Decomposing Heegaard splittings along separating incompressible surfaces in 3-manifolds

Kazuhiro Ichihara

Nihon University, College of Humanities and Sciences

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Makoto OZAWA (Komazawa University),
J. Hyam RUBINSTEIN (University of Melbourne)

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Heegaard splitting

Definition

M: closed orientable 3-manifold
S: closed orientable surface embedded in M
Heegaard splitting of $M :=$ splitting of M along S to 2 handlebodies
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Theorem 1

M : a closed irreducible orientable 3-manifold admitting a strongly irreducible Heegaard splitting.
J : a separating closed orientable incompressible surface in M, of which we denote the two sides as M_+, M_-.
$\{S_t\}_{0<t<1}$: a singular foliation of M associated to a height function for the Heegaard splitting so that J is in Morse position relative to S_t. Then either;

Theorem 1 (continued)

- there is some non critical level t so that $S_t \cap M_+$ is incompressible and $S_t \cap M_-$ has compressing disks on both sides of S_t, or the same with M_+, M_- interchanged.

Theorem

Theorem 1 (continued)

- there is some non critical level t so that $S_t \cap M_+$ is incompressible and $S_t \cap M_-$ has compressing disks on both sides of S_t, or the same with M_+, M_- interchanged.

- there is a critical level $\hat{t}$ so that $S_t \cap M_+$ is incompressible for $t < \hat{t}$ and t close to $\hat{t}$, and $S_{t'} \cap M_-$ is incompressible for $t' > \hat{t}$ and t' close to $\hat{t}$, or the same with M_+, M_- interchanged.

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Theorem 1 (continued)

- there is some non critical level t so that $S_t \cap M_+$ is incompressible and $S_t \cap M_-$ has compressing disks on both sides of S_t, or the same with M_+, M_- interchanged.

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- there is a critical level $\hat{t}$ so that both $S_t \cap M_+$ and $S_t \cap M_-$ are incompressible for $t < \hat{t}$ and t arbitrarily close to $\hat{t}$.

Corollaries

Corollary 1

Under the same setting as in Theorem 1, there exists a non-critical value t so that the level surface S_t satisfies one of $S_t \cap M_+$ or $S_t \cap M_-$ is incompressible in M_+ or M_- respectively.

This gives an alternative proof of [Kobayashi-Qiu, Prop.2.6, '08].

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Furthermore, we also have the following corollary, which also gives an alternative proof of a recent result obtained by T. Saito.

Corollary 2

Under the same setting as in Theorem 1, if the Heegaard splitting is of Hempel distance at least 4, then there is a non-critical value t so that both $S_t \cap M_+$ and $S_t \cap M_-$ are incompressible in each of M_+ and M_-.