Philosophy of Mathematics Workshop: From Leibniz to Modern Age
Nov. 20. 2017, Tokyo

Leibniz and H. Weyl on the Concept of Continuum

Shinji IKEDA
University of Toyama

Introduction

- It is very natural to compare Leibniz and H. Weyl, since Weyl's works ---especially his works concerning the concept of continuum--- are deeply inspired by Leibniz's Philosophy.
- In the preface to the Part I of his *Philosophy of Mathematics and Natural Science,* he writes: "Among the heroes of philosophy it was Leibniz above all who possessed a keen eye for the essential in mathematics, and mathematics constitutes an organic and significant component of his philosophical system." (Weyl 1949, p. 2)
- Although it is clear from above that Weyl is mathematically inspired by Leibniz, his philosophical influence is not so clear or to be doubted.

Introduction

- My object is to compare Leibniz and Weyl, focusing their doctrines on the concept of continuum, especially in a philosophical aspect.
- Questions:
  - Are they different or similar?

    If different, what is fundamentally changed?

    If similar, what is radically updated from Leibniz to Weyl?
  - What is the nature of the continuum?

Tables

I. Leibniz's concept of Continuum
II. Weyl's concept of Continuum
    1. Das Kontinuum
    2. Philosophy of Mathematics and Natural Science
III. Weyl's evaluation of Leibniz

- (Bold in the quotations is my emphasis)

I. Leibniz's concept of Continuum

(a brief summary and key notions)

The Labyrinth of the Continuum

- Traditionally, there are two important contributions to the problem of the continuum (Weyl 1949, 38):
  a. an analysis of the mathematical question of how to fix a single position in the continuum.
  b. the discovery of the philosophical paradoxes which have their origin in the intuitively manifest nature of the continuum (ex. Zeno's Paradox, Kant's antinomies, etc.).

The Labyrinth of the Continuum

- In the context of the problem of the continuum, Weyl refers to Leibniz:

  'Leibniz, among others, testifies that it was the search for a way out of the "labyrinth of the continuum" which first suggested to him the conception of space and time as orders of the phenomena. "From the fact that a mathematical solid cannot be resolved into primal elements it follows immediately that it is nothing real but merely an ideal construct designating only a possibility of parts" (correspondence Leibniz-De VoIder, Leibniz, *Philosophische Schriften,* II, p. 268).' (Weyl 1949, 41)

The Labyrinth of the Continuum

- The labyrinth of the continuum is the problem: “how can continuous things be composed from indivsibles? ; or inversely, “how can a line be divided into mere points?”.

The Labyrinth of the Continuum

- Leibniz's (negative) solution to the problem of the continuum in *Pacidius philalethi* [1676] :

  "We have concluded that the continuum can neither be dissolved into points nor composed of them, and that there is no fixed and determinate number (either finite nor infinite) of points assignable in it." (A VI, 3, 555; LC, 187)

The Nature of the Continuum

- From this, Leibniz concludes that the nature of the continuum is not real or substantial but only **apparent** or **phenomenal**.
- The Continuum also entails **indifference** or **indeterminacy** with respect to its (infinite) division.
- The continuum is said to be ***ideal,*** by which Leibniz wants to mean that its reality resides solely in being thought.

A Tension between Geometry and Metaphysics

- For Leibniz, the constitution of lines from points is different from the constitution of matters from substances:

  "Line is not an aggregate of points, but body is an aggregate of substances" (A VI, 4, 1674).

- Thus, there is a tension between A) the geometrical problem of continuum and B) the metaphysical problem of continuum.
- Geometry and Metaphysics are **not** analogical. At least, it is not a simple 1 to 1 isomorphic relation.

The Labyrinth of the Continuum

- For Leibniz, the labyrinth of the continuum is not a mere mathematical problem, nor a mere physical problem.
- It is also an epistemological problem concerning mind-body problem: “How can we reconcile our perceptual world with the conceptual world?”
- It is also a metaphysical problem: “How can non-extended monads constitute the extended bodies ?”

Labyrinth of the Continuum

- Referring to Leibniz's idea of the *monad*s as the absolute substances which gives a metaphysical foundation to the world of phenomena, Weyl continues the Leibnizian argument on the continuum:

  "Within the ideal or the continuum the whole precedes the parts . . . The parts are here only potential; among the real [i.e. substantial] things, however, the simple precedes the aggregates, and the parts are given actually and prior to the whole . . ." (letter to Remond, *Philosophische Schriften,* III, p. 622). (Weyl 1949, 41)

Labyrinth of the Continuum

- Breger[1986]: Leibniz's view of the continuum as a whole, which had to be stipulated in the intuition, is close to Weyl's semi-intuitionist understanding of the continuum.
- Scholz[1995]: In his shift from the semi-constructivist position to Brouwer's intuitionism (in early 1920's), Weyl wanted to link the mathematical concept of continuum to a "directly experienced continuity".

