12th International Congress on Mathematical Education
Topic Study Group 4
8 July – 15 July, 2012, COEX, Seoul, Korea

MATHEMATICAL ACHIEVEMENT AND CREATIVITY INHERENT IN CHILDREN WITH SPECIAL NEEDS

Tadato Kotagiri
University of the Ryukyus
kotagiri@edu.u-ryukyu.ac.jp

The assessment of children's mathematical learning achievement entails recognition of the child's human rights to learn Basic Mathematics: (1) to be able to fulfil his/her potential, and more importantly, (2) to be prepared for creative participation in his/her community, both in work and in other activities. Nonetheless, because many children with Special Needs face severe difficulties in obtaining the Basic Mathematical understanding and skills which they both deserve and need, they are effectively being denied their basic educational rights. This paper, based on years of using a clinical approach to remedial education, provides evidence of such children's remarkable possibilities for the achievement of Basic Mathematics, in particular exposing instances of significant creative response.
Key Words: Mathematical creativity, Task-based learning, Clinical intervention in education, Cognitive thinking.

INTRODUCTION

In previous studies (Kotagiri 2004A, 2004B), I verified the achievement of children with special needs in learning Basic Mathematics (Numeracy) by considering a number of cases, including that of a typical child with global type Learning Disabilities (LD). When Masa, the LD child, was a fourth grade pupil, he could not proceed in the learning of addition and subtraction of multi-digit numbers. Although he knew that a ten is ten ones and a twenty is twenty ones, and could numerate and count concrete things at least up to a hundred, and also knew the hundreds, he was nonetheless unable to perform correctly in carrying-up and carrying-down for addition and subtraction. One time, he happened to make mistakes in single-digit addition, specifically 3+9=17 and 8+9=15. Masa was then forced to receive many lessons in and after school until finally he simply rejected submitting to those lessons. He also refused to attempt memorization of a multiplication table. However, after a five-year period of remedial education that involved only a weekly 45-minute lesson plus daily homework amounting to no more than five minutes per day, the successful achievements of Masa can be seen in Figure 1, which shows his millennium-year sketched response that enabled him to answer correctly mathematical problems involving all mathematical operations.

The successful outcome of remedial instruction for Masa prompted extension of the methods for usage in helping children who were identified as slow-learners and children whose brains were damaged by hydrocephaly or meningitis. Presented here as evidence of achievement is Figure 2, which shows the "before" and "after eight years" sketches done by a child whose brain was severely damaged by meningitis.

