Unifying framework for mean field theories of asymmetric kinetic Ising systems

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Advances in high-throughput data acquisition technologies provide unprecedented possibilities for the statistical description of very large biological systems (e.g. recording of tens or hundreds of thousands of neurons). In this scenario, there is a pressing demand for mathematical tools that can cope with very large data sets. Asymmetric kinetic Ising models are powerful tools for studying the dynamics of complex systems and inferring the parameters that describe experimental recordings. Although the analytic study of the models' behaviour is usually intractable for large networks, mean field theories are frequently used to approximate their statistical properties. While being relatively simple to analyze, naive mean field approaches generally neglect the effects of correlations and fluctuations in a system, which can lead to inaccurate predictions of its behaviour. Here, we propose a unified framework for mean field theories of the dynamics of kinetic Ising systems based on information geometry (extending previous work as [1,2]). Our framework can describe classical naive mean field and TAP approximations, as well as novel mean field approaches that preserve the correlations of the system. This is done by applying Plefka expansions [3] around specific points on a sub-manifold of probability distributions with known properties. These expansions can approximate the behaviour of the system in transient and stationary states. Comparing our approximations with the exact numerical simulations we observe that they provide more accurate estimates than classical mean field approximations for the evolution of equal time and time delayed covariance matrices. Finally, we use the proposed approximations to solve the inverse Ising problem where we, similarly, observe better performance in estimation of interactions and external fields. Our framework clarifies and extends current approaches to mean field approximations of kinetic Ising models, and provides a framework for more accurate approximations.

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