

AD-AS ANALYSIS FROM THE PERSPECTIVE OF FUNCTIONAL FINANCE THEORY AND MMT

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Abstract

Research background: In the past few years, MMT (Modern Money Theory or Modern Monetary Theory) has been increasingly discussed in Japan as well as in the U.S. However, both in the U.S. and Japan, analysis using mathematics is lacking and analysis using IS-LM and AD-AS analysis methods in macroeconomics is almost non-existent.

Purpose: We present an AD-AS analysis from the perspective of Functional Finance Theory and MMT (Modern Monetary Theory).

Research methodology: Graphical analyses with some mathematical analyses. Mathematical analyses include graphic analysis using AD and AS curves and calculations to derive equations representing them based on models of consumer and firm behavior.

Results: Using an overlapping generations model under monopolistic competition we show the following results by calculations and graphical analyses.

1. The budget deficit (including interest payments on government bonds) equals an increase in the savings of consumers from period to period. This result means that the debt-GDP ratio would not diverge to infinity, but would remain at a finite value whether the interest rate of government bonds is larger or smaller than the growth rate.
2. We need a budget deficit (including interest payments) to maintain full employment without inflation under economic growth. However, if the interest rate of government bonds is larger than the growth rate, we need a budget surplus (excluding interest payments) to maintain full employment without inflation.
3. A return to full employment from a recession can be achieved by implementing the appropriate fiscal policies through increased government spending or tax cuts.
4. Excessive government expenditure or an insufficient tax under full employment induces inflation.

Novelty: This is probably the first AD-AS analysis done from the standpoint of Functional Finance Theory and MMT.

Keywords: AD curve, AS curve, Functional Finance Theory, MMT (Modern Monetary Theory), budget deficit

JEL classification: E12, E24

Introduction

Japan's outstanding government debt is said to be in a critical condition, exceeding 1,200 trillion yen. On the other hand, there are those who believe that there is no problem in accumulating a budget deficit if the debt is not owed to foreign countries, and that fiscal policy should be evaluated only on its effectiveness in preventing inflation and achieving full employment and stable economic growth. One such theory is the so-called Functional Finance Theory of Lerner (1943, 1944). MMT (Modern Monetary Theory – Wray, 2015; Mitchell *et al.*, 2019; Kelton, 2020), which has recently become popular in Japan as well as in the United States, is also a representative example. MMT proponents themselves admit that Lerner's theory is their source. However, it is often pointed out that MMT lacks mathematical and theoretical models for analysis, in contrast to mainstream economics, which is premised on a neoclassical framework. In Japan as well, there are various introductory books for the general public who are not familiar with economics, for example, Mochizuki (2020), Morinaga (2020), Nakano (2020), Park (2020), Shimakura (2019), but there is almost no academic literature on macroeconomic research from the standpoint of MMT. The author himself has developed a mathematical analysis of MMT in several recent studies (for example, Tanaka, 2021a; Tanaka, 2021b).

In this paper we present a graphical analysis using AD and AS curves for the arguments of MMT.

This paper is one of the attempts to give a theoretical basis to MMT. In particular, we provide a rationale for the following claims (Kelton, 2020). We refer to the summary of Kelton's book by Hogan (2021). In fact, Hogan criticizes that Kelton is wrong, but he summarizes Kelton's argument to the point.

1. The treasury creates new money.

If the savings are made by money, the money supply equals the savings. An increase in the money supply equals the increase in the net savings. An increase in the net savings from a period to the next period equals the budget deficit. The rate of an increase in the net savings, which equals the rate of an increase in the money supply, equals the rate of economic growth, and therefore the budget deficit and an increase in the money supply does not cause inflation in this case. In this paper we assume that the savings are made in government bonds, so the government bonds play the role of money. More generally, the savings may be made in both money and government bonds.

2. Inflation is caused by federal government deficit spending, not by Fed policy.

If the actual budget deficit is larger than the budget deficit that is necessary and sufficient to maintain full employment under economic growth with constant price, the price of the goods will rise.

3. Federal government spending is not related to taxes or borrowing.

Sustained budget deficits are necessary to maintain full employment under economic growth, and these budget deficits make it possible to maintain full employment. It is impossible to maintain full employment in a growing economy with a balanced budget. Therefore, even if the budget deficit to maintain full employment is financed by the national debt, it does not need to be repaid or redeemed, and must not be repaid or redeemed. Future budget surpluses need not and must not make up the deficits for growth.

