Cosmetic banding on knots

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joint work with
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Geometric Topology of low dimensions
2019/3/9 10:00–10:30

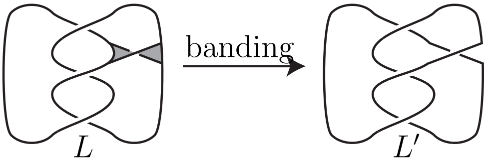
Banding

L : a link in S^3

$b\colon I \times I \to S^3$, embedding s.t. $b(I \times I) \cap L = b(I \times \partial I)$

Definition (banding)

$L' = (L - b(I \times \partial I)) \cup b(\partial I \times I)$ is called the link obtained from L by a banding along the band b.



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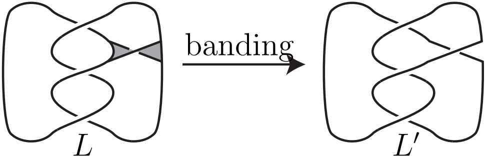

Question

For a link L, does a banding give the same link?

Cosmetic banding

L : a link in S^3, L' : a link obtained from L by a banding

Definition (cosmetic banding)

A cosmetic banding $\overset{\text{def}}{\Longleftrightarrow}$ ${}^{\exists}h\colon S^3 \to S^3$, homeo., $h(L) = L'$.

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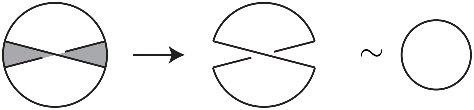
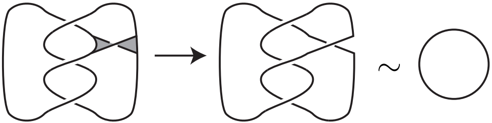
Example:



cosmetic



non-cosmetic

Pure/Chiral

$L \subset S^3$, L': a link obtained from L by a *cosmetic* banding
i.e., $^{\exists}h\colon S^3 \to S^3$, homeo. s.t. $h(L) = L'$.

Definition (purely / chirally cosmetic)

A banding is purely (resp. chirally) cosmetic
$\overset{\text{def}}{\Longleftrightarrow}$ h is orientation preserving (resp. reversing).

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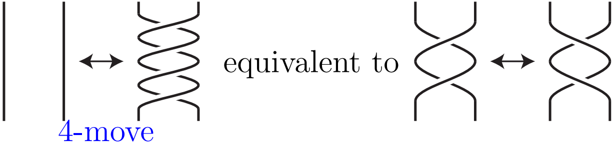
The aim of this talk:

- Find various (purely or chirally) cosmetic bandings.
- For a given cosmetic banding,
  reveal the reason why the banding is cosmetic.

banding and 4-move



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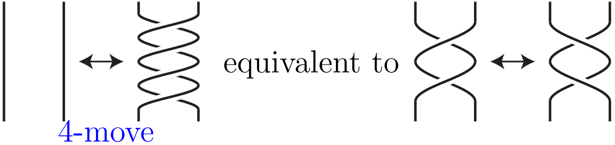
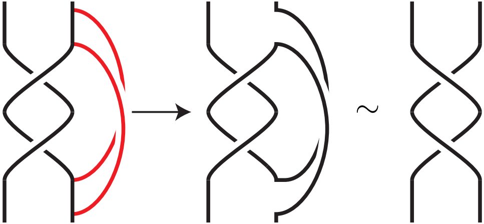


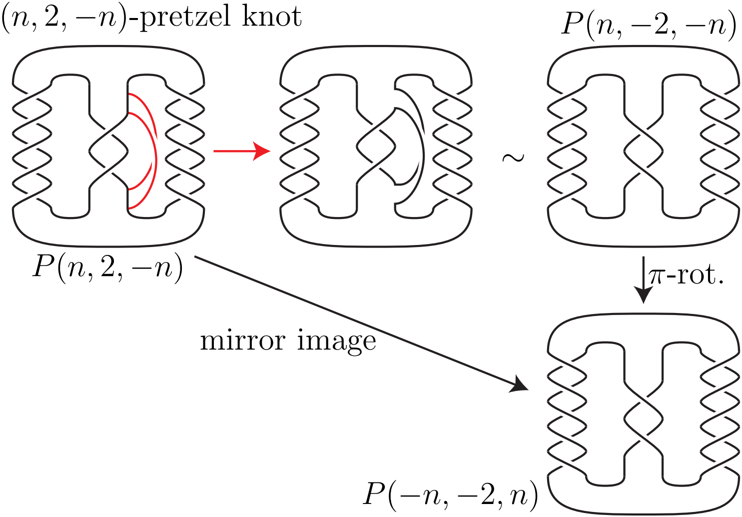
Lemma

A banding yields a 4-move.



