More Truthmakers for Vagueness

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Abstract. Towards the sorites paradox, many theorests have suggested their own solutions. Among these many, Sorensen suggests his version of epistecism solution, *truthmaker gap epistecism*, according to which the sorties paradox arises because there are unground truths – true with no truthmaker. Nevertheless, Sorensen's version has been criticized for abandoning *higher-order* vagueness. This paper suggests a novel version of truthmaker theory for vague predicates, which not only solves the sorites paradox but also encompasses higher-order vagueness.

Keywords: truthmaker · vagueness · the Sorites paradox · higher-order vagueness

1 Introduction

When vagueness matters, truth matters a lot. A well known paradox of the sorites has cast a question on the concept of truth. For example, a solution by degree theorists [1] to the sorites paradox questions our widely-believed presupposition on truth by suggesting that truth values do not have to be just false (truth value 0) nor just true (truth value 1) but somewhere between them such as 0.2 or 0.9876. Given that truth matters when we talk about vagueness, *truthmakers* may matter as well. At least, Sorensen [5] employs the idea of truthmakers to present a variant of the epistemicist solution (e.g. [7]) to the sorites paradox. According to his account labeled *truthmaker gap epistemicism*, there are sentences which are true but *ungrounded* i.e. has *no* truthmaker. In the sorites case, an epistemicist of this type insists on the existence of some natural number k such that a sentence (K) "a person with k hairs is bald" is true without any truthmaker. To solve the sorites paradox, we need to explain how someone turns into non-bald from bald. In terms of truthmaking, we need to explain what makes (K) true and (K+1) "a person with $k+1$ hairs is bald" false. Sorensen's answer is *nothing*. Such a natural number k exists but (K) has no truthmaker at all. Sorensen holds a version of epistemicism, claiming that we do not know the exact value of k because (K) has nothing that makes it true.

However, Sorensens solution calls for an unwanted byproduct. [2] As Jago [2] has already pointed out, Sorensen's approach is incompatible with higher-order

[1] For example, see [3].

[2] Another important issue, which is rather metaphysical, is about *truthmaker maximalism*. Most truthmaker theorists assume truthmaker maximalism, that is, every

vagueness: vagueness about whether something is a borderline case, vagueness on whether something is a borderline case of being a borderline case and so on. Sorensen's framework cannot explain this higher-order vagueness because his account of truthmaking is understood as clear-cut: either truth is *wholly* made true or truth is not made *at all*. Sorensen allows borderline cases in the first order but no higher-order borderline cases. This is problematic because we have many instances of higher-order vagueness. We – even Sorensen himself [6] [4] – often say that "vague is vague". We can reasonably discuss a borderline case of a borderline case, and a borderline case of a borderline case of a borderline case, and so on. Any reasonable theory of vagueness should explain this phenomenon.

This paper suggests a more fine-grained account to the sorites paradox, adopting several theoretical resources from truthmaker theories (*cf.* [1]). My approach is inspired by Sorenson's idea: truth-makers play a significant role in sorites scenarios. My proposition is richer and more fine-grained which better captures higher-order vagueness. To reconcile higher-order vagueness, I posit considerations that are neither subversive nor challenging, rather, I wish to utilize vocabulary with which we have already been familiar with in the ongoing truthmaking community. Instead of the black-and-white notion of "lack" and "existence" of truthmakers as Sorensen employed, we can utilize the richer toolkits of truthmakers such as distinction between *partial* and *full* truthmakers.

The rest of this paper is constructed as follows. The next sections explains an updated version of truthmaker theory for vague predicates (§2) and demonstrates this solves the sorites paradox (§3). The following section (§4) examines how the new one can deal with higher-order vagueness, which Sorensne fails.

2 More truthmakers and truthmakings

This section offers my version of truthmaker theory for the sorites paradox. Following my theory, it turns out that the sorites paradox arises only when you believe two unreasonable assumptions on certain facts (about the number of hairs). Unless we adopt these conditions, we avoid the counterintuitive conclusion such as "a person with 2,000,000 hairs is bald".