The structure of this paper in the subsequent sections is as follows. The next section is the Materials and Methods section. In that section we present our model about consumer and firm behavior, and present mathematical calculations to derive equations for the AD and AS curves. In Section 2, the Results section, we present the main results of this paper, some propositions with graphical analyses using AD and AS curves. Section 3 is the Discussion and Conclusion section. In this section we summarize the results, and consider their implications for economic policy.

The novelty of this research is that we present formal analyses of the Functional Finance Theory (and MMT), especially, we analyze fiscal policy to restore full employment from a recession, and inflation caused by excessive budget deficits by AD and AS curves based on an analysis of firm and consumer behavior using an overlapping generations model under imperfect competition.

1. Materials and methods

This research is a theoretical research with mathematical and graphical analyses about the relation between budget deficit and full employment. Detailed calculations are included in the appendices.

1.1. The model

We consider an overlapping generations (OLG) model of two generations of consumers. The consumers live over two periods, Period 1 (the younger or working period) and Period 2 (the older or retired period). In Period 1 they supply a unit of labor, earn a wage and receive profits from firms, consume goods and have savings for their retirement. They are employed or unemployed. Savings are not in money, but in interest-generating government bonds. In Period 2 they consume goods with savings carried over from Period 1. There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0, 1]$.

Goods z is monopolistically produced by firm z with decreasing returns to scale technology. Each consumer inherits ownership of the firms from the previous generation. Corporate profits are distributed equally to consumers. Employment in a period is denoted by L , and that in the previous period is denoted by \bar{L} . Employment in the full employment state is L_f . The economy grows by technological progress at the rate of $\gamma - 1 > 0$. Our model is based on the model of Tanaka (2011), Tanaka (2013), Otaki (2007), Otaki (2009), Otaki (2015) and Tanaka (2021a).

1.2. Consumers' utility maximization

We consider the utility maximization of employed consumers and that of unemployed consumers. Such a model is according to Dixit and Stiglitz (1977). Denote consumption of goods z of an employed consumer in Period 1 by $c_1^e(z)$, and that in Period 2 by $c_2^e(z)$. The utility function of an employed consumer is:

$$U^e = \alpha \ln C_1^e + (1 - \alpha) \ln C_2^e, 0 < \alpha < 1 \quad (1)$$

where

$$C_1^e = \left(\int_0^1 c_1^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{\eta}{\eta-1}}, \quad C_2^e = \left(\int_0^1 c_2^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{\eta}{\eta-1}}$$

They are consumption baskets for employed consumers. The utility function of an unemployed consumer is:

$$U^u = \alpha \ln C_1^u + (1 - \alpha) \ln C_2^u \quad (2)$$

where

$$C_1^u = \left(\int_0^1 c_1^u(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{\eta}{\eta-1}}, \quad C_2^u = \left(\int_0^1 c_2^u(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{\eta}{\eta-1}}$$

They are consumption baskets for unemployed consumers. $c_1^u(z)$ is the consumption of goods z of an unemployed consumer in Period 1, and $c_2^u(z)$ is that in Period 2. The value of α or $1 - \alpha$ includes the discount of consumption in the older period, and $\eta > 1$.

Let $p_1(z)$ be the price of goods z in Period 1, and $p_2(z)$ be that in Period 2. Then, the budget constraint for an employed consumer is:

$$\int_0^1 p_1(z) c_1^e(z) dz + \frac{1}{1+r} \int_0^1 p_2(z) c_2^e(z) dz = W + \Pi - T \quad (3)$$

W is the nominal wage rate, T is the (lump-sum) tax, and Π is the profit of firms that is allocated to each younger consumer, whether he/she is employed or not. Also, each consumer pays a tax whether he/she is employed or not. r is the interest rate of government bonds. We assume that the interest rate is exogenously given. But, assuming that the savings are made by government bonds and money, and money supply is determined by the government (or central bank), we can consider an endogenously determined interest rate.

The budget constraint for an unemployed consumer is:

$$\int_0^1 p_1(z) c_1^u(z) dz + \frac{1}{1+r} \int_0^1 p_2(z) c_u^e(z) dz = \Pi - T \quad (4)$$

Let

$$P_1 = \left(\int_0^1 p_1(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}}, \quad P_2 = \left(\int_0^1 p_2(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}}$$

They are price indices of the goods, or the prices of consumption baskets.

