Epistemic Closures and the Normativity of Logic

Shigaku Nogami

Department of Philosophy Graduate School of Humanities and Sociology
University of Tokyo
(shigaku-nogami@g.ecc.u-tokyo.ac.jp)
Outline

1. Normativity of Logic and Bridge Principles

2. KNOWLEDGE CLOSURE and REASON CLOSURE

3. REASON CLOSURES Defended from Objections

4. Reasons Characterized

5. Conclusions and Remaining Questions
1. Normativity of Logic

logic is a normative subject: it is supposed to provide an account of correct reasoning. [Priest (1979)]

What does it mean to say logic is normative?
Normativity of Logic and BRIDGE PRINCIPLES

MacFarlane (2004) has offered an answer: the normativity of logic is embodied by principles connecting logical consequence and epistemic norms.

MacFarlane calls such principles Bridge Principles:

**BRIDGE PRINCIPLE (BP)**

If \((P_0, P_1, \ldots, P_n \Rightarrow Q)\), then (normative claim about believing \(P_0, P_1, \ldots, P_n,\) and \(Q\)).
Examples of Bridge Principles (C-principles)

Where $P_0, \ldots, P_n \Rightarrow Q, \ldots$

C: Deontic operator embedded in consequent.

- **o**: Deontic operator is strict obligation (ought).
  - **Co**: if you believe $P_0, \ldots, P_n$, you **ought to** believe $Q$.

- **p**: Deontic operator is permission (may).
  - **Cp**: if you believe $P_0, \ldots, P_n$, you **may** believe $Q$.

- **r**: Deontic operator is “has (defeasible) reason for.”
  - **Cr**: if you believe $P_0, \ldots, P_n$, you **have reason to** believe $Q$. 


**Examples of Bridge Principles (B-principles)**

B: Deontic operator embedded in both antecedent and consequent.

o: Deontic operator is strict obligation (ought).

\[ \text{Bo: if you \textit{ought to} believe } P_0, \ldots, \text{ you \textit{ought to} believe } P_n, \text{ then you \textit{ought to} believe } Q. \]

p: Deontic operator is permission (may).

\[ \text{Bp: if you \textit{may} believe } P_0, \ldots, \text{ you \textit{may} believe } P_n, \text{ then you \textit{may} believe } Q. \]

r: Deontic operator is “has (defeasible) reason for.”

\[ \text{Br: if you \textit{have reason to} believe } P_0, \ldots, \text{ you \textit{have reason to} believe } P_n, \text{ you have reason to believe } Q. \]
Examples of Bridge Principles (W-Principles)

W: Deontic operator scopes over whole conditional.

o: Deontic operator is strict obligation (ought).
   \textbf{Wo}: you \textbf{ought to see to it that} (if you believe $P_0$, ..., you believe $P_n$, then you believe $Q$).

p: Deontic operator is permission (may).
   \textbf{Wp}: you \textbf{may see to it that that} (if you believe $P_0$, ..., you believe $P_n$, then you believe $Q$).

r: Deontic operator is “has (defeasible) reason for.”
   \textbf{Wr}: you have \textbf{reason to see to it that that} (if you believe $P_0$, ..., you believe $P_n$, you believe $Q$).
Two Important Bridge Principles

**Wo PRINCIPLE**

If $P_0, \ldots, P_n \Rightarrow Q$, then you **ought to see to it that** (if you believe $P_0$, ..., you believe $P_n$, you believe $Q$).

✓ Variants of Wo PRINCIPLE are endorsed by some philosophers. [Broome (2000), MacFarlane (2004), and Field (2009, 2017), cf. Restall (2004)]

**Br PRINCIPLE**

If $P_0, \ldots, P_n \Rightarrow Q$, then if you **have reason to** believe $P_0$, ..., you **have reason to** believe $P_n$, then you **have reason to** believe $Q$.

✓ In this talk, I will mainly focus on variants of Br PRINCIPLE.
Consider McGee’s ‘counter example’ to Modus Ponens (McGee 1985):

Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed, with good reason:

If a Republican wins the election, then if it’s not Reagan who wins it will be Anderson. \([A \rightarrow (B \rightarrow C)]\)

A Republican will win the election. \([A]\)

Yet they did not have reason to believe

If it’s not Reagan who wins, it will be Anderson. \([B \rightarrow C]\).
Consider “reductio” of classical rule of Explosion:

(1) If $X$ is logically inconsistent, then $X \Rightarrow P$, for any $P$. [EXPLOSION]

(2) If your belief set $B$ entails a proposition $P$ (and you know that $B \Rightarrow P$), then you would have a reason to believe $P$.