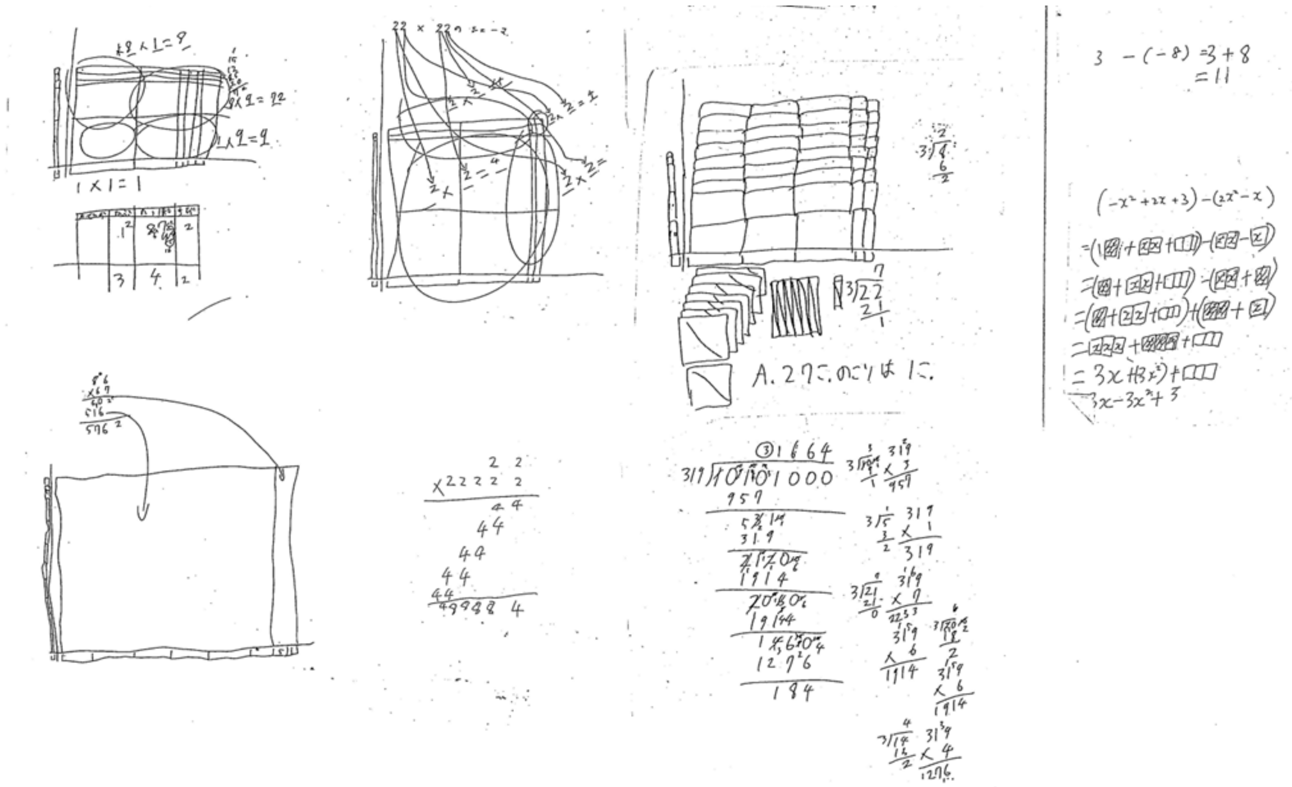

Figure 1. This series was done over a 3-year period
by a child assessed to be LD in Mathematics

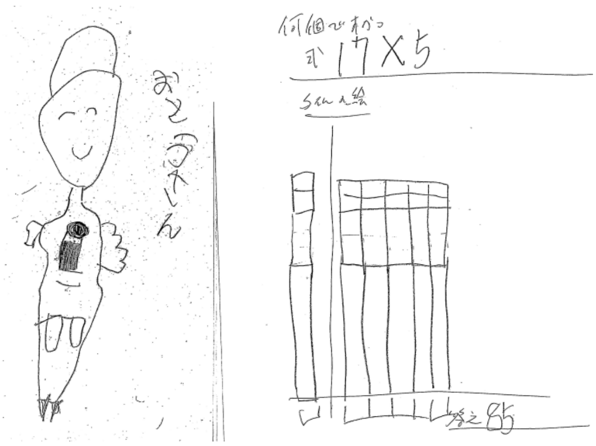

Figure 2. Results of 7-8 years of clinical intervention in education can be seen at the right.

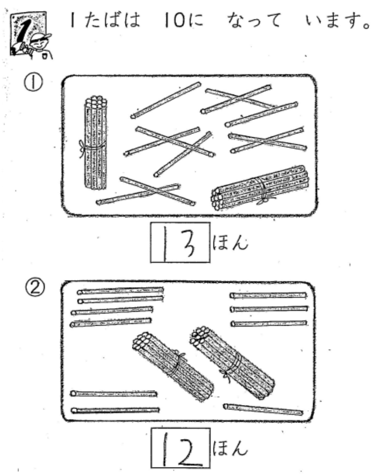

Figure 3. The child first expressed awareness of two tens, but ended by ‘counting’ each bundle as ‘one’.