Part-Whole Relation

Weyl sees the essence of the continuum in the priority of the whole to its parts.

a. A general concept, the whole (continuum), has to be presupposed in order to give meaning to an individual determination, the particular, or the part (point).
b. The whole, the general concept (continuum), is constituted in a process of common generation by the particles. (Scholz, 1995)

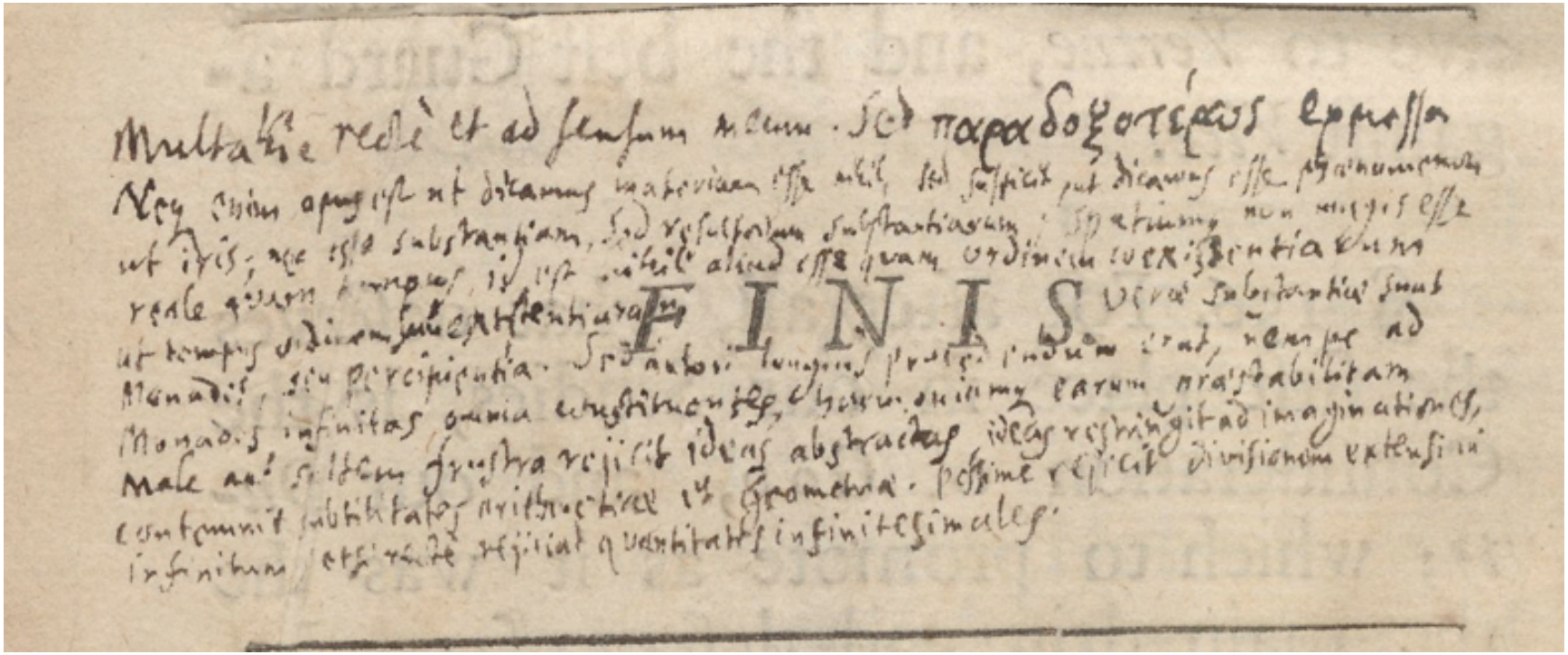
For Weyl, to avoid impredicativity which causes a paradox, the whole should be constituted from the parts without presupposing the whole itself.

The Principle of Continuity

Weyl sees that Leibniz's principle of continuity is sound from a contemporary point of view.