(3) Even if you know that your belief set $B$ is inconsistent (and $B \Rightarrow P$, for any $P$), there are (nonetheless) some $P$’s such that it is not the case that you would have a reason to believe $P$.

(4) Therefore, since (1)–(3) lead to absurdity, our initial assumption (1) must have been false.
2. **Knowledge Closure and Reason Closure**

Br Principle is closely related to the Principle of Knowledge Closure, one of Epistemic Closures:

**Simple Knowledge Closure**

If $P \Rightarrow Q$, then if you know $P$, then you know $Q$.

**Br Principle**

If $P \Rightarrow Q$, then if you have reason to believe $P$, then you have reason to believe $Q$. 
Problems of Simple Knowledge Closure

Simple Knowledge Closure

If \( P \Rightarrow Q \), then if you know \( P \), then you know \( Q \).

(1) We do not believe all logical consequences of our knowledge. If Simple Knowledge Closure were correct, we would believe all theorems of, e.g., set theory. But this is absurd, for there are too many theorems of set theory to be believed by finite beings like us [cf. Harman (1986), p. 14].

(2) Even if we believe logical consequences of our knowledge, it may be because we are just guessing them without basing our beliefs on our knowledge. But to know (or to be justified in believing) logical consequences of our knowledge, we need to deduce them from our knowledge.
**Knowledge Closure Reformulated**

**Knowledge Closure (KC)**

If $P \Rightarrow Q$, then if (you know $P$) & (you competently deduce $Q$ from $P$, thereby forming a belief that $Q$ on this basis while retaining your knowledge that $P$), then you know $Q$.

[Pritchard (2016), Hawthorne (2004), and Williamson (2000)]

**REASON CLOSURE Formulated**

**REASON CLOSURE (RC)**

If \( P \rightarrow Q \), then (you have a reason to believe \( P \)) \& (you competently deduce \( Q \) from \( P \), thereby forming a belief that \( Q \) on this basis while retaining your belief that \( P \)), then you have a reason to believe \( Q \).

*Given KNOWLEDGE CLOSURE, we need to REASON CLOSURE to save the intuitive idea that ‘inferential’ knowledge must be based on reasons (inferential knowledge without any reasons is impossible).*
From KNOWLEDGE CLOSURE to REASON CLOSURE (1)

Given some intuitive assumptions, KNOWLEDGE CLOSURE is true only if REASON CLOSURE is true:

1. Suppose, for reductio, REASON CLOSURE is false.
2. Then, there is a possible world $W'$ where you have a reason to believe $P$ and come to believe $Q$ by inferring $Q$ from $P$, but still you don’t have any reason to believe $Q$. [From 1]

* ✓ You have a reason $R$ to believe that $P$
* ✓ You come to believe that $Q$ by inferring $Q$ from $P$
* ✓ You don’t have any reasons to believe $Q$
From **Knowledge Closure to Reason Closure** (2)

3. Let us suppose further that, if it is possible that you have a reason $R$ to believe $P$, then it is possible that, without obtaining new reasons, you have know $P$ on the basis of the reason $R$.

4. Then, there is a possible world $W'$ where you know $P$ on the basis of the reason $R$ and you come to believe $Q$ by inferring $Q$ from $P$, but still you lack any reason to believe $Q$.

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✓ You have a reason $R$ to believe that $P$
✓ You come to believe that $Q$ by inferring $Q$ from $P$
✓ You don’t have any reasons to believe $Q$

✓ You have a reason $R$ to believe that $P$
✓ You come to believe that $Q$ by inferring $Q$ from $P$
✓ You **know $P$ on the basis of $R$**
5. But the difference between \( W \) and \( W' \) does not lie in what reasons you have. (Rather, it lies in other external conditions such as truth of \( P \) or reliability of belief forming process.)

6. Therefore, since you don’t have any reasons to believe \( Q \) at \( W \), you don’t have any reason to believe \( Q \) in \( W' \).

- You have a reason \( R \) to believe that \( P \)
- You come to believe that \( Q \) by inferring \( Q \) from \( P \)
- You don’t have any reasons to believe \( Q \)
- You have a reason \( R \) to believe that \( P \)
- You come to believe that \( Q \) by inferring \( Q \) from \( P \)
- You know \( P \) on the basis of \( R \)
- You don’t have any reasons to believe \( Q \)
From Knowledge Closure to Reason Closure (4)

7. Since you know $P$ at $W'$, you know $Q$ at $W'$. [Knowledge Closure]

8. Thus, while your knowledge that $Q$ is inferential knowledge, you don’t have any reason to believe $Q$ at $W'$. That is, you have inferential knowledge without any reasons.