The children whose achievements are shown Figures 1 and 2 are among those who can successfully learn to conceptualize the decimal numeration system during a comparatively short period even though they had shown difficulty in learning addition or subtraction. For example, they can answer “twenty three” for “two tens and three ones” despite their confusion about the procedure of carrying-up or carrying-down. Development of such fundamental numeral conception is an important benchmark for educational intervention for any children exhibiting difficulty in learning numbers. Children with intellectual difficulties are seemingly transfixed in front of the decimal numeration system. Despite undertaking many exercises in numerating aloud and in reading and writing decimal numerals, they can be perplexed when confronting the task of finding a whole number that corresponds to what they know in terms of the amount of tens and ones they see. This is revealed in Figure 3. By the way, Figure3, and others here like that, were not materials included in my mathematical intervention, but are commercially available and popular as study materials in Japan. Figure 3, with the child’s penciled response, was provided to me by her parents.

As for memorization, to a certain extent such children can learn to read and write each numeral much as they can memorize how to read and write simple Chinese characters used in the writing of Japanese. However, such memorization should be distinguished from development of the ability to understand the idea of the decimal numeration system. Thus it appears there are five fundamental conditions that must be fulfilled before we can claim that the child understands the decimal numeration system:

(1) Use of the words ″one″, ″two″, and ″three″ in the child′s everyday-life situation.
(2) Recognition that ″four″ and ″five″ are comprised of different combinations of ″three,″ ″two,″ and ″one″.
(3) Recognition that ″six″ to ″nine″ can be understood by using the chunk ″five″.
(4) Recognition that chunks of ″ten″ and ″five″ can be utilized for counting a set in order to ascertain the cardinal number of the set.
(5) Awareness that the number of tens and the number of ones provide a means for determining cardinal numbers larger than ″ten″.

When the child has succeeded consistently in achieving at this level of numerical literacy, we can confidently claim that the child understands the idea of the decimal numeration system. The child’s ability to read and write two-digit numbers such as twelve and twenty-three is not sufficient. This stage, supported by evidence of achievement of the preceding five steps, must be attained if we are to acknowledge the child’s understanding of place-value notation rather than being misled by a child’s performance in the reading and writing of numbers, a skill based on rote memorization.

The question at hand is whether this series of five steps or conditions can be recognized as a process of cognitive development for children with intellectual difficulties, enabling them to understand the idea of the decimal numeration system. Can such children learn Basic Mathematics as creatively as is expected of other children? This question is pursued in the next section.

SPROUTING AND GROWTH OF NUMERAL CONCEPTION - CASES OF CHILDREN WITH INTELLECTUAL DISABILITIES

This section considers the results of remedial-education planned for two females, Ayu and Iku. Ayu was a nineteen-year-old with intellectual disturbance who had “successfully” completed a twelve-year educational program. Iku was a ten-year-old with Down’s syndrome. Both had difficulty distinguishing “four things” and “five things” from “several things”. For example, neither could answer “twenty three” when confronted with “two tens and three ones” even though they could numerate aloud numbers from one to a hundred in sequence and both were aware that there were indeed two tens and three ones. Evidence of Ayu’s cognition prior to clinical and remedial educational intervention is shown in Fig. 3, which also corresponds to Iku’s reaction.

The sequence of Figure 4 through Figure 8 reflects the developmental phases of Iku’s numeral cognition in response to educational intervention as remedial education. Ayu’s numeral development is very similar to Iku’s. Figure 4 is Iku’s drawing when she was a third grader. Although Iku could draw a picture appropriate for illustrating four or five concrete tiles, she drew the picture shown here in Figure 4 when confronted with eight concrete tiles. She simply drew many tiles from left to right, stopping at the edge of the paper.

