tau)w_1 - m_1 - b_1}{p_1} - c_1^y \right] L_1.$$

It is smaller than (4) given p_1 , w_1 and τ . By (6) the equilibrium value of the rate of return is determined by the employment. It does not depend on the investment in capital. On the other hand, by (7) the wage rate is increasing in the capital in the same period. Therefore, issuance of the government bonds in a period decreases the wage rate in the next period.

In this case instead of (4) and (5) we have

$$K_2 = \left[\frac{1 - \beta}{p_1} (1 - \tau)w_1 - \frac{b_1}{p_1} - \frac{m_1}{p_1} \right] L_1,$$

$$p_1 c_1^o L_0 = (1 + r_0)p_0 K_1 + (1 + r_0)b_0 L_0 + m_0 L_0.$$

b_0 is the government bonds held by each consumer in Period 0. Then, instead of (12), (13), and (14), we get

$$\begin{aligned} & \beta(1 - \tau)w_1 L_1 + (1 + r_0)p_0 K_1 + (1 + r_0)b_0 + m_0 + p_1(K_2 - K_1) + G_1 = p_0 r_1 K_1 + w_1 L_1, \\ & (1 + r_0)p_0 K_1 + (1 + r_0)b_0 + m_0 + p_1(K_2 - K_1) + G_1 - \tau w_1 L_1 \\ & = p_0 r_1 K_1 + p_1 K_2 + b_1 L_1 + m_1 L_1, \end{aligned}$$

and

$$G_1 - \tau w_1 L_1 + r_0 b_0 = m_1 L_1 - m_0 + b_1 L_1 - b_0 + p_0(r_1 - r_0)K_1 + (p_1 - p_0)K_1.$$

The left-hand side is the budget deficit including interest payments on government bonds. This is positive when $p_1 = p_0$, $r_1 = r_0$ and $L_1 = 1$.

5 | CONCLUSION

In this paper, we have mainly proved that the budget deficit is necessary and inevitable to maintain full employment under constant prices in a growing economy by incorporating consumers' desire to hold money into the overlapping generations endogenous growth model. As mentioned in the introduction, critically important to the conclusions of this paper is that it considers a growing economy and that it takes into account people's money holdings. Indeed, without money holdings, it cannot be said that budget deficits are essential. Also, it is crucial that money is not produced. This paper does not pursue the causes of involuntary unemployment. However, it is believed that deflation has not occurred to the extent that the real balance effect could realistically eliminate the recession.

AUTHOR CONTRIBUTIONS

Yasuhiro Tanaka: Conceptualization; formal analysis; investigation; methodology; supervision; validation; writing—original draft; writing—review and editing.

ACKNOWLEDGMENTS

The author would like to thank the anonymous referees of this journal for their valuable comments and suggestions which have substantially improved this paper. The author is entirely responsible for any remaining errors. This work was supported by the Japan Society for the Promotion of Science KAKENHI Grant Number 18K01594.

CONFLICT OF INTEREST STATEMENT

The author declares no conflict of interest.

ETHICS STATEMENT

None declared.

ORCID

Yasuhiro Tanaka  <http://orcid.org/0000-0003-1435-462X>

REFERENCES

- Arai, Real. 2011. "Productive Government Expenditure and Fiscal Sustainability." *FinanzArchiv* 67: 327–51.
- Arrow, Kenneth J. 1962. "The Economic Implications of Learning by Doing." *Review of Economic Studies* 29: 155–73.
- Diamond, Peter A. 1965. "National Debt in a Neoclassical Growth Model." *American Economic Review* 55: 1126–50.
- Grossman, Gene M., and Noriyuki Yanagawa. 1993. "Asset Bubbles and Endogenous Growth." *Journal of Monetary Economics* 31: 3–19.
- Keynes, John Maynard. 1936. *The General Theory of Employment, Interest and Money*. Cham, Switzerland: Macmillan.
- Lucas, Robert E., Jr. 1988. "On the Mechanics of Economic Development." *Journal of Monetary Economics* 22: 3–42.
- Maebayashi, Noritaka, and Jumpei Tanaka. 2021. "Limited Asset Market Participation and Fiscal Sustainability." The Society for Economic Studies, The University of Kitakyushu, Working Paper Series No. 2020-4. https://www.kitakyu-u.ac.jp/economy/uploads/2020_4.pdf
- Otaki, Masayuki. 2007. "The Dynamically Extended Keynesian Cross and the Welfare-Improving Fiscal Policy." *Economics Letters* 96: 23–9. <https://www.sciencedirect.com/science/article/abs/pii/S0165176506004277>

