Epistemic Closures and the Normativity of Logic

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Bridge Principle and Knowledge Closure

**BRIDGE PRINCIPLE**
If \( P_1, P_2, \ldots, P_n \models Q \), then (normative claim about believing \( P_1, P_2, \ldots, P_n \), and \( Q \)).

**\( Br^+ \)-PRINCIPLE**
If \( P_1, P_2, \ldots, P_n \models Q \), then (if \( S \) has reasons to believe \( P_s \), \( S \) has a reason to believe \( Q \)).

**SIMPLE KNOWLEDGE CLOSURE**
If \( P_1, P_2, \ldots, P_n \models Q \), then necessarily (if \( S \) knows \( P_s \), then \( S \) knows \( Q \)).

**KNOWLEDGE CLOSURE**
If \( P_1, P_2, \ldots, P_n \models Q \), then necessarily (if \( S \) knows \( P_s \) and competently deduce \( Q \) from \( P_s \), thereby forming a belief \( Q \) on this basis while retaining \( S \)’s knowledge \( P_s \), then \( S \) knows \( Q \)).

Two Distinctions among Reasons

**SUFFICIENT REASONS**
\( R \) is a sufficient reason to believe \( P \) iff there could be a subject who knows \( P \) on the basis of \( R \).

(Here ‘could’ expresses *metaphysical possibility.*) For \( S \) to know \( P \) on the basis of \( R \), at least, \( S \) has to base her belief on \( R \) and \( R \) itself must be strong enough to justify \( P \). \( S \)’s having a sufficient reason to believe \( P \) does not ensure \( S \)’s being in a position to know \( P \), because \( P \) may be false and other necessary conditions for \( S \) to know \( P \) may not be satisfied.

**PRO-TANTO REASONS**
A pro-tanto reason to believe \( P \) is a consideration which counts in favor of \( P \).

Pro-tanto reasons are defeasible in the sense that they may be either undermined or opposed by other considerations. A pro-tanto reason \( R \) to believe \( P \) is undermined by another reason \( R^* \) when \( R \) is made to count less in favor of \( P \) by \( R^* \). A pro-tanto reason \( R \) to believe \( P \) is opposed by another reason \( R^* \) when \( R^* \) counts in favor of something incompatible with \( P \). When reasons are undermined or opposed, they are defeated.

**ALL-THINGS-CONSIDERED REASONS**
A pro-tanto reason \( R \) is an all-things-considered reason iff \( R \) is not defeated (that is, neither undermined nor opposed) even when all other reasons you have are taken into consideration.

Reason Closure

Let’s say ‘\( S \) AS-reasonably believes \( P \)’ when \( S \) believes \( P \) and \( S \) has an all-things-considered sufficient reason to believe \( P \).
ALL-THINGS-CONSIDERED SUFFICIENT REASON CLOSURE (REASON CLOSURE)
If $P_1, P_2, \ldots, P_n \models Q$, then if $S$ AS-reasonably believes $P$s and $S$ competently deduces $Q$ from $P$s, thereby forming a belief that $Q$ on this basis while retaining $S$’s AS-reasonable beliefs that $P$s, then $S$ AS-reasonably believes $Q$.

Internalism about Epistemic Reasons
Let’s say ‘$S$ and $S^*$ are internal twins’ when they have the same non-factive mental states

INTERNALISM ABOUT EPISTEMIC REASONS (INTERNALISM)
If $S$ and $S^*$ are internal twins, $S$ and $S^*$ have the same reasons.

Because sufficient reasons are reasons, it follows from (INTERNALISM) that internal twins $S$ and $S^*$ have the same sufficient reasons. Furthermore, since internal twins have the same considerations, they have the same all-things-considered reasons. Thus, given (INTERNALISM), internal twins have the same AS reasons.

Inferential Knowledge and Reasons

IMPOSSIBILITY OF INFERENTIAL KNOWLEDGE WITHOUT AS REASONS (IMPOSSIBILITY)
There couldn’t be a subject who inferentially knows $P$ but has no AS reason to believe $P$.

Argument from (KNOWLEDGE CLOSURE) to (REASON CLOSURE)
Given (INTERNALISM) and (IMPOSSIBILITY), we can derive (REASON CLOSURE) from (KNOWLEDGE CLOSURE) as follows:

Assume for reductio that (REASON CLOSURE) is false. Then, for some $P$s and $Q$ such that $P_1, \ldots, P_n \models Q$, (1) $S$ AS-reasonably believes $P$s, (2) $S$ competently deduces $Q$ from $P$s, thereby forming a belief that $Q$ on this basis while retaining $S$’s AS-reasonable beliefs that $P$s, but (3) $S$ doesn’t AS-reasonably believe $Q$. Since (2) entails that $S$ believes $Q$, it follows from (3) that $S$ doesn’t have any AS-reason to believe $Q$.

Let $R$s be $S$’s AS reasons to believe $P$s. (That is, $S$ has an AS reason $R_1$ to believe $P_1$, $S$ has an AS reason $R_n$ to believe $P_n$.) $S$’s internal states such as beliefs, experiences, and inferences are compatible with knowing $P$s based on $R$s. This is because, first, since $R$s are sufficient reasons to believe $P$s, $R$s are able to base knowledge that $P$s and, second, since $R$s are all-things-considered reasons, they are not defeated by other considerations which $S$ has to take into account. (If $R$s were just pro-tanto, some of $R$s might have been defeated by other considerations. In that case, $R$s may be incompatible with knowing that $P$s.) Therefore, there could be an $S$’s internal twin $S^*$ who knows $P$s on the basis of $R$s.

Since $S^*$ knows $P$s, $S^*$ retains her knowledge that $P$s iff $S^*$ retains her reasonable beliefs that $P$s. Therefore, the following equivalence holds: $S^*$ competently deduces $Q$ from $P$s, thereby forming a belief that $Q$ on this basis while retaining $S^*$’s reasonable beliefs that $P$s iff $S^*$ competently deduces $Q$ from $P$s, thereby forming a belief that $Q$ on this basis while retaining $S^*$’s knowledge that $P$s. Since $S$ and $S^*$ are internal twins, $S$ and $S^*$ do the same inference from $P$s to $Q$. Therefore, it follows from (2) that $S^*$ also competently deduces $Q$ from $P$s, thereby forming a belief that $Q$ on this basis while retaining $S^*$’s knowledge that $P$s.

$S^*$ knows $P$ and $S^*$ competently deduces $Q$ from $P$s, thereby forming a belief that $Q$ on this basis while retaining $S^*$’s knowledge that $P$s. Therefore, by (KNOWLEDGE CLOSURE), $S^*$ knows $Q$. Furthermore, since $S^*$ forms her belief that $Q$ inferentially, $S^*$ inferentially knows $Q$.

But, by (INTERNALISM), since $S^*$ is an internal twin of $S$, $S$ and $S^*$ have the same AS reasons. Since, by (3), $S$ doesn’t have any AS reason to believe $Q$, neither does $S^*$.

Thus, $S^*$ has no AS reason to believe $Q$ but inferentially knows $Q$. However, this contradicts (IMPOSSIBILITY). Therefore, we must reject the initial assumption that (REASON CLOSURE) is false. Thus, (REASON CLOSURE) is true.