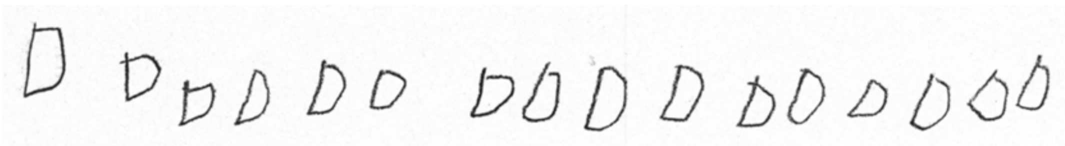

Figure 4. This is the child’s representation of eight concrete tiles which had been presented to her lined up and adjacent.

Figures 5 and 6 are Iku’s response to two requests: “how many?” and “draw the picture that you see.” These were drawn when she was a fourth grader.

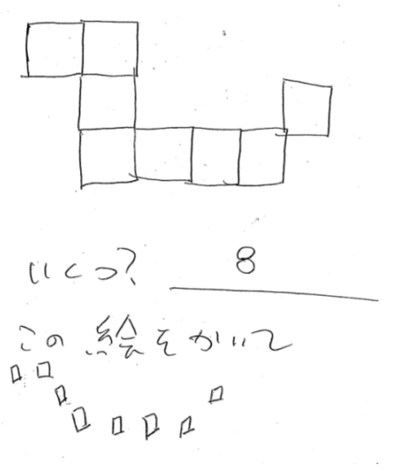

Figure 5. The teacher’s drawing of eight tiles is deconstructed and then reconstructed by the child.

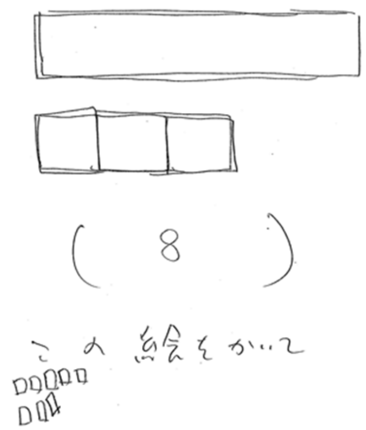

Figure 6. The child confronts the undivided “chunk-five” and then reconstructs it as five tiles.

Figures 7 and 8 were drawn a year later in response to the now-familiar task of writing an appropriate number (in the bracketed space) and drawing a suitable figure. Here Iku was a fifth grader. For each figure, she first provided the number at the top and then drew a picture below the question mark before writing in the required numbers at the bottom. Unprompted, Iku cleverly devised the technique of understanding "8" by placing the four fingers of her left hand on the 5-chunk and then using the index finger of her right hand to complete the 5-chunk, filling the remaining tiles with the other three fingers.

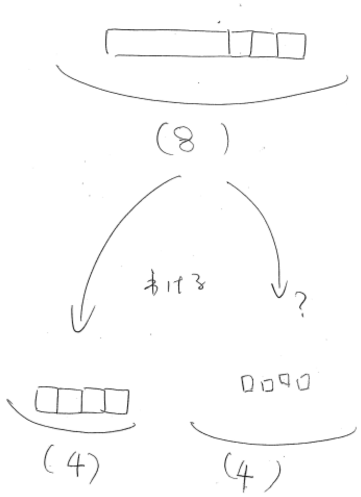

Figure 7. Iku's response shows she recognizes eight as five-and-three and then she created eight as four-and-four.

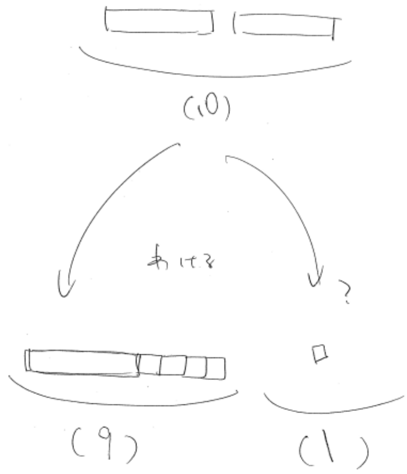

Figure 8. Iku recognized that five-and-five is ten. She then created ten as nine (five-and-four) and one.