By calculations in Appendix A we obtain the demand functions for goods z of an employed consumer in Periods 1 and 2 as follows:

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1} \right)^{-\eta} C_1^e = \left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{\alpha(W + \Pi - T)}{P_1} \quad (5)$$

and

$$c_2^e(z) = \left(\frac{p_1(z)}{P_2} \right)^{-\eta} C_2^e = \left(\frac{p_2(z)}{P_2} \right)^{-\eta} \frac{(1+r)(1-\alpha)(W + \Pi - T)}{P_2} \quad (6)$$

Similarly, the demand functions for goods z of an unemployed consumer in Periods 1 and 2 are

$$c_1^u(z) = \left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{\alpha(\Pi - T)}{P_1} \quad (7)$$

and

$$c_2^u(z) = \left(\frac{p_2(z)}{P_2} \right)^{-\eta} \frac{(1+r)(1-\alpha)(\Pi - T)}{P_2} \quad (8)$$

1.3. Government expenditure

Next consider government expenditure. The government decides its expenditure to goods z , $g(z)$, to maximize

$$\mathcal{G} = \left(\int_0^1 g(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{\eta}{\eta-1}}$$

subject to

$$\int_0^1 p_1(z) g(z) dz = G$$

G is the total government expenditure. By calculations in Appendix B, we get

$$g(z) = \left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{G}{P_1} \quad (9)$$

1.4. Firms' profit maximization and market equilibrium

The total demand for goods z by younger generation consumers is

$$\left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{\alpha(W + \Pi - T)L}{P_1} + \left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{\alpha(\Pi - T)(L_f - L)}{P_1} = \left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{\alpha(WL + \Pi L_f - TL_f)}{P_1}$$

This is the sum of the demand of employed consumers and that of unemployed consumers. On the other hand, since the economy grows at the rate of $\gamma - 1$, the demand for goods z of the older generation consumers is:

$$\left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{\frac{1+r}{\gamma}(1-\alpha)(\bar{W}L + \bar{\Pi}L_f - TL_f)}{P_1}$$

The profit of firms that is allocated to each consumer in the previous period is $\frac{1}{\gamma}\bar{\Pi}$, and the tax in the previous period is $\frac{1}{\gamma}T$. The nominal wage rate in the previous period equals $\frac{1}{\gamma}\bar{W}$. Without inflation \bar{W} equals W . The savings of the older generation consumers is:

$$\frac{1}{\gamma}(1-\alpha)(\bar{W}L + \bar{\Pi}L_f - TL_f)$$

This generates interest at the rate of r .

The total demand for goods z is represented as follows:

$$y(z) = \left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{\alpha (WL + \Pi L_f - TL_f) + \frac{1+r}{\gamma} (1-\alpha) (\overline{WL} + \overline{\Pi} L_f - TL_f) + G}{P_1} \quad (10)$$

The profit of the firm producing goods z is:

$$\pi(z) = p_1(z)y(z) - WL(z)$$

Each firm produces $y(z)$ according to its technology. $L(z)$ is employment by the firm producing goods z . The production function is represented as follows:

$$y(z) = AL(z)^a \quad (11)$$

A is a positive constant, and it increases at the rate of $\gamma - 1$ from period to period by technological progress. We assume $1/2 < a < 1$. This production function is according to Tanaka (2011), and Tanaka (2013).

At the equilibrium $P_1 = p_1(z)$ for all z . By calculations in Appendix C according to the profit maximization for the firms, we obtain the equations for the AD curve and the AS curve. The equation for the AD curve is (C.6), and that for the AS curve is (C.4) in Appendix C as follows:

AD-Curve

$$P_1 = \frac{\frac{1+r}{\gamma} (1-\alpha) (\overline{WL} + \overline{\Pi} L_f - TL_f) + G - \alpha TL_f}{(1-\alpha)y(z)} \quad (12)$$

AS-Curve

$$P_1 = A^{\frac{1}{a}} \frac{\eta}{\eta-1} \frac{W}{a} y(z)^{\frac{1}{a}-1}$$

From them we obtain the following equilibrium values of the output and the price:

$$y(z) = A \left(\frac{\eta}{\eta-1} \frac{W}{a} \right)^{-a} \left(\frac{1}{1-\alpha} \right)^a \left[\frac{1+r}{\gamma} (1-\alpha) \overline{WL} + \overline{\Pi} L_f - TL_f + G - \alpha TL_f \right]^a$$

and

$$P_1 = A^{-1} \left(\frac{\eta}{\eta-1} \frac{W}{a} \right)^a \left(\frac{1}{1-\alpha} \right)^{1-a} \left[\frac{1+r}{\gamma} (1-\alpha) (\overline{WL} + \overline{\Pi} L_f - TL_f) + G - \alpha TL_f \right]^{1-a}$$

These are increasing in the government expenditure G and decreasing in the tax T . We can assume that the size of the industry is one. Then, $y(z)$ equals the total output, and $L(z)$ equals L at the equilibrium.

