# 3 次元多様体の双曲性判定

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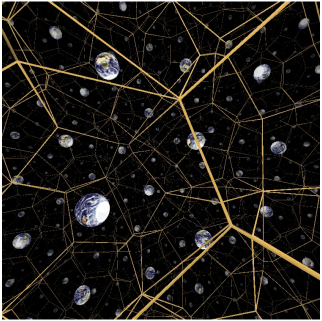
**「精度保証付き数値計算の基礎」チュートリアル**

早稲田大学，2018.9.10

# 3-manifold is ...

## 3-dimensional manifold (3-manifold)

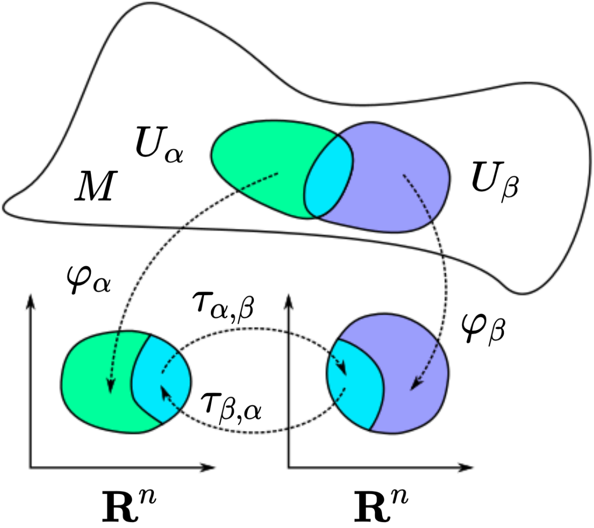
A space locally modeled on $\mathbb{R}^3$ (like our UNIVERSE)



Curved Spaces by J. Weeks

`http://geometrygames.org/CurvedSpaces/index.html`

# Manifold & Charts



# Classification of 3-manifolds

As a consequence of $\boxed{\text{Geometrization Conjecture}}$

including famous Poincaré Conjecture (1904)

conjectured by Thurston (late '70s)

established by Perelman (2002-03)

**Theorem [Perelman]**

The interior of every compact 3–manifold has a canonical decomposition into pieces which have geometric structures.

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**Theorem [Perelman]**

The interior of every compact 3–manifold has a canonical decomposition into pieces which have geometric structures.

A "geometric structure" can be defined as a complete Riemannian metric which is locally isometric to one of the eight model structures.

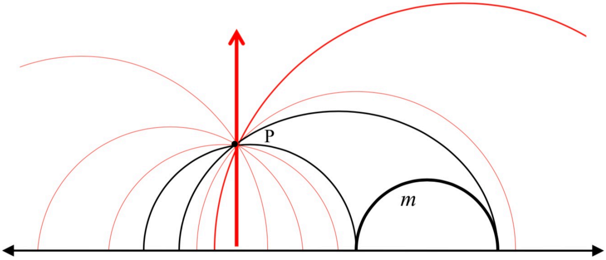
**The most interesting and richest one;**

**Hyperbolic** structure (Riem.metric of const.curv.$-1$)

# Hyperbolic Geometry

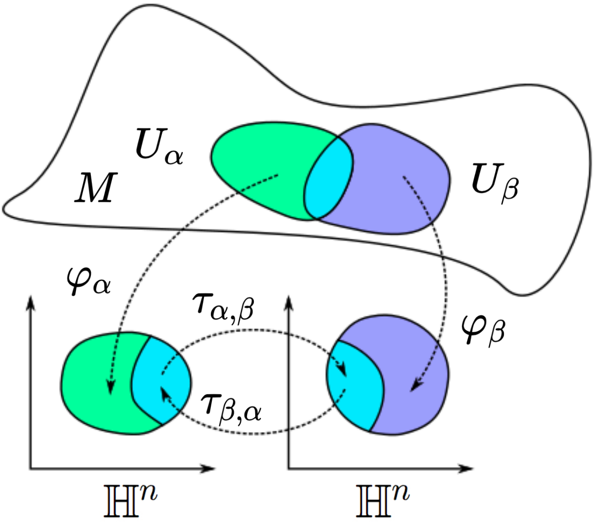
**Playfair's axiom ($\equiv$ parallel postulate)**

For any given line $m$ and point $P$ not on $m$, there are at least two distinct lines through $P$ that do not intersect $m$.