Figures 9 and 10 were drawn when Iku was a sixth grader and both show evidence of her cognition. In Figure 9, it is recognizable that she has acquired a sense of the target number because of the way she counted. She began by counting the unseen "ones" in the first two ten-chunks, and then she skipped the five-chunk and proceeded to count again the unseen "ones" in the next two ten-chunks. In that way, she continuously counted up to forty. After that, she returned to the five-chunk and counted the unseen "ones", beginning with "forty-one" until she reached "forty-five". In confronting Figure 10, she was asked "How many tens are there?" to which she answered "20". It is important to note that her thought is correct, but the form of her answer is incorrect. Her thinking is evidence of her creativity because it is her own, independent response, not something which she had been taught.

Figures 11 and 12 here show Iku's understanding as a seventh grader. She has almost reached an understanding of the idea of the decimal numeration system, though not yet sufficiently well. The creativity of her response is seen in Figure 12 where she has chosen to give a number to each individual pencil such that her number includes awareness of chunk-ten. However, her spontaneous individual enumeration did not incorporate both chunk-tens in the second task, so her answer is incorrect.

Figure 13 and 14 are Ayu's, at age 25, after six years of similar educational intervention. She too has almost achieved the goal of understanding the decimal numeral system, but has not yet reached it. Her creative response was to write "10" for each ten-chunk.

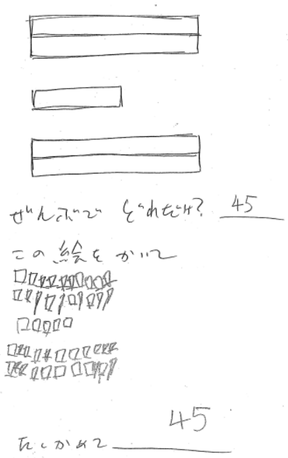

Figure 9. How much all together?
Draw this picture. Make sure.

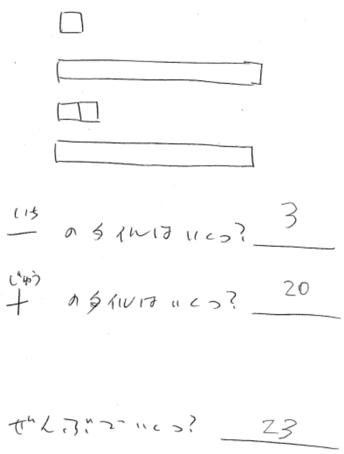

Figure 10. How many ones?
How many ten? How much all together?

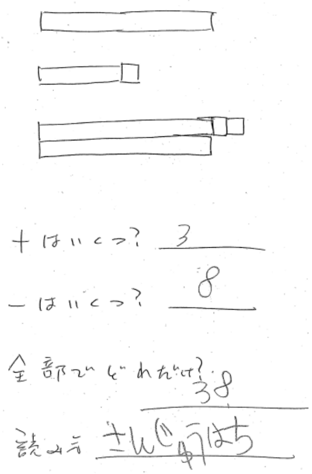

Figure 11. How many tens?
How many ones? How much all together?

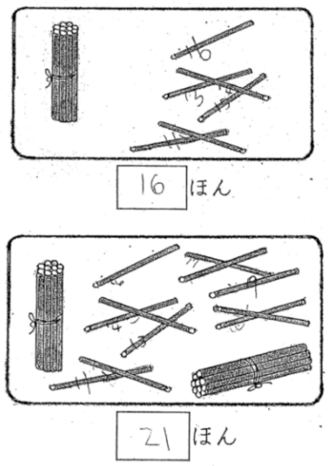

Figure 12. How many pencils
are there?