9. But, ‘inferential’ knowledge without reasons is impossible.

10. Therefore, we should reject our initial assumption that Reason Closure is false.

- You have a reason $R$ to believe that $P$
- You come to believe that $Q$ by inferring $Q$ from $P$
- You don’t have any reasons to believe $Q$
- You have a reason to believe that $P$
- You come to believe that $Q$ by inferring $Q$ from $P$
- You know $P$ on the basis of $R$
- You know $Q$. [Knowledge Closure]
- You don’t have any reasons to believe $Q$
3. Objections to REASON CLOSURE?

1. Harman’ Objection from Belief Revision
2. Lottery Objection
Objection from Belief Revision (Harman 1986)

Consider the following case:

Mary believes that if she looks in the cupboard, she will see a box of Cheerios \([L \rightarrow C]\). She comes to believe that she is looking in the cupboard \([L]\) and that she does not see a box of Cheerios \([\sim C]\). [...] she abandons her first belief \([L \rightarrow C]\), concluding that it is false at all. [Harman (1986), p. 5]
Objection from Belief Revision

Mary has a reason to believe $L & L \rightarrow C$. If Mary deduces $C$ from her belief that $L & L \rightarrow C$, then, according to REASON CLOSURE, she has a reason to believe $C$.

However, if Mary has a reason (e.g., her visual experience) to believe $\sim C$, she may not obtain a reason to believe $C$ by inference. After all, she obtains a reason to reject her initial belief that $L & L \rightarrow C$ rather than to accept $C$. 
Mary has a pro-tanto reason to believe $L \& L \rightarrow C$. And, Mary may have a pro-tanto reason to believe $C$.

However, Mary doesn’t have an all-things-considered reason to believe $C$. Neither does she have a all-things-considered reason to believe $L \& L \rightarrow C$. 
Pro-tanto Reasons

To answer Objection from Belief Revision, we need to distinguish pro-tanto reasons and all-things-considered reasons:

**Pro tanto Reason** [cf. Pryor (2012, 2013)]

A pro-tanto reason to believe \( P \) is a consideration which counts in favor of \( P \)

Pro-tanto reasons may be either undermined or opposed by other reasons. Both undermining and opposing reasons are defeating reasons.
Defeating Reasons: Undermining Reasons

**UNDERMINING REASON** [cf. Pryor (2012, 2013)]

A pro-tanto reason $R$ to believe $P$ is **undermined** by another reason $R'$ when $R$ is made to count less in favor of $P$ by $R'$.

**EXAMPLE**

You see a wall which looks red. Your visual experience $R$ may be your reason to believe that the wall is red ($P$). However, you find that the room lighting is red ($R'$). Your reason ($R$) to believe $P$ is made to count less in favor of $P$ by the fact that the room lighting is red ($R'$). $R$ is undermined by $R'$. 

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Defeating Reasons: Opposing Reasons

OPPOSING REASON [cf. Pryor (2012, 2013)]
A pro-tanto reason $R$ to believe $P$ is opposed by another reason $R'$ when $R'$ counts in favor of what is incompatible with $P$.

EXAMPLE
You see a bird that looks like a swan. Your visual experience $R$ may be your reason to believe that the bird is a swan ($S$). However, your friend ornithologist says ‘It’s a duck’. The fact that the ornithologist says that it’s a duck ($R'$) is a reason to believe that the bird is a duck ($D$). $S$ and $D$ are incompatible. Thus, $R$ is opposed by $R'$. 
**All-things-considered Reasons**

**ALL-THINGS-CONSIDERED REASON** [cf. Broome (2000)]

An all-things-considered reasons to believe $P$ is a consideration which counts in favor of $P$ when all reasons you have are taken into consideration.

A pro tanto reason $R$ will be an all-things-considered reason when $R$ is not **defeated** (that is, neither undermined nor opposed by other reasons).
Pro-tanto Reason Closure (PRC)

If $P \Rightarrow Q$, then if (you have a **pro-tanto** reason to believe that $P$) & (you competently deduces that $Q$ from $P$, thereby forming a belief that $Q$ on this basis while retaining your belief that $P$), then you have an **pro-tanto** reason to believe that $Q$.