The upper half space model of Hyperbolic plane $\mathbb{H}^2$.

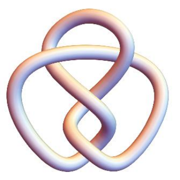
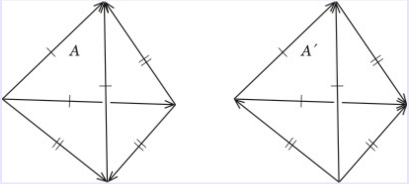
# Hyperbolic manifold



# Thurston's observation

## Figure-eight knot complement can be decomposed...

(hyperbolic ideal tetrahedra)

$S^3-$  $\cong$ 

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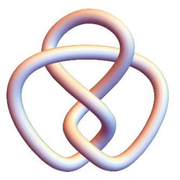
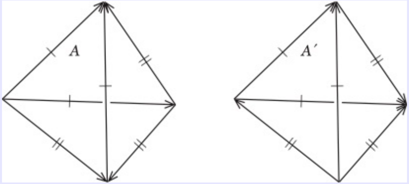
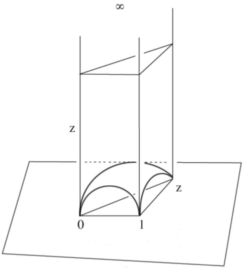
$S^3 -$  $\cong$ 


Each Hyperbolic Ideal tetrahedron is parametrized by a complex variable $z$.



# To find Hyperbolic Structure

## [W. Thurston]

$\forall M$: triangulated 3-manifold, possibly with torus boundary.

$\exists$ equation s.t. whose solution (IF ANY) gives rise to a hyperbolic structure on $M$. (Gluing equation)

$$\prod_{j=1}^{n} (z_j)^{(a_{j,m} - c_{j,m})} \cdot (1 - z_j)^{(-b_{j,m} + c_{j,m})} = \prod_{j=1}^{n} (-1)^{c_{j,m}}$$

for $m = 1, \cdots, n + 2k + h$ and

$$\sum_{j=1}^{n} \arg((z_j)^{(a_{j,m} - c_{j,m})}) + \arg((1 - z_j)^{(-b_{j,m} + c_{j,m})}) = \epsilon_m - \sum_{j=1}^{n} c_{j,m} \cdot \pi i.$$

$\boxed{\text{How to solve?}}$ $\Rightarrow$ Use Verified Numerical Computations!

# HIKMOT

[ Hoffman, Ichihara, Kashiwagi, Masai, Oishi, & Takayasu ]
**Verified computations for hyperbolic 3-manifolds**
Experimental Mathematics, 25 (2016), Issue 1, 66–78.
http://www.oishi.info.waseda.ac.jp/~takayasu/hikmot/

It can possibly give us a rigorous certification
for a given (triangulated) 3-manifold to be hyperbolic.

The python module is available on

`http://www.oishi.info.waseda.ac.jp/~takayasu/hikmot/`

# Application

[ Kazuhiro Ichihara and Hidetoshi Masai ]

**Exceptional surgeries on alternating knots**

Communications in Analysis and Geometry, 24 (2016), 337–377.

We recursively applied `hikmot` to obtain a purely mathematical result, and also used;

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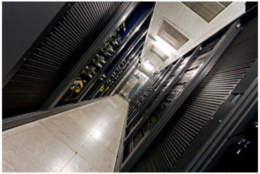
We recursively applied `hikmot` to obtain a purely mathematical result, and also used;

- TSUBAME; the supercomputer of Tokyo Tech. providing large-scale parallel computing.



In total, i.e. the sum of the computation time of all nodes, computation time was approximately $\boxed{\text{512 days}}$, and
the number of manifolds we applied `hikmot` is $\boxed{5{,}646{,}646}$.

Thank you for your attention!

ありがとうございました.

Danke schön!