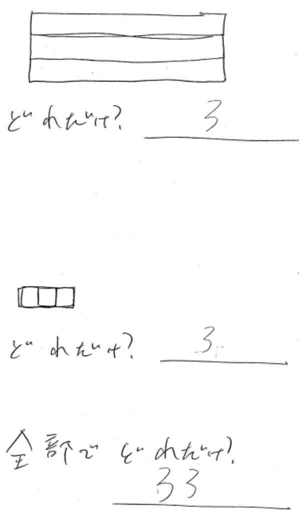

Figure 13. Ayu counted “ten, twenty,
thirty, thirty-one, thirty-two, thirty-three.”

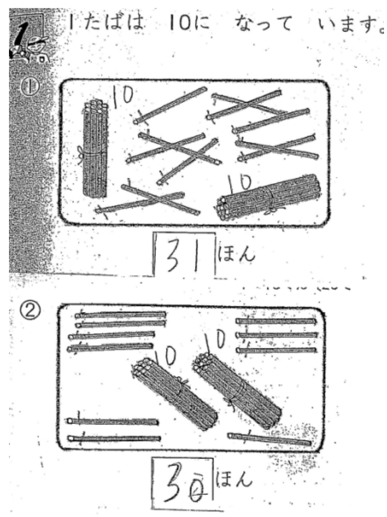

Figure 14. Ayu counted “ten, twenty,
twenty-one…thirty-one.”

PROCEDURAL THINKING, COGNITIVE THINKING, AND CREATIVE TRIALS

When confronting each picture, both Iku and Ayu counted the concrete objects one-by-one as they might do in everyday-life. This reflects their procedural way of thinking. In this manner, they could gradually become comfortable handling numeral concepts such as two, three, five, and ten. This also indicates that they have re-learnt the numerals, internalizing the numerals that they had previously memorized by numeration aloud. When trying to handle numeral concepts that are not familiar to them, they attempted to perform the task creatively according to their own manner of approach, even though the end result was that they sometimes happened to get a wrong number. These trials are creative.

The extent of creativity of thinking becomes more visible according to the progression of learning. This is more clearly seen in in the next examples, Figs. 15-18, taken from another case. These show the efforts of Nac, a 16-year-old boy with autism. It can be seen that new algorisms are coming into being in the process of his responding to these trials.

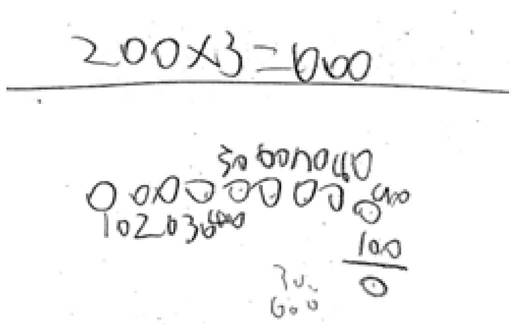

Figure 15. Make sure each of 200 students get 3 candies. How many candies do you need?

Figure 16. There are 102 bags. Each bag has 123 tiles. All together, how many tiles?

Figure 17. Three people want to share 2000 yen equally. How do they do it?

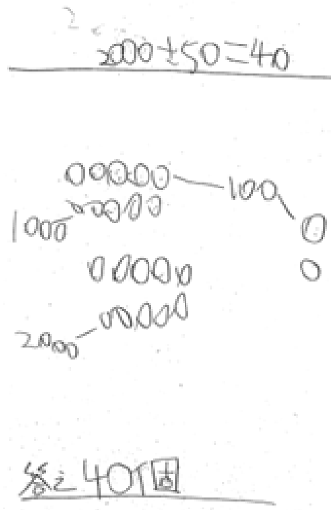

Figure 18. Divide 2000 candies among 50 children equally.

ACHIEVING THE GOAL TO UNDERSTAND THE DECIMAL-NOTATION IDEA

How can we know that a child has achieved the goal of understanding the idea of decimal notation? Mentally-challenged children seem to stumble over that goal even if they, like Ayu and Iku, can numerate by memorization. Although Ayu and Iku were able to answer correctly the questions “How many tens? How many ones? How much in all?”, they could not operate tens and ones conceptually, but they could do so procedurally. Figure 19 shows that Iku failed to get the answer when she tried to operate a number of tens and a number of ones. However, when she was counting “one, two, three.., ten, eleven.., twenty, twenty-one.., thirty, thirty-one.., fifty, fifty-one...,” she could surely be expected to reach the right answer.