All-things-considered Reason Closure (ARC)

If $P \Rightarrow Q$, then if (you have an **all-things-considered** reason to believe that $P_0$ & ... & you have an **all-things-considered** reason to believe that $P_n$) & (you competently deduce that $Q$ from $P$, thereby forming a belief that $Q$ on this basis while retaining your belief that $P$), then you have an **all-things-considered** reason to believe that $Q$. 27
If $P \implies Q$, then if (you have a reason to believe that $P$) & (you competently deduces that $Q$ from $P$, thereby forming a belief that $Q$ on this basis while retaining your belief that $P$), then you have a reason to believe that $Q$. 
Multiple Premises in Inference

Consider Modus Tollens:

You believe that if it was raining, the ground is wet now (A→B). You look to the street, which is dry now. You come to believe that the street is not wet now (~B). Then, you infer that it was not raining (~A) from the two premises A→B and ~B. Given that you have a reason to believe A→B and also have a reason to believe ~B, it seems that, you have a reason to believe ~A.
MULTI-PREMISE REASON CLOSURE

MULTI-PREMISE REASON CLOSURE (MRC)

If $P_0, \ldots, P_n \Rightarrow Q$, then if (you have a reason to believe that $P_0$) & \ldots & (you have a reason to believe that $P_n$) & (you competently deduce that $Q$ from $P$, thereby forming a belief that $Q$ on this basis while retaining your beliefs that $P_0, \ldots, P_n$), then you have a reason to believe that $Q$. 
### REASON CLOSURES

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Lottery Case

Lottery Objection is targeted at Multi-premise Reason Closure. Consider the following case:

Suppose you draw lots. You know that the lottery contains only one winning ticket. Since the lottery contains 10,000 tickets, the probability that your ticket is a winner is extremely low. So, you believe that it is a loser. And, it seems, you have a pro-tanto reason to believe it. And since there is no defeater of it, you have an all-things-considered reason to believe it.

But, by parity of reasoning, of each ticket $i$, you come to believe that $i$ is a loser ($L_i$). And, it seems, of each $i$, you have a pro-tanto reason to believe $L_i$. And, it seems, since there is no defeater of it, you have an all-things-considered reason to believe $L_i$.  

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Lottery Objection to **MULTI-PREMISE ALL things-CONSIDERED** **REASON CLOSURE** [cf. Kyburg (1971), Steinberger (2016)]:

1. \( L_0, L_1, \ldots, L_{10,000} \Rightarrow L_0 \& L_1 \& \ldots \& L_{10,000} \) [**CONJUNCTION INTRODUCTION**]
2. You have an all-things-considered reason to believe \( L_0 \& \) you have an all-things-considered reason to believe \( L_1 \ldots \& \) you have an all-things-considered reason to believe \( L_{10,000} \).
3. Suppose you infer \( L_0 \& \ldots \& L_{10,000} \) from \( L_0, L_1, \ldots, L_{10,000} \).
4. Then, according to **MULTI-PREMISE REASON CLOSURE**, you have a reason to believe that \( L_0 \& \ldots \& L_{10,000} \). This is absurd, for you know that the lottery contains one winning ticket.
5. Therefore, we should reject **MULTI-PREMISE REASON CLOSURE**.
Lottery Objection to MULTI-PREMISE ALL-THINGS-CONSIDERED RC


1. $L_0, L_1, \ldots, L_{10,000} \Rightarrow L_0 \& L_1 \& \ldots \& L_{10,000}$ [CONJUNCTION INTRODUCTION]

2. You have an all-things-considered reason to believe $L_0$ & you have an all-things-considered reason to believe $L_1$ \ldots & you have an all-things-considered reason to believe $L_{10,000}$.

3. Suppose you infer $L_0 \& \ldots \& L_{10,000}$ from $L_0, L_1, \ldots, L_{10,000}$.

4. Then, according to MULTI-PREMISE REASON CLOSURE, you have a reason to believe that $L_0 \& \ldots \& L_{10,000}$. This is absurd, for you know that the lottery contains one winning ticket.

5. Therefore, we should reject MULTI-PREMISE REASON CLOSURE.\textsuperscript{34}
Lottery Objection

In the lottery case, you have no reasons which defeat a pro-tanto ‘reason’ to believe $L_i$.

Therefore, to deny that you have an all-things-considered reason to believe $L_i$ is to deny that you have a pro-tanto reason to believe $L_i$.

We need to find a characterization of pro-tanto reason such that you do not have a pro-tanto reason to believe $L_i$. 
Practical Reasoning in Lottery Case

Consider the following case:

You’ve bought a ticket which costs a dollar. Before the winner announcement, you are offered a cent for a lottery ticket and reason as follows:

The ticket is a loser. [Call this premise $L$]
If I keep the ticket, I will get nothing.
If I sell the ticket, I will get a cent.
So I ought to sell the ticket.