2. Results

In this section we present the results of this paper.

2.1. Budget deficit for growth and recession

From (12) and

$$P_1 y(z) = WL + \Pi L_f \quad ((C.5) \text{ in Appendix C})$$

we have

$$(1 - \alpha)(WL + \Pi L_f) = \frac{1+r}{\gamma}(1 - \alpha)(\bar{W}L + \bar{\Pi}L_f - TL_f) + G - \alpha TL_f$$

From this we get

$$G - TL_f + \frac{r}{\gamma}(1 - \alpha)(\bar{W}L + \bar{\Pi}L_f - TL_f) = (1 - \alpha)(WL + \Pi L_f - TL_f) - \frac{1}{\gamma}(1 - \alpha)(\bar{W}L + \bar{\Pi}L_f - TL_f)$$

$\frac{r}{\gamma}(1 - \alpha)(\bar{W}L + \bar{\Pi}L_f - TL_f)$ is the payments of interest on the government bonds. The left-hand side of this equation is the budget deficit (including interest payments), $(1 - \alpha)(WL + \Pi L_f - TL_f)$ is the savings of the younger generation consumers, and $\frac{1}{\gamma}(1 - \alpha)(\bar{W}L + \bar{\Pi}L_f - TL_f)$ is the savings of the older generation consumers. Thus, the following result is obtained.

Proposition 1

The budget deficit (including interest payments on government bonds) equals an increase in the savings of consumers from period to period (or generation to generation).

This result does not imply the Ricardian equivalence theorem, since the increase in consumer savings is for future consumption, not for future tax liability.

If $L = \bar{L} = L_f$, that is, full employment is maintained without inflation, then $W = \bar{W}$, the price P_1 and the profit $\Pi = \bar{\Pi}$ are also constant.¹ Then, we have:

¹ Note that in the previous period the nominal wage rate is $\frac{1}{\gamma}\bar{W}$, and the profit for each consumer is $\frac{1}{\gamma}\bar{\Pi}$.

$$G - TL_f + \frac{r}{\gamma}(1 - \alpha)(WL_f + \Pi L_f - TL_f) = \left(1 - \frac{1}{\gamma}\right)(1 - \alpha)(WL_f + \Pi L_f - TL_f) > 0 \quad (13)$$

or

$$G - TL_f = \left(1 - \frac{1+r}{\gamma}\right)(1 - \alpha)(WL_f + \Pi L_f - TL_f)$$

If $\bar{L} = L_f$ but L does not necessarily equal L_f (and Π does not necessarily equal the profit under full employment), we have:²

$$G - TL_f = (1 - \alpha)(WL + \Pi L_f - TL_f) - \frac{1+r}{\gamma}(1 - \alpha)(WL_f + \Pi_f L_f - TL_f)$$

Then,

$$\begin{aligned} (1 - \alpha)(WL + \Pi L_f - TL_f) &= G - TL_f + \frac{r}{\gamma}(1 - \alpha)(WL_f + \Pi_f L_f - TL_f) + \\ &\quad + \frac{1}{\gamma}(1 - \alpha)(WL_f + \Pi_f L_f - TL_f) \end{aligned} \quad (14)$$

In this equation the independent variable is G , and the dependent variables are L and Π given L_f and T . L and Π are increasing in G .

They mean the following results.

Proposition 2

1. (By (13)) We need a budget deficit (including interest payments on government bonds) to maintain full employment without inflation under economic growth. However, if the interest rate of government bonds is larger than the growth rate ($r > \gamma - 1$), we need a budget surplus (excluding interest payments) to maintain full employment without inflation. This is because interest payments on the bonds become income for the bond holders and stimulate consumption demand.

2. (By (14)) If the budget deficit is smaller than (13), full employment is not achieved, that is, $L > L_f$ and a recession occurs.

We present a graphical explanation about 2 of this Proposition in Section 2.3.1.

² We denote the profit under full employment by Π_f .

2.2. Inflation with full employment

Assume $\bar{L} = L_f$ and full employment is continuously maintained until the previous period. If the budget deficit in a period is larger than (13), inflation will occur. In Section 2.3.2 we present a graphical explanation.