Notation. Before diving into the discussion, let me note several notations. Let B be a 1-ary vague predicate "is bald". Write a natural number n indicating a person with n hairs. Hence, read $B(n)$ as "a person with n hairs is bald".

Let $f, g, h, ...$ be *facts*. If a single fact f fully or partially makes a proposition ϕ true, then f is a (full or partial) truthmaker for ϕ. For example, the fact that I am being a student is the full truthmaker for a proposition "I am a student". Similarly, the fact that I am being a student is a partial truthmaker for a proposition "I am a student in Tokyo" since we need other facts to make it true (like the fact that I am living in Tokyo).

truth has its truthmaker, as a default setting. Sorense does not seem to provide enough reasons to reject this default principle. I will leave this issue for another project.

Let Γ be a set of facts $\Gamma = \{f, g, h, ...\}$. A fact can be about anything. We introduce *types* of facts. In particular, let f^n be a fact that the person in question has n hairs. Facts other than this type such as facts about allocations of hairs, facts that a person puts wig are written in different alphabets. Call a collection of facts (truthmakers) a *truthmaking*, highliting the difference between each fact and a collection of facts. [3] Read $\Gamma \vdash B(n)$ "a truthmaking Γ makes $B(n)$ true". We often drop brackets of a truthmaking for the sake of simplicity. For example, write $f, g \vdash \phi$ instead of $\{f, g\} \vdash \phi$.

The sorites paradox reformalized. Now, let us write the sorites paradox in the truthmaking terminology. (Seemingly) reasonable assumptions (1,2) lead to an unintuitive conclusion (3).

1. Base case. $f^0 \vdash B(0)$. *A person with 0 hair (no hair at all) is bald.*

2*. Tolerance principle. There is no $n \in N$ such that $f^n \vdash B(n)$ and $f^{n+1} \vdash \neg B(n+1)$. *A small change such as pulling a single hair does not make a person bald from non-bald.*

3. Conclusion. $f^{2,000,000} \vdash B(2,000,000)$. *A person with 2,000,000 hair is bald.*

The spirit of tolerance principle, a key assumption of the sorites paradox, is "small change does not matter". Intuitively, pulling one single hair seems to change nothing about whether somebody is bald or not bald. Its equivalent claim also sounds reasonable as well: We do not have the clear threshold between baldness and non-baldness. If we do, we have the exact number k – which is the largest number of hairs for being bald and $k+1$ is the smallest number for being non-bald. Most non-truthmaker theories formulate the tolerrance in a normal if-then clause such as: if $B(n)$, then $B(n+1)$ for an arbitrary n. Or, equivalently, non-existence claim such as there is no n satisfying both $B(n)$ and $\neg B(n+1)$.

From the truthmaker perspective, we can understand the sorites scenario in greater detail. First, the base case is now understood in the terms of truthmaking as "the fact of her having no hair at all is sufficient to make the person bald". With no help of other facts, it is enough to determine that the person is bald just by the fact about her number of hair, which is, zero. Second, this truthmaking talk gives a closer view to the tolerance principle –the key clause of this paradox. We should understand the principle as follows: there is no $n \in N$ such that $f^n \vdash B(n)$ and $f^{n+1} \vdash \neg B(n+1)$. This principle translated into the truthmaker terminology is read as: you cannot find a pair of facts f^n "having n hairs" which makes someone bald and f^{n+1} "having $n+1$ hairs" which makes someone not bald. [4]

[3] You can reasonably claim that truthmaking – a collection of truthmakers – is also a (kind of) truthmaker. But the metaphysical debate on this does not affect our discussion on the sorites in this paper.

[4] Note this is weaker than the following version.

2!. The tolerance principle (stronger). If $f^n \vdash B(n)$, then $f^{n+1} \vdash B(n+1)$.

This stronger version claims that if the fact f^n of having n hairs makes someone bald, then another fact f^{n+1} of having $n+1$ hairs also makes someone bald.