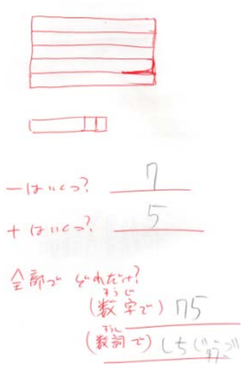

Figure 19. Iku incorrectly answered “seventy-five.”

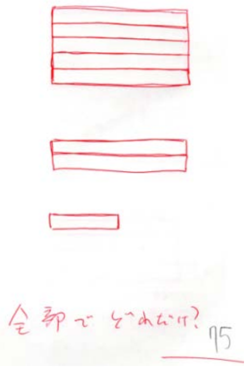

Figure 20. Iku reached the correct answer by counting, one-by-one.

With Iku, it was possible to witness the moment when she had achieved the goal of operating numbers of tens and ones cognitively in order to find the number corresponding to the total although she happened to make mistakes, depending on the situation. This is shown in the sequence of figures 21 through 28, during which sequence she herself modified her thinking, all within one lesson period. At this step in her learning, she needed to become familiar with this way of thinking. As seen in figures 27 and 28, she still needed to learn the idea of place-value notation even though she could correctly put the answer in each box. It can be seen that Iku’s learning was progressing along the path of Masa’s learning.

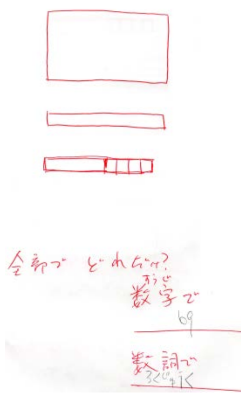

Figure 21. She counted the tiles individually from starting fifty.

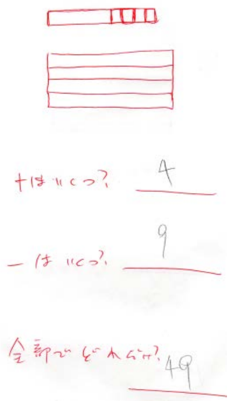

Figure 22. After answering (How many tens? How many ones?), she quickly responded “49” without enumerating.

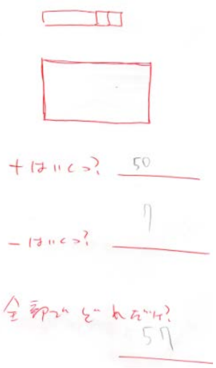

Figure 23. When she could not answer the first question "How many tens?", she was shown a block of five tens, and she answered "50," technically wrong though perceptually appropriate. She then got the "all together" answer correct.

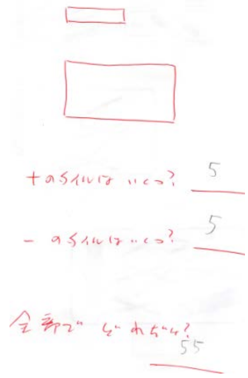

Figure 24. This was done immediately after Fig. 23. She said "This time, I'll do it" and wrote "5" for "How many" tens?" She then correctly wrote "55" without numeration.

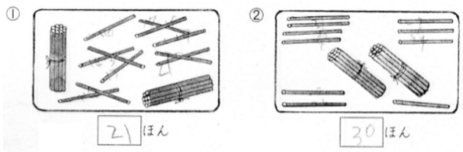

Figure 25. She answered these in sequence, the first incorrectly. However, she was told the results in reverse order: "right" and then "wrong."