- Otaki, Masayuki. 2009. "A Welfare Economics Foundation for the Full-Employment Policy." *Economics Letters* 102: 1–3. <https://www.sciencedirect.com/science/article/abs/pii/S0165176508002292>
- Otaki, Masayuki. 2015. *Keynesian Economics and Price Theory: Re-orientation of a Theory of Monetary Economy*. Japan: Springer.
- Romer, Paul M. 1986. "Increasing Returns and Long-Run Growth." *Journal of Political Economy* 94: 1002–1037.
- Sidrauski, Miguel. 1967. "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review* 51: 534–44.
- Tamai, Toshiki, and Akira Kamiguchi. 2019. "Deficit-Financed Public Investment, Economic Growth, and Welfare in an Overlapping Generations Model." Economic research Center discussion paper, E-Series, E19-1, Economic Research Center, Graduate School of Economics, Nagoya University, Japan.
- Tanaka, Jumpei. 2010. *Basics of Keynesian Economics (in Japanese)*. Kyushu Daigaku Shuppankai.
- Tanaka, Jumpei. 2011a. "Macroeconomic Theory Based on "Non-walrasian Approach" Chapter 2 IS-LM Model and its Application (in Japanese)." The Society for Economic Studies, The University of Kitakyushu, Working Paper Series No. 2011-2. https://www.kitakyu-u.ac.jp/economy/uploads/2011_01.pdf
- Tanaka, Jumpei. 2011b. "Macroeconomic Theory Based on "Non-walrasian Approach" Chapter 3 AD-AS Model And Its Application (in Japanese)." The Society for Economic Studies, The University of Kitakyushu, Working Paper Series No. 2011-4. https://www.kitakyu-u.ac.jp/economy/uploads/2011_03.pdf
- Tanaka, Jumpei. 2013. "Welfare Analysis of Fiscal Policies in a Fixed Price Overlapping Generations Model." The Society for Economic Studies, The University of Kitakyushu, Working Paper Series No. 2012-11. https://www.kitakyu-u.ac.jp/economy/uploads/2012_11.pdf
- Tanaka, Yasuhito. 2022. "Budget Deficit in a Growing Monetary Economy." *Applied Mathematical Sciences* 16: 359–66. <http://www.m-hikari.com/ams/ams-2022/ams-5-8-2022/916812.html>
- Uzawa, Hirofumi. 1965. "Optimum Technical Change in an Aggregative Model of Economic Growth." *International Economic Review* 6: 18–31.
- Yakita, Akira. 2008. "Sustainability of Public Debt, Public Capital Formation, and Endogenous Growth in an Overlapping Generations Setting." *Journal of Public Economics* 92: 897–914.

How to cite this article: Tanaka, Yasuhito. 2023. "Monetary Economy and Budget Deficit in Endogenous Growth Model." *International Studies of Economics* 1–14. <https://doi.org/10.1002/ise3.49>

APPENDIX A: WHEN MONEY IS PRODUCED

This paper is an attempt of an analysis by a very simple model of the following statement by John Maynard Keynes.

"Unemployment develops, that is to say, because people want the moon; — men cannot be employed when the object of desire (i.e. money) is something which cannot be produced and the demand for which cannot be readily choked off. There is no remedy but to persuade the public that green cheese is practically the same thing and to have a green cheese factory (i.e. a central bank) under public control."

In this appendix we suppose that money is produced by capital and labor. The production function is

$$Y_1 + \frac{\overline{M}_1}{p_1} = K_1^\alpha (\theta \overline{K}_1 L_1)^{1-\alpha}.$$

\overline{M}_1 is the supply of money in Period 1. It is the total cost to produce money in Period 1. The market equilibrium condition for the goods is

$$p_1 c_1^y L_1 + p_1 c_1^o + p_1 (K_2 - K_1) + G_1 = p_1 Y_1 - \overline{M}_1 = p_0 r_1 K_1 + w_1 L_1 - \overline{M}_1.$$

The condition for the money market equilibrium is that the supply of money equals an increase in money holdings. Therefore,

$$\overline{M}_1 = m_1 L_1 - m_0.$$

Then, under full employment and constant prices, (15) is rewritten as

$$G_1 - \tau w_1 = 0.$$

Thus, if money as well as goods are produced by capital and labor, budget deficit is not necessary for full employment under constant prices.

However, if money circulates with a value greater than the cost of production, a budget deficit will be necessary. In that case we have

$$\overline{M}_1 < m_1 L_1 - m_0.$$

Then,

$$m_1 L_1 - m_0 - \overline{M}_1$$

is the *seigniorage*. A moderate seigniorage to economic growth without inflation is necessary unless the production of money is quite costly.