I am not claiming that such $B(k+1)$ is indeterminate. f^{K+1} is not enough but it may have its truthmaking. With extra facts, say Δ, $B(k+1)$ (or its negation) would

The paradox avoided. Following this formalization, the sorites paradox does not necessarily happen. Let us assume the base case **1**. From the truthmaking perspective, this says more than a person with no hair at all is bald; it claims that the single fact f^0 about the number of hair is enough to make the person bald. Let us also assume the tolerance **2***. Notice that the tolerance here only claims that there is no pair of a single fact f^n about the number of hair n which makes $B(n)$ true and another single fact f^{n+1} about $n+1$ hairs which make $\neg B(n)$ true. In other words, the truthmaking of a single fact f^k about the number of hair k for $B(k)$ does *not* promise another fact f^{k+1} about $k+1$ hairs does the same job for $B(k+1)$. Note that I do not mean that such k is a threshold between baldness and non-baldness. $B(k+1)$ may be still true with other truthmakers (like $\{f^k, g, h\}$) or by other truthmakings (like $\{g, h, i\}$). In such cases, the tolerance does not block another pair of two facts, say, g^m which makes $B(m)$ true and g^{m+1} which makes $\neg B(m+1)$ true because the tolerance is only about facts about the number of hair.

Conditions needed. Now, we avoid the sorites paradox by offering my trutuhmaker picture. But this is not the end of the story. We also need to explain why this is a paradox – why people find the sorites paradox paradoxical at all. For this purpose, we name the two extra conditions over truthmakers behind the paradox. Namely, the facts about the number of hairs should be (i) *full* and (ii) *obligatory* trutuhmakers. In other words, the sorites paradox arises because people (mistakingly) adopt these questionable and unjustified assumptions saying the number of hair is (i) sufficient and (ii) necessary for someone to be bald.

Condition 1: full. Firstly, to reach the problmatic conclusion, the truthmakers f^n – facts about the number of hairs – need to be *full* with respect to the baldness predicate B. By f^n being full with respect to $B(n)$, I mean that either f^n suffices to make $B(n)$ true or f^n suffices to make $\neg B(n)$ by itself and no other fact $g \neq f$ is needed. That is, the number of hairs sufficiently determines the truth value of $B(n)$.

What happens if we drop this fullness character? The sorites paradox does not pop up! Let us see in detail. Assume the base case **1**. The fact of having no hair f^0 is enough to make a person with no hair at all bald. Formally written, $f^0 \vdash B(0)$. However, our tolerance **2*** does not block us to move forward to the following case: $f^n \vdash B(n)$ and $f^{n+1} \nvdash B(n+1)$ and $f^{n+1} \nvdash \neg B(n+1)$. This says that when the number of hair is n, the fact about the number of hair is enough to make true $B(n)$. But as for $n+1$ and $B(n+1)$, f^{n+1} is not enough. We need extra facts about the person with $n+1$ hairs, say, g, to make $B(n+1)$ true: $f^{n+1} \nvdash B(n+1)$ but $f^{n+1}, g \vdash B(n+1)$. It is easy to see that the problematic consequence ("a person with 2,000,000 hairs is bald") never arises after such n. The fact that a person has $2,000,000$ hairs is not sufficient to make $B(2,000,000)$ true.

be determinated as $\Delta, f^{k+1} \vdash B(n)$ (or $\Delta, f^{k+1} \vdash \neg B(n)$). There may not be such supporting Δ. This would be Sorensen's picture of absolute borderline case. I will discuss this later.

Condition 2: obligatoriness. We have just observed that the sorites paradox presupposes that the number of hair should be a *full* trutuhmaker for baldness. The other required condition is *obligatoriness*: the number of hair should be *obligatory* in the sense that the number of hair appears in any truthmaking Γ for $B(n)$. In other words, the number of hairs *always* matters. More formally speaking, f^n is obligatory for a predicate B if $\Gamma \vdash B(n)$ implies $f^n \in \Gamma$.