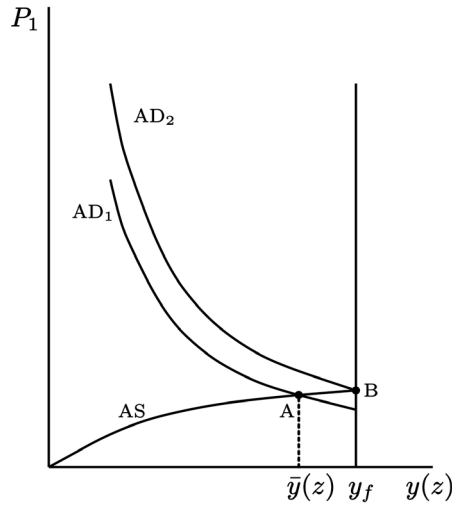


Figure 1. Restoring full employment from a recession

Source: own elaboration.

2.3. AD-AS analysis

In this subsection we present our main results by graphical analyses with AD and AS curves.

2.3.1. Restoring full Employment from a recession

Suppose that in a period government expenditure is insufficient, or the tax is excessive, and a recession with involuntary unemployment has occurred. If fiscal policy through increased government expenditure or tax cuts is not implemented in the next period, the state is expressed by A in Figure 1. The output of each the goods and the total output in this case is $\bar{y}(z)$. An appropriate fiscal policy by an increase in G or a decrease in T shifts the AD curve from AD_1 to AD_2 , and full employment is restored. At that time, prices will rise. However, this is due to decreasing returns to scale, and the nominal wage rate does not change.

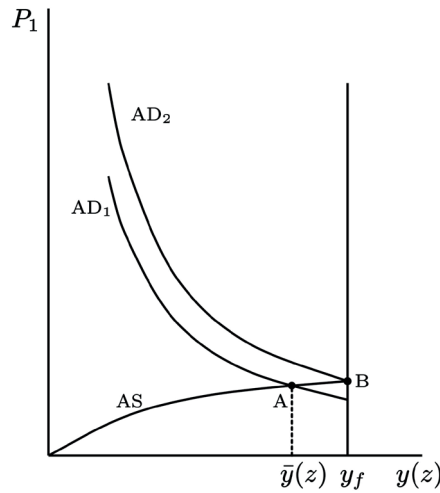


Figure 2. Inflation by excessive government expenditure or insufficient tax

Source: own elaboration.

2.3.2. Inflation by excessive government expenditure or tax cut

Suppose that full employment without inflation is achieved in a period. If government expenditure increases, or the tax is reduced, the AD curve shift from AD_1 to AD_2 in Figure 2. Then, the AD and AS curves do not intersect. However, in this case the excess demand for goods generates excess demand for labor, and the nominal wage rate W is expected to rise. Then, the AS curve shifts from AS_1 to AS_2 in Figure 2. The equilibrium point shifts from B to C, and inflation occurs.

3. Discussion and conclusion

The main results of this paper are as follows.

1. The budget deficit (including interest payments on government bonds) equals an increase in the savings of consumers from period to period (or generation to generation) (Proposition 1).

This result does not imply the Ricardian equivalence theorem, since the increase in consumer savings is for future consumption, not for future tax liability.

2. We need a budget deficit (including interest payments) to maintain full employment without inflation under economic growth. If the budget deficit is smaller than this value,

full employment is not achieved and a recession occurs. However, if the interest rate of government bonds is larger than the growth rate, we need a budget surplus (excluding interest payments) to maintain full employment without inflation (Proposition 2).

3. A return to full employment from a recession can be achieved by implementing appropriate fiscal policies through increased government spending or tax cuts (see Figure 1).

Discretionary fiscal policies such as public investment, tax cuts, and benefits to compensate for declining revenues are in mind, but since there is a time lag between the recognition of the situation, policy decisions, and policy implementation for discretionary policies, an automatic stabilizer that is automatically triggered according to the economic situation at the time is also possible. The job guarantee program advocated by Wray, Kelton, and other leading advocates of MMT may be such an automatic stabilizer.

4. Excessive government expenditure or an insufficient tax under full employment induces inflation (see Figure 2).

Cochrane (2022) argues that recent inflation in the U.S.A. and Europe is fiscal inflation. According to the spirits of MMT, excessive government expenditure (or an excessive budget deficit due to insufficient tax) induces inflation.