> "It rests upon the impossibility of proper division of a uniform continuum … Rest is not contradictory to motion, but a limiting or special case of motion. Leibniz says that by virtue of that principle "the law of bodies at rest is, so to speak, only a special case of the general rule for bodies in motion, the law of equality a special case of inequality, the law for the rectilinear a subspecies of the law for the curvilinear," and he calls manifolds "homogenous if one can be transformed into the other by a continuous change" (Initia rerum Mathematicarum metaphysica, *Mathematische Schriften,* VII, pp. 25, 20)." (Weyl 1949, 161)

Lebniz Marginalen 52



Leibniz's marginal remarks to
Berkeley's *A treatise concerning the Principles of Human Knowledge* (1710)

Lebniz Marginalen 52

"There is much here that is correct and close to my own view. But it is expressed paradoxically. **For it is not necessary to say that matter is nothing, but it is sufficient to say that it is a phenomenon, like the rainbow; and that it is not a substance, but the result of substances, and that space is no more real than time, that is, that space is nothing but the order of coexistents, just as time is the order of things that have existed before. True substances are monads, that is perceivers.** [continues]

Lebniz Marginalen 52

[continued] **But the author should have gone furthur, to the infinity of monads, constituting everything, and to their pre-established harmony.** Badly or at least in vain, he rejects abstract ideas, restrict ideas to imaginations, and condemns the subtleties of arithemetic and geometry. The worst thing is that he rejects the division of extension to infinity, even if he might rightly reject infinitesimal quantities."(Leibniz 1989, 307)

- “I believe the nature of extension[space] can be explained. … Extension is an abstraction from the extended, and the extended is a continuum whose parts are coexistent, i.e. exist at the same time.” (NE II, 13, 15)
- “Thus viewed, space is no more a substance than time is, and if it has parts it cannot be God. It is a relationship: an order, not only among existents, but also among possibles as though they existed. But its truth and reality are grounded in God, like all eternal truths.” (NE II, 13, 17)
- “Then the best way of putting it is that space is an order but that God is its source.” (NE II, 13, 17)

reciprocal indispensability

ratio essendi (onto-logical order)

- Space(Condition of possibility)→ extended things

  "the concrete one is as it is only by virtue of the abstract one (le concrete n'étant tel que par l'abstrait)."

- Bodies cannot move to other places, before there are possibilities to change their places.

ratio cognoscendi (epistemic-idealistic order)

- External perception → abstract extension

  "indeed knowledge of concrete things is always prior to that of abstract ones – hot things are better known than heat."(NE II, 13, 6)

Two orders

- 1) the order of possibles precedes the order of actuals, and concrete extension realizes space.

- 2) the extended concrete does not realize the abstract, but the abstract extension is drawn out of it.

- 1 → space as a purely logical condition
- 2 → extension as abstracted from extended body

Pre-established harmony

- Abstract things depends its reality on concrete things, while concrete things demand its possibility to abstract things.
- This reciprocal indispensability between concrete and abstract is the system called `**pre-established harmony**', which is intended to establish the representative relation between our mind and the world by **the theory of expression**.

Leibniz notion of Space

- *Leibniz – Clarke Correspondence*
- What Space **is**:
  - something ideal
  - something relational
  - a possible
  - an order of coexistents
- What Space **is not**:
  - an attribute
  - an absolute
  - a substance

Leibniz notion of Space

- How the concept of space is formed:
  - From a constitution of situations which are abstracted from things.

- Ontological base of Space:
  - Possibility → human understanding and God's intellect.
  - Actuality → body or concrete things and God's creation.

II. Weyl's concept of continuum

1. Das Kontinuum

Das Kontinuum

- 1918-1921: wrestled with the problem of constructing the mathematical continuum (the real number line)
- set-theoretical approach involves vicious circle
- Rejected Impredicative definition which implies implicit cyclicity.
- Restricted analysis to the predicative definition

Impredicativity

- A definition is said to be *impredicative* if it generalizes over a totality to which the entity being defined belongs. Otherwise the definition is said to be *predicative*.

  Ex. 1.) The least upper bound property of Reals (Dirichlet's principle or the definition of Dedekind cut).

  Ex. 2.) Richard's Paradox.

  Ex. 3.） Russell's Paradox.

- Weyl's Motivation is to construct a consistent theory of analysis which does not include any impredicativity.
- How to avoid impredicative definitions?

→Restricting concepts and objects of analysis to the elementary definables (*i.e.* restricting the use of "for all" and "exists" only to the natural numbers.)

Provable & Improvable Theorems

In Weyl's restricted procedure,

- the monotone convergence theorem
- the nested interval theorem

are provable, but

- Dedekind's cut principle
- least upper bound property/greatest lower bound property
- Bolzano-Weierstrass Theorem (Every bounded infinite set of real numbers has an accumulation point)

are improvable. (Weyl 1994, 76f.)