Let us check that the sorites paradox needs to presuppose this condition. For the sake of the purpose of this current argument (to highlight the necessity to reach the sorites paradox), we assume that the number of hairs is full for baldness. Let us start with the base case $f^0 \vdash B(0)$. And suppose $f^n \vdash B(n)$ for some n. Here let us drop the obligatoriness. Then, we can have another fact $g \neq f$ such that $g \vdash B(n)$. Now we have two independent truthmakings for the same proposition $B(n)$: $f^n \vdash B(n)$ and $g \vdash B(n)$. The tolerance **2*** states only about the facts about the number of hairs. So it has nothing to do with facts with another type g, h, i or anything other than f, which is not about the number of hairs. Hence, some non-f fact may make $\neg B(2,000,000)$ true — $g_{\neq f} \vdash \neg B(2,000,000)$. Or, at least, may not make $B(2,000,000)$ true — $g_{\neq f} \nvdash B(2,000,000)$

3 Arguments against idealized properties

In the previous section, we specified two conditions over truthmakers required to make the sorites scenario paradoxical. This section presents less-formal arguments claiming that these two conditions are too ideal in most cases. Our starting point (a person with no hair at all is bald because she has no hair at all) and ending point (a person with 2,000,000 hairs is not bald because she has 2,000,000 hairs) are just exceptional cases where the numbers of hairs solely matter.

3.1 Fullness?

Let us start with the condition to be full i.e. the fact about the number of hair is sufficient to determine truth value of the bald claim. This principle is not always the case for baldness and other vague predicates [5] because we often rely on facts other than the number of hairs when we evaluate the predicate. In fact, the number of hair alone does not tell us enough information about the

[5] For example, consider “is tall”. In some cases, the number of centimeters (or any units of heights) is enough to tell whether the person is tall or not. 230 cm is tall. We can say so without any fact other than the height in centimeters. However, in many cases, the number in centimeters does not determine whether the person is tall or not. Say, 173 cm. S/he may be tall for the first grader. But it is not sure when she is in the middle age. We need other facts to consider. A similar arguement goes for “is a heap” and the number of grains. In some apparent cases like zero, the number of grains is enough for us to tell a heap from a non-heap. But for many numbers, we may need information on other facts such as arrangement or form of sand grains.

baldness. Suppose you are given the exact number of someones hair, say, $5,302$. Do you have a clear idea whether s/he is bald or not? Maybe, in some exceptional apparent cases (e.g. where the person has 0 hair or 10^{80} hairs), the exact number of hairs by itself provides enough information to determine the baldness. But it is highly questionable to think that the number of hairs is always sufficient to know whether someone is bald.

3.2 Obligatoriness?

Canceling the obligatory condition means that things independent of the number of hairs would make the baldness claim true. Someone is bald due to things which have little to do with the number of hairs.

This is possible and plausible in many cases. First of all, we seem to make a clear and unquestionable evaluation on baldness predicate without knowing the number of hairs. My father is certainly not bald. But I do not know the exact or even approximate number of his hair. My grandfather was certainly bald. But I do not know the exact or even approximate number of his hair. You have a clear evaluation of whether someone is bald or not. But when you have such a clear vision, do you know the exact value of the number of hairs? The number of hairs may play an important role in many cases. Not only ordinary people but professional philosophers have assumed, mistakenly, the number of hair is a *necessary* component of what makes baldness predicates true or false. Facts which make the baldness true include allocation, length, compositions, and so on. And sometimes, such facts, which are independent of the number of hairs, make a person bald or non-bald. I am *not* claiming here that the number of hair *never* matters when we evaluate someone is bald. Rather, I claim that it is compatible that the fact about the number of hairs f^n makes $B(n)$ true and another fact g independent of f^n makes $B(n)$ true at the same time.