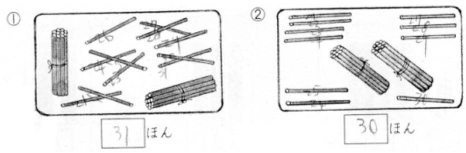

Figure 26. She was asked to try again. She then confronted these two and correctly modified her numeration.

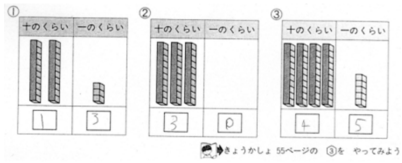

Figure 27. Again, she was told the results, but in reverse order of her responses.

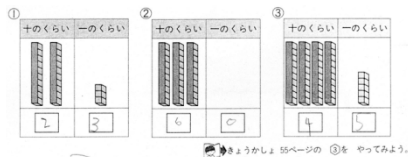

Figure 28. When offered a chance to try again, she modified her approach and got all answer right.

CONCLUSION

Presented here is evidence of the success of remedial educational intervention for children with Special Needs, utilizing an approach similar to one now familiar to many under the term "Microgenetic Method." However, at the time that the research reported here was originally undertaken, in the mid 1980s and reported initially in Hungary in 1988, there were no published methods utilizing the approach that the author of this research was struggling to develop. The research and remedial education presented here is an expression of the strategy of intervening in a child's learning through the presentation of meaningful learning tasks.

That is, this method incorporates Task-based Learning, and Clinical Intervention in Education. Tasks created on the basis of a series of learning steps, as described above, are given to a child. The child then tries to perform each task with knowledge acquired before, from previous tasks, or from reliance on thoughts and approaches used in everyday-life. The learning is thus context-based and not rote, but meaningful. After performing his/her trials, the child's achievement or cognitive development is diagnosed, and the results are presented to the child in a manner that encourages self-awareness of progress. The child is then offered the chance to attempt another, similar task.

The remedial strategy for intervening in a child's learning, as presented here, does not involve teaching the child. Instead, it involves a patient approach to giving the child opportunities to discover how to learn. If a child is "taught" how to solve a problem, the child stops searching for his/her own creative approach, the child stops "thinking," and instead is likely to attempt to memorize the method of the solution being presented. It is important that a child be asked to solve a problem or to perform a task by himself/herself, without prodding or assistance. In the situation of intervention, if any explanation is to be made of what the child is doing, it should be the child's own, self-expressed explanation. It is of fundamental importance that we change our view of learning and teaching, especially when the teacher's intervention in the children's learning is crucial and desperately needed, as is the case for children even in a classroom situation (Kotagiri 2008).

By combining evidence of the learning process shown by the achievements of Iku and Ayu with understanding of the learning process displayed Fig.1, we can glean insight into a model of the cognitive development of learning numbers. These "Microgenetic" learning records of the children who were subjects in this study are to be made into a Web-Database that will be open and accessible for teachers and researchers.

Access to such a Web-Database enables this thesis to be practically observed and convincingly confirmed: The children in this study have shown that they can and do think sufficiently and that their development of thinking can be linked to their development of mathematical cognition. Their learning is as creative as that of peer children who can and do think, and learn, about mathematical concepts. As has been shown here, educational intervention for children with Special Needs must be a higher quality of education, individualized according to the needs of the child. The children can enjoy, and benefit from, this educational chance. Moreover, the benefits of such educational intervention are likely to prove valuable, even necessary, for the peer children.

References

Kotagiri, T. (2004A). Developing Pictorial Ideas in Learning Numbers and Calculations. http://www.icme-organisers.dk/tsg16/.

Kotagiri, T. (2004B). How Basic Arithmetic Skills are Obtained by Children with Learning Difficulties? http://www.icme-organisers.dk/tsg08/.

Kotagiri, T. (2008). Theory and Practice of Educational Intervention in the Learning of Numbers and Arithmetic Operations by Special Needs Children. http://tsg.icme11.org/tsg/show/8.