- From a contemporary viewpoint, Weyl's System can be regarded as a system based on $ACA_{0.}$
- In $ACA_{0,}$ most of the theorems of analysis --- such as the least upper bound property and the Bolzano-Weierstrass Theorem--- are provable.
- So, Weyl's method is vindicated. (S. Feferman)

- Although *Das Kontinuum* is intended to provide a consistent logical foundation to the concept of mathematical continuum, it contains a philosophical analysis of the continuum.
- →Notably, in Ch. 2, §6: “The Intuitive and the Mathematical Continuum”

Intuitive Continuum & Mathematical Continuum

"To the criticism that the intuition of the continuum in no way contains those logical principles on which we must rely for the exact definition of the concept "real number", we respond that the conceptual world of mathematics is so foreign to what the intuitive continuum presents to us that **the demand for coincidence between the two must be dismissed as absurd**." (Ibid., 108)

Intuitive Continuum & Mathematical Continuum

- **There is an unbridgeable gap between the intuitive continuum** given by our ordinary sense-perception **and the mathematical continuum** constructed from the discrete real numbers.

Intuitive Continuum & Mathematical Continuum

"Certainly, **the intuitive and the mathematical continuum do not coincide; a deep chasm is fixed between them**. But there are rational motives which push science from the experientially constituted reality in which we live as natural human beings over toward the "truly objective", exact, non-qualitative, physical world-from the chromatic qualities of visual things, e.g., to the oscillations of the ether or the corresponding mathematical descriptions of electro-magnetic fields. So one might say that our construction of analysis contains a theory of the continuum which must establish its own reasonable physical theory."(Ibid., 93)

Intuitive Continuum & Mathematical Continuum

- "When our experience has turned into a real process in a real world and our phenomenal time has spread itself out over this world and assumed a cosmic dimension, **we are not satisfied with replacing the continuum by the exact concept of the real number, in spite of the essential and undeniable inexactness arising from what is given.**" (Ibid., 93)
- → It is in principle impossible to give an exact mathematical formulation to the continuum presented to our perception.

obvious nonsense

- For Weyl, it is an "**obvious nonsense**" to connect our intuition of time (or phenomenal time) with the world of mathematical concepts. Intuitive continuum cannot supply the foundation of mathematical discipline.
- Cf. Bergson's idea of "la durée" (or the immediately experienced continuity of phenomenal time).

Weyl relation to Intuitionism

- 1918- Das Kontinuum →semi-constructivism
- 1920- committed to Brouwer's Intuitionism
- 1923 *Raum-Zeit-Materie*
- 1925- disappointed to Intuitionism
- 1926 intensive studies of Leibniz
- 1927 "Philosophie der Mathematik und Naturwissenschaft" (in *Handbuch der Philosophie*) [English tr. in 1949]
  - → Science as "Symbolic construction".
  - → Influence of/similarity to Hilbert's Formalism and Cassirer's philosophy of Symbolic Forms[1923-29]

II. Weyl's concept of continuum

2. Philosophy of Mathematics and Natural Science

Intuition and Symbolic construction

- Weyl regards Mathematics of the infinite as *symbolic construction*:

  "the human mind for the first time senses its full power to fly, through the use of the symbol, beyond the boundaries of what is attainable by intuition." (Weyl 1949, 36)

Constructive Cognition

- Weyl presents "constructive cognition" as the basic feature of arithmetic. (Ibid., 37f.)
  1. Certain characters are not manifest in the phenomena but are arrived at as the result of certain mental operations;
  2. The assertions are relatively independent from the reality by the intruduction of symbols;
  3. "Characters are not individually exhibited as they actually occur, but their symbols are projected on the background of an ordered manifold of possibilities which can be generated by a fixed process and is open into infinity."

Constructivism or Idealism of the continuum

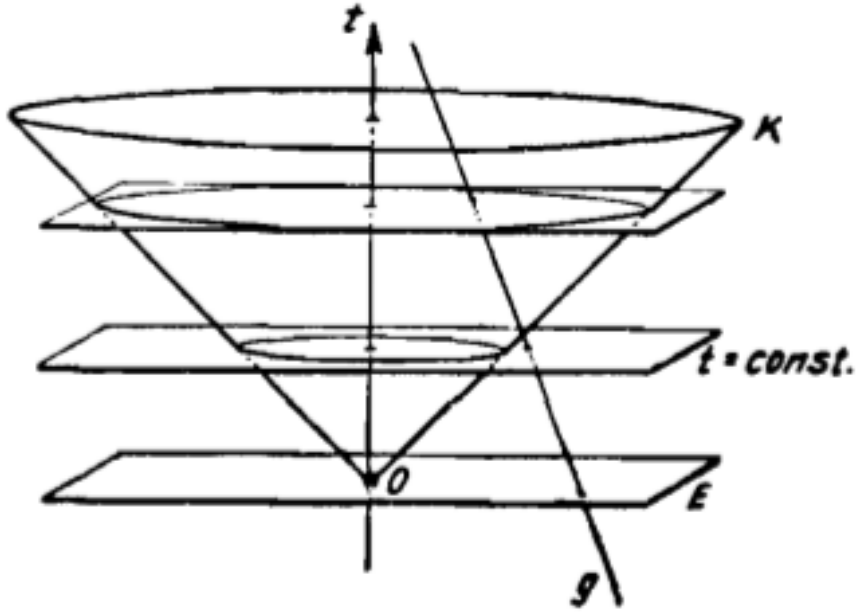
- Scholz sees Weyl did not withhold his constructivist sympathies in his discussion of number and continuum.
- "The determination of localizations in a continuum stood, for Weyl, in the tension between "the real" and "the ideal". And this could be understood as paradigmatic for gaining (ideal) knowledge of (real) things. He insisted that a "real thing" can never be given, but has to be "unfolded" by an infinitely continued process." (Scholz 2012)
- → In this idealistic context, Weyl quoted Leibniz's argument on continuum.