4 Higher-order vagueness

Now, the sorites has been solved in the truthmaking terms: the sorites paradox arises only when we assume the two unreasonable assumptions over truthmakers. My remedy to the paradox is simple. To overcome the paradox, just cancel one of these unjustified assumptions. However, any solution has its competitors. In fact, Sorensen also solves the paradox in his more simpler way. So we need to show what points my solution is better than others (at least Sorensen's).

To this end, this section argues that importing different types of truthmakers and truthmaking gives enough space for *higher-order vagueness*, which Sorensen's system fails to capture. Instead of the number of hairs, I employ the *number of truthmakers (facts) which ground.* Some sentences just need a single truthmaker to determine its truth value. Others may need two. Still others need more. The main idea here is that the number of truthmakers corresponds to how higher-order its vagueness is. The more facts needed to support, the more vague it is. Here are some quick examples for this idea. "is unemployed" seems less vague than "is poor". Why? According to my truthmaker oriented explanation, it is because while the former needs only single type of facts (just check whether she is unemployed or not) the latter needs more types of facts such as posessions of properties, the amount of spending, prices of commodities, and so on. "is good" is another instance. Being good is, without question, not only ambiguous but vague. My view tells why so – because it needs to consider many different types of facts about the person to determine whether she is good.

This line of thought offers us some space for higher-order vagueness in the sorites scenario — higher-order borderline cases. Even if we consider only the single type of facts (in the sorites scenario, the number of hairs), as we have already seen, we face many instances of the first-order vague i.e. first-order borderline cases like 72 hairs or 5789 hairs. Taking another type of facts (like allocation of hairs) into consideration dissolves some borderline cases into non-borderline cases. Others still remain borderline cases. These remaining ones are now called the higher(second)-order borderline cases. Taking consideration to another type of facts, we eliminate second-order borderline cases and face third-order borderline cases.

Borderlines at first glance – first-order borderlines. As we have already observed, in some instances of the sorites series, the number of hair f^n is insufficient to determine $B(n)$ or $\neg B(n)$ for some n. When we consider only these factors about the numbers of hairs, we will face many indeterminate cases. For example, consider a person with j hairs such that $f^j \nvdash B(j)$ and $f^j \nvdash \neg B(j)$. Considering another fact with another type g which is about other than the number of hairs (such as the area covered by hair?) may desolve the indeterminacy. For some k, it may be indeterminate whether $B(k)$ or $\neg B(k)$ by considering only facts f^k about the number of hair. But considering another type of facts, say g such as the allocation of hair, may fix the truth value. $f^k, g^k \vdash B(k)$. Then, this case k is vague cases in a *higher-order* than n.

Further borderlines. We expand this idea above – a *single* truthmaker is not suffice to make true but two truthmakers, if rightly selected, are suffcie – for defining the order of borderline cases. Let us see the first-order cases. When you cannot determine $B(n)$ or $\neg B(n)$ by considering *any* single fact, $B(n)$ is said to be a first-order vague instances (with respect to baldness predicate B). More formally written, n is a first-order borderline case with respect to B if $f \nvdash B(n)$ and $f \nvdash \neg B(n)$ for any fact f.

The idea of higher-order vagueness is naturally given by putting more facts (truthmakers) under consideration. For the second-order borderline cases, define as follows: n is a second-order borderline case if $\Gamma \nvdash B(n)$ and $\Gamma \nvdash \neg B(n)$ for any Γ such that $|\Gamma| = 2$.

More generally, the m-th order borderline case is defined case as follows: n is a m-th order borderline case if $\Gamma \nvdash B(n)$ and $\Gamma \nvdash \neg B(n)$ for any Γ such that $|\Gamma| = m$.