The first result implies that the accumulated budget deficit at the end of a period equals the savings in that period. A period in an OLG model may mean several years or decades. Even if this were the case, the debt-GDP ratio would not diverge to infinity, but would remain at a finite value whether the interest rate of government bonds is larger or smaller than the growth rate. About an analysis of debt-to-GDP ratio please see Y. Tanaka (2021a). In this paper's model, savings are for consumption in retirement, and not left to future generations. Even if savings are left to future generations, the same conclusion would be reached if they were used for consumption. Therefore, we should not be concerned about the accumulation of government debt, but rather about the loss of production and people's lives due to a recession.

One of the things this paper shows is that MMT is not necessarily heterodox, and it can be discussed by widely used analytical methods such as AD-AS analysis.

In this paper we have considered production by labor only. More generally we should consider production by capital and labor. Tanaka (2022) analyzed such a case and proved the necessity of a budget deficit when consumers hold money.

Appendix

A. Consumers' utility maximization

The conditions for utility maximization of an employed consumer are:

$$\frac{\alpha}{C_1^e} \left(\int_0^1 c_1^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{1}{\eta-1}} c_1^e(z)^{\frac{-1}{\eta}} - \lambda_e p_1(z) = 0 \quad (\text{A.1})$$

$$\frac{1-\alpha}{C_2^e} \left(\int_0^1 c_2^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{1}{\eta-1}} c_2^e(z)^{\frac{-1}{\eta}} - \lambda_e \frac{1}{1+r} p_2(z) = 0 \quad (\text{A.2})$$

From them,

$$\begin{aligned} \frac{\alpha}{C_1^e} \left(\int_0^1 c_1^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{1}{\eta-1}} c_1^e(z)^{\frac{\eta-1}{\eta}} - \lambda_e p_1(z) c_1^e(z) &= 0 \\ \frac{1-\alpha}{C_2^e} \left(\int_0^1 c_2^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{1}{\eta-1}} c_2^e(z)^{\frac{\eta-1}{\eta}} - \lambda_e \frac{1}{1+r} p_2(z) c_2^e(z) &= 0 \end{aligned}$$

Integrating them, we get

$$\begin{aligned} \frac{\alpha}{C_1^e} \left(\int_0^1 c_1^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{1}{\eta-1}} \int_0^1 c_1^e(z)^{\frac{\eta-1}{\eta}} dz - \lambda_e \int_0^1 p_1(z) c_1^e(z) dz &= 0 \\ \frac{1-\alpha}{C_2^e} \left(\int_0^1 c_2^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{1}{\eta-1}} \int_0^1 c_2^e(z)^{\frac{\eta-1}{\eta}} dz - \lambda_e \frac{1}{1+r} \int_0^1 p_2(z) c_2^e(z) dz &= 0 \end{aligned}$$

They mean

$$\begin{aligned} \lambda_e \int_0^1 p_1(z) c_1^e(z) dz &= \alpha \\ \lambda_e \frac{1}{1+r} \int_0^1 p_2(z) c_2^e(z) dz &= 1-\alpha \end{aligned}$$

Therefore, we obtain

$$\int_0^1 p_1(z) c_1^e(z) dz + \frac{1}{1+r} \int_0^1 p_2(z) c_2^e(z) dz = W + \Pi - T = \frac{1}{\lambda_e} \quad (\text{A.3})$$

Again from (A.1) and (A.2),

$$\left(\frac{\alpha}{C_1^e} \right)^{1-\eta} \left(\int_0^1 c_1^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{-1} c_1^e(z)^{\frac{\eta-1}{\eta}} - \lambda_e^{1-\eta} p_1(z)^{1-\eta} = 0$$

$$\left(\frac{1-\alpha}{C_2^e} \right)^{1-\eta} \left(\int_0^1 c_2^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{-1} c_2^e(z)^{\frac{\eta-1}{\eta}} - \lambda_e^{1-\eta} \left(\frac{1}{1+r} \right)^{1-\eta} p_2(z)^{1-\eta} = 0$$

Integrating them,

$$\left(\frac{\alpha}{C_1^e} \right)^{1-\eta} \left(\int_0^1 c_1^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{-1} \int_0^1 c_1^e(z)^{\frac{\eta-1}{\eta}} dz - \lambda_e^{1-\eta} \int_0^1 p_1(z)^{1-\eta} dz = 0$$

$$\left(\frac{1-\alpha}{C_2^e} \right)^{1-\eta} \left(\int_0^1 c_2^e(z)^{\frac{\eta-1}{\eta}} dz \right)^{-1} \int_0^1 c_2^e(z)^{\frac{\eta-1}{\eta}} dz - \lambda_e^{1-\eta} \left(\frac{1}{1+r} \right)^{1-\eta} \int_0^1 p_2(z)^{1-\eta} dz = 0$$