Physical World as a Four Dimensional Continuum

- For Weyl, the possible space-time locations (world-points) form a four dimensional continuum.
- Time and Space has a metrical structure: the Equality of Time intervals and the Congruency of Spatial configurations.

Physical World as a Four Dimensional Continuum

"we are able to draw a picture of the world in intuitive space; a picture in which the layers of simultaneous world-points all appear as horizontal planes while the fibers of equally located world-points are represented by vertical straight lines." (Weyl 1949, 95)



t: time
g: uniform translation
K: Light cone

Physical World as a Four Dimensional Continuum

This stratification determines the *causal connection of the world.*

Weyl sees that this stratified causal connection was already recognized by Leibniz, and quotes his "Initia rerum mathematicarum metaphysica".(Weyl 1949, 101)

"If of two elements which are not simultaneous one comprehends the cause of the other, then the former is considered as *preceding,* the latter as *succeeding."* (GM VII, 18)

K: Light cone
L: life line

Physical World as a Four Dimensional Continuum

- Scholz[2012]: “Weyl thus invoked Leibniz just as if he were a contemporaneous (or time-less) dialogue partner who could be asked for advice in questions pertaining to most recent modern physics.”
- Based on the general theory of relativity, Weyl concludes the final result of the historical development of the structural problem of space and time as follows: *“The world is a four-dimensional Riemannian space”*. (Weyl 1949, 108)

Weyl's notion of Space

- Weyl seems to distinguish four notions of space.
  - Mathematical Space (possible structure)
  - Physical Space (symbolic construction of the objective world)
  - Space of Intuition [*i.e.* perceptual space]
  - Objective world
- Located at the Base of this distinction is the epistemology of *subject and object*, i.e. the problem of idealism versus realism.

Weyl's notion of Space

- Leibniz: idealistic view of the continuum.

  "Concerning the bodies I am able to prove that not only light, color, heat, and the like, but motion, shape, and extension too are only apparent qualities"(GP VII, 322)

- Weyl: distinguishes the subjective space from the objective space.

  "Intuitive space and intuitive time are thus hardly the adequate medium in which physics is to construct the external world. No less than the sense qualities must the intuitions of space and time be relinquished as its building material; **they must be replaced by a four-dimensional continuum in the abstract arithmetical sense**."(Weyl 1949, 113)

Symbolic Construction of Space-Time

- For Weyl, space-time is nothing but a *Symbolic construction* (same kind as Hilbert's formalism).
- For example, colors "now appear merely as mathematical functions of periodic character depending on four variables that as coordinates represent the medium of space-time." (Weyl 1949, 113)
- The distillation (or abstraction) of this objective world from what is given to our intuition, is only possible by symbolic representation.

Reconciliation of idealism and realism

- "Within the natural sciences the conflicting philosophies of idealism and realism signify principles of method which do not contradict each other."
- "We construct through science an objective world which, in order to explain the sense data, must satisfy the following fundamental principle […] : *A difference in the perceptions offering themselves to us is always founded on a difference in the real conditions* (Helmholtz). […] Here the natural sciences proceed realistieally."

Reconciliation of idealism and realism

"On the other hand science concedes to idealism that its objective reality is not given but to be constructed (nicht gegeben, sondern aufgegeben), and that it cannot be constructed absolutely but only in relation to an arbitrarily assumed coordinate system and in mere symbols. Above all the central thought of idealism comes into its own in the converse of the above fundamental principle: *the objective image of the world may not admit of any diversities which cannot manifest themselves in some diversity of perceptions;* an existence which as a matter of principle is entirely inaccessible to perception is not admitted." (Weyl 1949, 117)

Reconciliation of idealism and realism

- We can summarize Weyl's reconciliation of idealism and realism in Science as follows:
  - Realism: Our perceptual world is based on the objective world.
  - Idealism: This objective world is representable only in symbols.