Pretzel knots (symmetric unions)

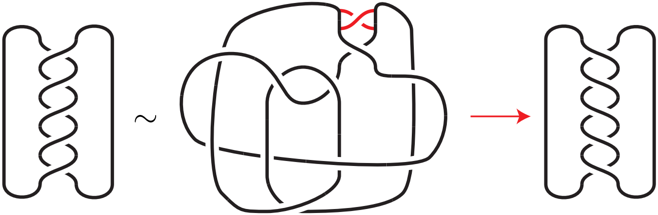
- $(n, 2, -n)$-pretzel knot



(2,5)-torus knot

Proposition [Zeković '14]

The $(2, 5)$-torus knot admits a cosmetic banding.



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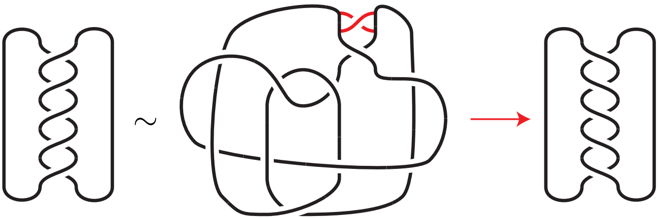
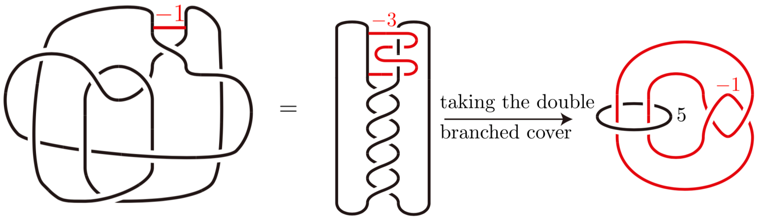


- This banding **cannot be realized by a 4-move.** (shown by using signature)

Figure-eight sibling

Remark

- The complement of the red colored knot in $L(5,1)$ is called the "figure-eight sibling" which is amphicheiral.

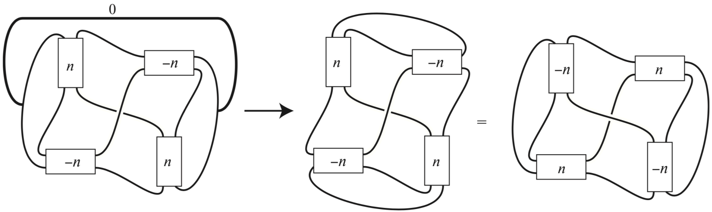


Generalization

Let K_n be the link in S^3 shown in the figure below.

Proposition [I.-Jong-Taniyama, '18]

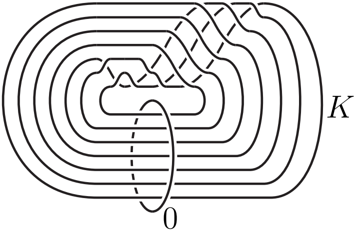
Each of K_n admits a chirally cosmetic banding ($n \neq 0, 1$).
In particular, $K_2 = T(2,5)$ does.



Bleiler-Hodgson-Weeks's example

Proposition [Bleiler-Hodgson-Weeks, '99]

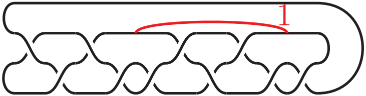
$\exists$a hyperbolic knot $K \subset S^2 \times S^1$ s.t. $E(K)$ admits chirally exotic cosmetic fillings yielding $L(49, \pm 18)$.
$(L(49, -18) = L(49, 19))$

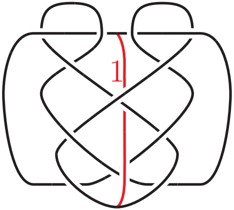


On 9_{27}

Observation

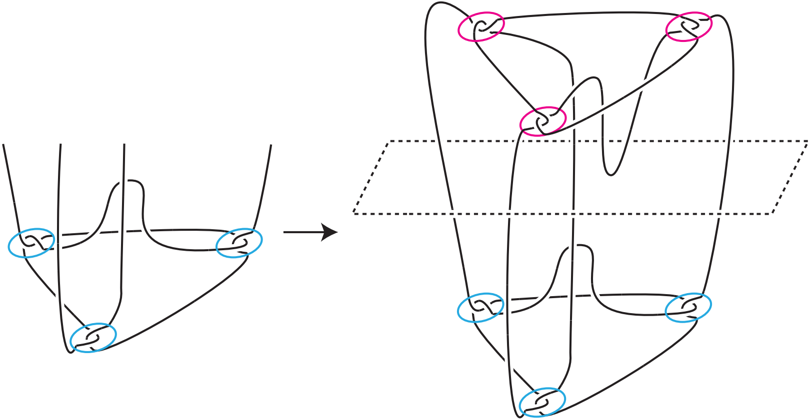
$9_{27} = S(49, 18)$ admits a chirally exotic cosmetic banding.