(cf. Hawthorne 2004, p. 29)

It would be irrational to sell the ticket. But the reasoning itself seems to have no flaw.
Explanation of Irrationality

One explanation would be that you don’t know the premise $L$ (‘the ticket is a loser’) of the practical reasoning (Hawthorne 2004):

That one does not know a lottery proposition […] seems to prohibit one from using it as a remise in one’s deliberations about how to act.

But, since the fact that you do not know the premise $L$ does not play an essential role in explaining your irrationality, better explanation is that you have no reason to believe the premise $L$. 
Irrationality Explained by Lack of Reasons

Consider the following case:

You heard the winner announcement that the ticket is a loser. Then, you have a pro-tanto reason to believe the premise $L$ of the practical reasoning. Your reason to believe $L$ is that you have heard an announcement that your ticket is a loser. (In many cases, this reason would give you knowledge that $L$ when $L$ were true and the announcement were reliable.) However, unknown to you, the announcement was incorrect and your ticket was a winner. Thus, you don’t know $L$. 
Lack of Knowledge Explained by Lack of Reasons

It is commonly supposed, in the lottery case, you cannot know of any ticket $i$ that $i$ is a loser ($L_i$) (before winner announcement.) [Hawthorne (2004), Pritchard (2004)]

At the same time, you do not have any (pro-tanto) reasons to believe that $i$ is a loser (before winner announcement.) This fact explains your irrationality of the practical reasoning.

This fact also explains your lack of knowledge that $i$ is a loser!
4. Pro-tanto Reasons Characterized

Pro-tanto reasons must be considerations whose lack explains the following two facts in the lottery case:

✓ You cannot know $L_i$,
✓ You cannot use $L_i$ as a premise of a practical reasoning.

I propose a necessary condition on reason:

**PRO-TANTO REASON**

You have a pro-tanto reason $R$ to believe $P$ only if it is possible that you know $P$ on the basis of $R$. 
2. You have an all-things-considered reason to believe $L_0$ & you have an all-things-considered reason to believe $L_1$ …. & you have an all-things-considered reason to believe $L_{10,000}$.

In the lottery case, (1) you cannot know $L_i$ and (2) you cannot use $L_i$ as a premise of a practical reasoning.

Best explanation of both of these is that you don’t have a pro-tanto reason to believe $L_i$. Therefore, you don’t have an all-things-considered reason to believe $L$. Thus, we can reject the second premise of the Lottery Objection. **MULTI-PREMISE ALL-THINGS-CONSIDERED RC** can be defended.
Caveats on Knowledge and Pro-tanto Reasons

✓ Having a pro-tanto/all-things-considered reason to believe $P$ is not sufficient for knowing $P$ on the basis of $R$. To have knowledge that $P$ may (at least) need:

(1) $P$ must be true.

(2) Your belief that $P$ may need to be safe in the sense that your belief that $P$ ‘could not have easily been false.’ [Sosa (1999), Pritchard (2005, 2015), Williamson (2000)].

✓ There may be knowledge without reasons. Our characterization does not exclude such possibility.
MULTI-PREMISE PRO-TANTO RC Fails

Under our characterization, MULTI-PREMISE PRO-TANTO REASON CLOSURE fails:

You see a bird that looks like a swan. Your visual experience $R$ may be your pro-tanto reason to believe that the bird is a swan ($S$). However, your friend ornithologist says ‘It’s a duck’. The fact that the ornithologist says that it’s a duck ($R'$) is a pro-tanto reason to believe that the bird is a duck ($D$).

You have a pro-tanto reason $R$ to believe $S$ and a pro-tanto reason $R'$ to believe $D$. But even if you infer $S \& D$, you cannot know $S \& D$. You cannot know $S \& D$ on the basis of any reason. Thus, you don’t have a reason to believe $S \& D$. 
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**Reason Closures**
1. When you have inferential belief that $Q$ by inferring $Q$ from $P$, what is your reason to believe $Q$? (Suppose you have a reason $R$ to believe $P$.)
   
i. The reason to believe $P$, $R$? [cf. Pryor (2012)]
   
   
iii. Anything else?

This question is important in thinking how REASON CLOSURE is related (equivalent under (i)) to REASON TRANSMISSION [cf. Wright (2004)]:

**REASON TRANSMISSION (Rough)**

If $P \implies Q$, then if you have a reason $R$ to believe that $P$, then you have $R$ to believe that $Q$. 
2. How is Reason Closure related to other Bridge Principles, in particular, Wo Principle: ‘If \( P \Rightarrow Q \), then if you believe \( P \), you ought to believe \( Q \)?’ Are they irreducible to each other?

3. Given our characterization of reasons, are arguments against classical logic (e.g., McGee’s argument, the argument against Explosion) viable?
References

References


Thank you for your Attention!