Algebraic model

"Real observer and real object, I, thou, and the external world arise, so to speak, in unison and correlation with one another by subjecting the sphere of 'algebraic appearances' to the viewpoint of invariance. On this issue our theory bears out Leibniz (compare, for instance, *Nouveaux Essais,* Libre IV, Chap. 11) as opposed to Descartes, who through his" cogito ergo sum" assigns to the reality of the ego a precedence in principle over the reality of the external world." (Weyl 1949, 124)

→ Leibniz's theory of expression

What is beyond human knowledge

"Postulation of the external world does not guarantee that such a world will rise from the phenomena through the cognitive work of reason which attempts to create concordance. For this to take place it is necessary that the world be governed throughout by simple elementary laws. Thus the mere positing of the external world does not really explain what it was meant to explain, the question of the reality of the world mingles inseparably with the question of the reason for its lawful mathematical harmony. The latter clearly points in another direction of transcendency than that of a transcendental world; towards the origin rather than the product. **Thus the ultimate answer lies beyond all knowledge, in God alone**; …." (Weyl 1949, 125)

What is beyond human knowledge

- "**The postulation of the ego**, of the 'thou,' **and of the external world is without influence upon the cognitive treatment of reality. It is a matter of metaphysics**, not a judgment but an act of acknowledgment or belief […] **Knowledge is incapable of harmonizing the luminous ego with the dark erring human being that is cast out into an individual fate**." (Weyl 1949, 125)

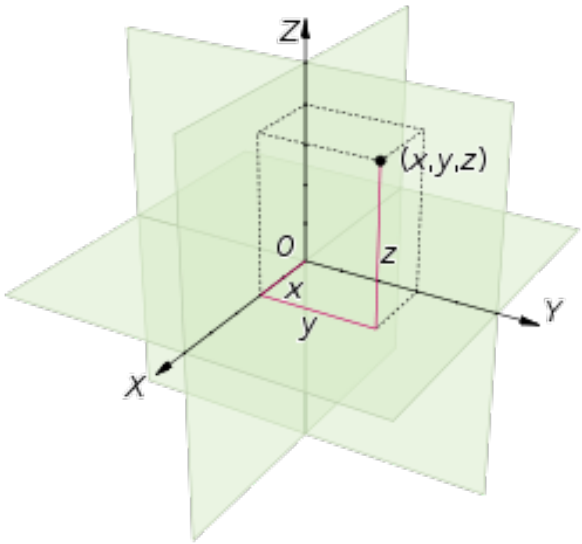
Weyl's Monadology

- There is no doubt that Weyl's view is inspired by Leibniz's Monadology.
- But Weyl's Monadology eliminates the hypothesis of the pre-established harmony.
- While Leibniz requires a metaphysical foundation to explain the relation between our mind and the world, Weyl remains in the realm of natural science.

Representation of Space

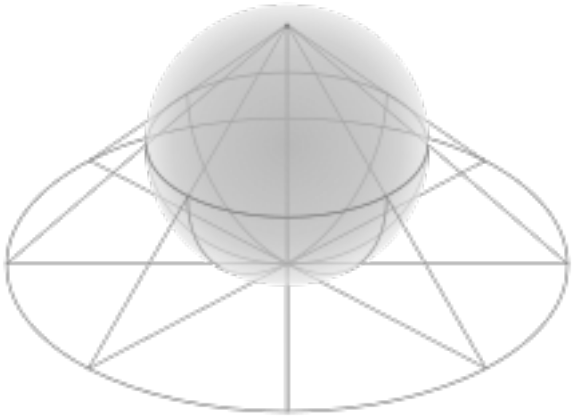
Intuitive Space ⟷ Objective Space

isomorphism



metrical structure

- Euclidean
- Riemannian
- …



Representation of Space

"If we confront the 'objective' space on one side and my intuitive space on the other, and if we assume both to bear a Euclidean metrical structure, then the utmost in faithfulness that could be demanded of the correspondence between objective thing and its image given in my intuition is an **isomorphic** (or similar) mapping […]."(Weyl 1949, 126)

objective-symbolic / subjective-intuitive

Isomorphism

Weyl also sees that Leibniz has already the idea of isomorphism:

> "Thus even if we do not *know* the things in themselves, still we have just as much *cognition* about them as we do about the phenomena. **The same idea of isomorphism clarifies the problem which Leibniz**, stimulated by Hobbes' nominalistic theory of truth, **treats in his dialogue on the connection between things and words**; Leibniz evidently wrestles with giving expression to that idea *(Philosophische Schriften,* VII, pp. 190-193)" (Weyl 1949, 25).