From them we get

$$\frac{\alpha}{C_1^e} = \lambda_e \left(\int_0^1 p_1(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}} = \lambda_e P_1 \quad (\text{A.4})$$

and

$$\frac{1-\alpha}{C_2^e} = \lambda_e \left(\int_0^1 p_2(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}} = \lambda_e \frac{1}{1+r} P_2 \quad (\text{A.5})$$

By (A.3) we obtain

$$P_1 C_1^e = \alpha (W + \Pi - T) \quad (\text{A.6})$$

and

$$\frac{1}{1+r} P_2 C_2^e = (1-\alpha) (W + \Pi - T) \quad (\text{A.7})$$

By (A.1), (A.2), (A.4) and (A.5), we get

$$\left(\frac{p_1(z)}{P_1} \right)^{-1} \left(C_1^e \right)^{\frac{1}{\eta}} = c_1^e(z)^{\frac{1}{\eta}}$$

and

$$\left(\frac{p_2(z)}{P_2} \right)^{-1} \left(C_2^e \right)^{\frac{1}{\eta}} = c_2^e(z)^{\frac{1}{\eta}}$$

With (A.6) and (A.7), they imply

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1} \right)^{-\eta} C_1^e = \left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{\alpha(W + \Pi - T)}{P_1}$$

and

$$c_2^e(z) = \left(\frac{p_2(z)}{P_2} \right)^{-\eta} C_2^e = \left(\frac{p_2(z)}{P_2} \right)^{-\eta} \frac{(1+r)(1-\alpha)(W + \Pi - T)}{P_2}$$

Similarly, the demand functions for goods z of an unemployed consumer in Periods 1 and 2 are

$$c_1^u(z) = \left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{\alpha(\Pi - T)}{P_1}$$

and

$$c_2^u(z) = \left(\frac{p_2(z)}{P_2} \right)^{-\eta} \frac{(1+r)\beta(\Pi - T)}{P_2}$$

B. Government expenditure

The condition for maximization of G is:

$$\left(\int_0^1 g(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{1}{\eta-1}} g(z)^{\frac{-1}{\eta}} - \lambda_g p_1(z) = 0 \quad (\text{B.1})$$

Then, we obtain

$$\left(\int_0^1 g(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{1}{\eta-1}} \int_0^1 g(z)^{\frac{\eta-1}{\eta}} dz - \lambda_g \int_0^1 p_1(z) g(z) dz = 0$$

This means

$$\left(\int_0^1 g(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{1}{\eta-1}} g(z)^{\frac{-1}{\eta}} - \lambda_g p_1(z) = 0 \quad (\text{B.2})$$

Integrating (B.1) yields

$$\left(\int_0^1 g(z)^{\frac{\eta-1}{\eta}} dz \right)^{-1} \int_0^1 g(z)^{\frac{\eta-1}{\eta}} dz - \lambda_g^{1-\eta} \int_0^1 p_1(z)^{1-\eta} dz = 1 - \lambda_g^{1-\eta} \int_0^1 p_1(z)^{1-\eta} dz = 0$$

From this

$$\lambda_g \left(\int_0^1 p_1(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}} = \lambda_g P_1 = 1$$

Therefore,

$$\lambda_g = \frac{1}{P_1} \quad (\text{B.3})$$

By (B.2)

$$P_1 \mathcal{G} = \int_0^1 p_1(z) g(z) dz = G$$

From (B.1) and (B.3), we have

$$\left(\int_0^1 g(z)^{\frac{\eta-1}{\eta}} dz \right)^{\frac{\eta}{\eta-1}} g(z)^{-1} = \left(\frac{p_1(z)}{P_1} \right)^{\eta}$$

Thus, we get the following demand function of the government.

$$g(z) = \left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{G}{P_1}$$

C. Firms' profit maximization and market equilibrium

Let

$$Y = \alpha(WL + \Pi L_f - TL_f) + \frac{1+r}{\gamma}(1-\alpha)(\bar{WL} + \bar{\Pi}L_f - TL_f) + G$$

Then,

$$y(z) = \left(\frac{p_1(z)}{P_1} \right)^{-\eta} \frac{Y}{P_1} \quad (\text{C.1})$$