We can capture Sorensen's version of truthmaker gap epistemicism as a tiny subset of my grand theory. My truthmaking formulation can capture what

Sorensen mean by *absolute borderline cases*. Absolute vagueness is written as the limit of this scheme of adding further facts to determine $B(n)$ or $\neg B(n)$. Formally written, n is an *absolute* borderline case (i.e. $B(n)$ is absolutely vague) if $\Gamma \nvdash B(n)$ and $\Gamma \nvdash \neg B(n)$ for *any* Γ (hence any $|\Gamma|$). In other words, an absolute borderline case is where no collection of truthmakers makes true $B(n)$ nor $\neg B(n)$. My formulation and this paper leaves it open the existence of such a limit point. But my system is capable of describing Sorensen's opinion comparing it with others.

Higher-order apparent case! My framework can do even more than the higher-order of vague cases. We can account for higher-order *apparent* cases, which enable us to depict the more fine-grained graduation between apparent cases (0 hair and 2,000,000 hairs). When you think of borderline cases, you may also think of apparent cases. Suppose $B(567)$ is true, but not for completely sure. $B(15)$ is true more certainly than $B(567)$. But $B(2)$ is true further more seemingly.

To give grades of being apparent, we can use the formal idea of truthmakers. Our formal setup allows us to take two (or more) different collections of truthmakers to make the same statement true. How apparent the claim is corresponds to the number of the collections of truthmakers which make the claim true. Put more formally, n is a first-order apparent case for B if there is only one Γ such that $\Gamma \vdash B(n)$. Naturally, higher-order apparent case is defined: n is a mth-order apparent case for B if there are exact m $\Gamma_1, ..., \Gamma_m$ such that $\Gamma_1 \vdash B(n) ... \Gamma_m \vdash B(n)$.

For example, let $B(2034)$ have 5 different collections of truthmakers $\Gamma_1, ...\Gamma_5$. Let, also $B(2)$ have more truthmakers, say, 23 of them – $\Delta_1, ..., \Delta_{23}$. Given that, we can describe how apparent $B(2034)$ and $B(2)$ and we can say the latter is more apparent than the former in terms of the number of collections of truthmakers.

Together with the higher-order borderline cases, we can describe the sorites paradox in the following reasonable way. The paradox begins with assuming that the fact of having no hair at all f^0 is enough for determining a person is bald $B(0)$. In this sense, this case with zero hair is not vague at all (given you consider the number of hairs). But also, many other facts such as allocation, how it looks and so on, make them true as well. $B(0)$ is a many-order apparent case; So no one is against this assumption of the paradox. The following case $B(1)$ may be similar to $B(0)$ with respect to not only its truth but also its apparency. But adding further number, it becomes less and less apparent – less collections of truthmakers are available.

A key observation is that being apparent and being vague are independent from each other. Apparent cases and borderline cases can overlap. For example, consider $B(k)$ such that $f, g \vdash B(k)$ and $h, i \vdash B(k)$ and no other Γ make it true. $B(k)$ is an apparent case because $B(k)$ has two different collections of truthmakers. But also $B(k)$ is (the first-order) vague in a sense that it requires more than one facts to support. This feature fits well the epistemicists' project because this overlaps explain why we often overlook thresholds in any order. We fail to find thresholds because such cases can be not only vague but also apparent.

5 Conclusion

Sorensen's importing truthmakers in the long-disputed topic of vagueness and the sorites paradox is a small but great step still in advancing our understanding of the nature of truth. However, unfortunately, his move sacrifices higher-order vagueness, which epistemicists have been very good at dealing with. I offer a more fine-grained interpretation of the sorites scenario using several common notions of truthmakers. I point out that the fact of the number of hairs does not necessarily play the sole and decisive role in truthmaking vague predicates. The fact about the number of hairs may need further extra facts to make the baldness claim true. Moreover, facts independent of the number of hairs may make the baldness claim true. The sorites paradox appears paradoxical because we often take for granted two conditions over truthmakers, which are questionable in many scenarios.

As a bonus, this richer framework of mine covers the higher-order vagueness. The last section demonstrates how my truthmaker formalization can talk about higher-order vagueness in an unified manner. This paper should not be perceived as a negative critique of Sorensen's work. His approach – and his goal to capture absolute vagueness – was also explained in a reasonable way under my scheme.

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