The Essence of Space

- "A thing exists only in the indissoluble unity of intuition and sensation, through the superimposition of continuous extension and continuous quality. Phenomenologically it is impossible to go beyond this."(Weyl 1949, 131)
- Leibniz: infers the ideality of space and time from the concepts of similarity and congruence, for they violate the principle of the identity of indiscernibles (PII).

The Essence of Space

Weyl's notion of four-dimensional continuum is close to Leibniz's notion of abstract space as an order of possible positions.

> "The dual nature of reality accounts for the fact that we cannot design a theoretical image of being except upon the background of the *possible*. Thus the four-dimensional continuum of space and time is the field of the *a priori* existing possibilities of coincidences. That is why Leibniz calls the "abstract space the order of all positions assumed to be possible" and adds that "consequently it is something ideal" (Leibniz's fifth letter to Clarke, §104)." (Weyl 1949, 131)

III. Weyl's Evaluation of Leibniz

- Scholz[2012]: “Weyl did not intend a historical reconstruction of Leibnizean thought, but rather read him in a presentist perspective (group theory as part of ars combinatoria etc.).”

Weyl's evaluation of Leibniz

- "Even if the opinion can thus be justified that the world is far more accurate than it appears to the senses, or even that it is absolutely accurate, nevertheless this absolutely accurate state could only be ascertained by me as the observer if I waited for the resulting developments till the end of time […]. Complete accuracy is therefore a limiting idea and by no means immediately given. **Leibniz's thought of preestablished harmony** ---which he himself illustrates by the example of two entirely independent clocks that are synchronous, not because they regulating influence upon each other but because they are identically constructed--- **contradicts**, therefore, **the nature of the continuum**." (Weyl 1949, p.142f.)

Weyl's evaluation of Leibniz

- Weyl's Argument:

1. Leibniz's system of PH, implies complete accuracy.
2. The nature of the continuum is its observational indeterminacy.
3. Thus, PH contradicts the nature of the continuum.

Nature of Continuum for Weyl

- It is not the dynamics but the field theory of matter that reflects the essence of the continuum:

  "**This conception of the world can hardly be described as dynamical any more, since the field is neither generated by nor acting upon an agent separate from the field, but following its own laws is in a quiet continuous flow. It is of the essence of the continuum**. Even the atomic nuclei and the electrons are not ultimate unchangeable elements that are pushed back and forth by natural forces acting upon them, but they are themselves spread out continuously and are subject to fine fluent changes." (Weyl 1949, p. 171)

Weyl's evaluation of Leibniz

"The classical philosopher of the dynamical conception of the world, however, is Leibniz. To him, what is real in motion does not lie in the change of position as such, but in the moving force. *"La substance est un etre capable d'action, une force primitive"* --- transspatial and immaterial. "For not all truths relating to the world of bodies can be derived from merely arithmetical and geometrical axioms, that is, from axioms of larger and smaller, of shape and position," he says in criticism of Descartes *(Mathematische Schriften,* VI, p. 241) "but others must be added concerning cause and effect, activity and passivity, in order to give an account of the order of things."

Monadology reinterpreted by Weyl

"The ultimate element is the *monad*, an indecomposable unit without extension, from which the force bursts forth as a transcendental power. Only with regard to the distribution of the monads in space, which itself is merely a *phaenomenon bene fundatum*, is the body described as an *extended* agent. Pure activity, however, is all; preestablished harmony takes the place of such reciprocal effects as we think are carried by the field from particle to particle." (Weyl 1949, p. 174)

Monadology reinterpreted by Weyl

- Weyl attempted to unify Maxwell's Electromagnetism and Einstein's General Relativity. → Gauge Theory
- Weyl does not regard Leibniz's *Monad* as "a kind of fantastic fairy tale (Russell)", but rather accepted in a sense.
- Weyl reinterprets Leibniz's Monadology in the context of modern field theory: "Indeed general relativity does not prescribe the topology of the world, and it may therefore happen that the world has unattainable 'fringes' not only toward the infinite but also inwardly. **In line with Leibniz's ideas, the material particle, although imbedded in a spatial environment from which its field effects take their start, would itself then be a *monad* existing beyond space and time.**" (Weyl 1949, p. 175.)

On Complexity and Regularity of the World

- In the context of the Causal Law as a mathematical function, Weyl says: "The assertion of regularity becomes meaningless if complications of arbitrary degree are admitted. This was emphasized already by Leibniz in his "Metaphysische Abhandlung" (*Philosophische Schriften*, IV, p. 431). **What is decisive and at the same time astounding is the fact that the laws show such a simple mathematical structure, while the quantitative distribution of the state quantities in the world continuum is incredibly complicated.** This has the consequence, for our knowledge, that limited experience enables us to ascertain those laws while the unique quantitative course of events remains largely unknown. This distinction, for the naive realist only the vague one between simple and complicated, becomes one of principle, when the intuitionist or constructivist view is adopted in mathematics and physics."