From this

$$\frac{\partial y(z)}{\partial p_1(z)} = -\frac{\eta}{P_1} \left(\frac{p_1(z)}{P_1} \right)^{-\eta-1} \frac{Y}{P_1} = -\frac{\eta}{p_1(z)} y(z) \quad (\text{C.2})$$

By the production function (11) and (C.1),

$$L(z) = \left(\frac{y(z)}{A} \right)^{\frac{1}{a}}$$

Then, we get

$$\frac{dL(z)}{dy(z)} = \frac{1}{A^{\frac{1}{a}} a} y(z)^{\frac{1-a}{a}} \quad (\text{C.3})$$

The condition for profit maximization is

$$\frac{\partial \pi(z)}{\partial p_1(z)} = y(z) + \left[p_1(z) - W \frac{dL(z)}{dy(z)} \right] \frac{\partial y(z)}{\partial p_1(z)} = 0$$

By (C.2)

$$y(z) - \left[p_1(z) - W \frac{dL(z)}{dy(z)} \right] \frac{\eta}{p_1(z)} y(z) = 0$$

Then, from (C.3)

$$(1-\eta)y(z) + W \frac{1}{A^{\frac{1}{a}} a} y(z)^{\frac{1}{a}} \frac{\eta}{p_1(z)} = 0$$

This is rewritten as

$$(1-\eta)p_1(z) + W \frac{\eta}{A^a a} y(z)^{\frac{1}{a}-1} = 0$$

Further,

$$(1-\eta)p_1(z) + W \frac{\eta}{A^a a} \left(\frac{p_1(z)}{P_1} \right)^{\eta \frac{\eta}{a}} y(z)^{\frac{1}{a}-1} = 0$$

Since at the equilibrium the prices of all goods are equal, we have

$$p_1(z) = P_1$$

Then,

$$(1-\eta)P_1 + W \frac{\eta}{A^a a} y(z)^{\frac{1}{a}-1} = 0$$

and so

$$P_1 = A^{\frac{1}{a}} \frac{\eta}{\eta-1} \frac{W}{a} y(z)^{\frac{1}{a}-1} \quad (\text{C.4})$$

This is the equation for the AS curve.

From (C.1),

$$y(z) = \frac{Y}{P_1}$$

By $p_1(z) = P_1$,

$$y(z) = \frac{Y}{P_1} = \frac{\alpha(WL + \Pi L_f - TL_f) + \frac{1+r}{\gamma}(1-\alpha)(\bar{WL} + \bar{\Pi} L_f - TL_f) + G}{P_1}$$

Since

$$P_1 y(z) = WL + \Pi L_f \quad (\text{C.5})$$

we have

$$(1-\alpha)P_1y(z) = \frac{1+r}{\gamma}(1-\alpha)(\overline{WL} + \bar{\Pi}L_f - TL_f) + G - \alpha TL_f$$

$$y(z) = \frac{\frac{1+r}{\gamma}(1-\alpha)(\overline{WL} + \bar{\Pi}L_f - TL_f) + G - \alpha TL_f}{(1-\alpha)P_1}$$

or

$$P_1 = \frac{\frac{1+r}{\gamma}(1-\alpha)(\overline{WL} + \bar{\Pi}L_f - TL_f) + G - \alpha TL_f}{(1-\alpha)y(z)} \quad (C.6)$$

This is the equation for the AD curve.

Combining (C.4) and (C.6),

$$A^{\frac{1}{a}} \frac{(1-\alpha)\eta}{\eta-1} \frac{W}{a} y(z)^{\frac{1}{a}} = \left[\frac{1+r}{\gamma}(1-\alpha)(\overline{WL} + \bar{\Pi}L_f - TL_f) + G - \alpha TL_f \right]$$

Thus, we obtain

$$y(z) = A \left(\frac{\eta}{\eta-1} \frac{W}{a} \right)^{-a} \left(\frac{1}{1-\alpha} \right)^a \left[\frac{1+r}{\gamma}(1-\alpha)(\overline{WL} + \bar{\Pi}L_f - TL_f) + G - \alpha TL_f \right]^a$$

Similarly,

$$P_1 = A^{-1} \left(\frac{\eta}{\eta-1} \frac{W}{a} \right)^a \left(\frac{1}{1-\alpha} \right)^{1-a} \left[\frac{1+r}{\gamma}(1-\alpha)(\overline{WL} + \bar{\Pi}L_f - TL_f) + G - \alpha TL_f \right]^{1-a}$$

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