On 9_{27}

9_{27} can be obtained as follows:



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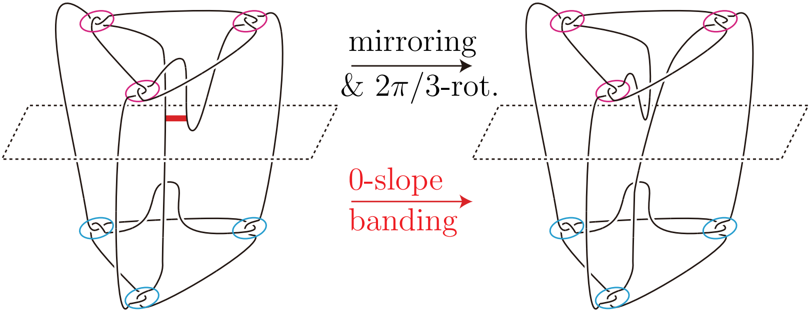

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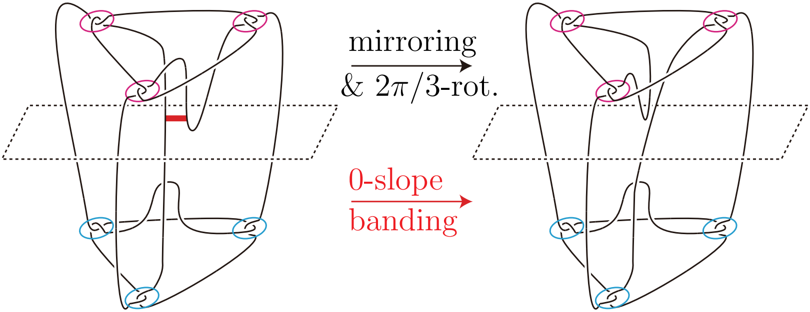


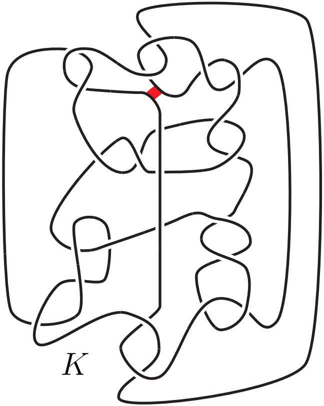
This yields many chirally cosmetic banding by

- adding twists of the tangles, or
- increasing the number of tangles from 3×2 to $n \times 2$, ...

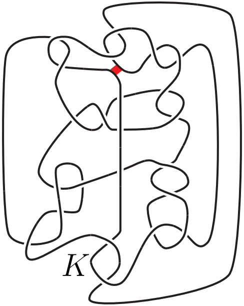
Generalization

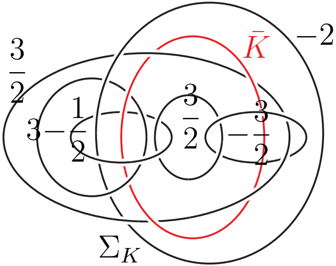
Theorem [I.-Jong-Masai, '18]

The hyperbolic knot K admits a chirally exotic cosmetic banding.



Related to Cosmetic surgery conjecture



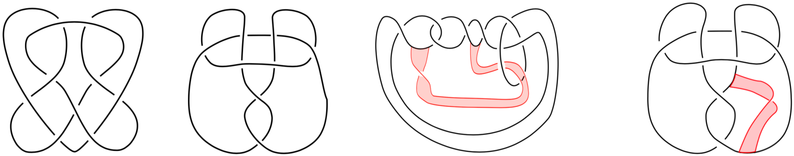


By using the computer-programs *SnapPy* and *hikmot*, it is shown that

- Σ_K and $\Sigma_K \setminus \bar{K}$ are hyperbolic.
- The slopes 3 & 1/0 on $\partial N(\bar{K})$ in Σ_K are inequivalent.

New example

Allison H. Moore, Mariel Vazquez
A note on band surgery and the signature of a knot
preprint, arXiv:1806.02440



(A) The knots 8_8 and 8_{20}.

(B) Chirally cosmetic banding along 8_8^* and 8_{20}.

FIGURE 4. (A) The knots 8_8 and 8_{20} are symmetric unions, and as such their determinants are squares. (B) Chirally cosmetic bandings relating the pair 8_8^* and 8_8 (left) and the pair 8_{20} and 8_{20}^* (right). The banding exhibited for 8_8^* was discovered via computer simulation as described in section 4.3. The banding for 8_{20} induces a "4-move."