Leibniz[1686], *Discours de Métaphysique*, § 6:

"**Ce qui est si vrai, que non seulement rien n'arrive dans le monde, qui soit absolument irrégulier, mais on ne sçaurait mêmes rien feindre de tel**. Car supposons par example que quelqu'un fasse quantité de points sur le papier à tout hazard, …, je dis qu'il est possible de trouver une ligne géométrique dont la notion soit constante et uniforme suivant une certain règle, en sorte que cette ligne passe par tous ces points, et dans le même ordre que la main les avait marqués. … **Mais quand une règle est fort composée, ce qui lui est conforme, passe pour irrégulier.** Ainsi on peut dire que de quelque manière que Dieu aurait créer le Monde, il aurait tousjours été régulier et dans un certain ordre général. …" (GP IV, 431)

Difference and Similarity

- Weyl remains in the realm of mathematics and physics (and that of phenomenology).
- Leibniz requires God and Metaphysics
- Although their bases are different, they agree in the point that the complicated world can be represented by a simple mathematical structure.

Indeterminacy of the Continuum

- "**It will now be understandable that most of the physical concepts, especially those concerning matter with its atomic structure** (e.g. the density of a gas), **are not exact but statistical, that is, they represent mean values affected with a certain degree of indeterminacy. Similarly most of the usual physical 'laws,' especially those concerning matter, must not be construed as strictly valid laws of nature but as statistical regularities**." (Weyl 1949, p. 199)

Indeterminacy of the Continuum

- Leibniz, as a metaphysician or an idealist, bases the indeterminacy of the continuum on the structure of our perception or consciousness: "the consideration of myself which also furnishes to me other metaphysical concepts such as cause, effect, . . . " *(Philosophische Schriften,* VI, p. 502).
- Weyl, as a scientist, explains the indeterminacy of the continuum by statistical physics:"**The justification of statistical physics evidently derives from the fact that the hidden complicated molecular processes bear no direct relation to our perceptions**. The latter depend on certain mean values, and statistics teaches us how to determine these. **Our consciousness does not reflect the molecular chaos of the phenomena but exerts an integrating function with respect to both space and time, from which results the apparent homogeneity and continuity of the phenomena**." (Weyl 1949, p. 200)

Weyl's rejection of Leibniz's metaphysics

- In the context of mind-body problem, Weyl rejects Leibniz's mechanical conception of causality and his theory of PH.
- "It is an altogether too mechanical conception of causality which views the mutual effects of body and soul as being so paradoxical that one would rather resort,.., like Leibniz, to a harmony instituted at the beginning of time." (Weyl 1949, p. 215)

Double aspect of the Ego

- **"The real riddle**, if I am not mistaken, **lies in the double position of the ego**: **it is not merely an existing individual** which carries out real psychic acts **but also 'vision,' a self-penetrating light** (sense-giving consciousness, knowledge, image, or however you may call it); as an individual capable of positing reality, its vision open to reason […]. **But this secret, by its very nature, lies beyond the cognitive means of natural science.**"(Weyl 1949, p. 215f.)

Objective World and the Continuum

- While Leibniz postulates a hypothetical system which gives a conformity between our perception of the continuum and the aggregate of Monads, Weyl gives a negative answer to reconcile perfectly our intuition of the continuum with the objective world.
- "The objective world simply is, it does not *happen*. Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time." (Weyl 1949, p. 116)

Leibniz's influence

Scholz[2012] sees Leibniz's influence on Weyls view in the following topics:

- mathesis universalis realized in modern axiomatic mathematics,
- characteristica generalis and ars combinatoria realized in Hilbert's
- foundational approach to mathematics,
- Ausdehnungslehre and vector calculus considered as a variant of analysis situs,
- the discussion of the relativity of space,
- causality and time order in modern physics,
- and the dynamical character of inertia, including Weyl's own agens theory of matter, supported and upgraded by Leibnizian fragments.

Leibniz's influence

With Breger[1986], we may add to this list

- The concept of the continuum and
- Weyl's "purely infinitesimal geometry"

Leibniz was, for Weyl, an advisor, a supporter and a contemporaneous dialogue partner (Scholz 2012).

Conclusion

- Leibniz recognized the phenomenal or ideal character of space and time as consisting in the mere ordering of phenomena; however, space and time themselves do not have an independent reality.
- Weyl regards Space and Time as *Symbolic construction.* That is, Space and Time is a four-dimensional continuum, provided by the theory of relativity and the quantum theory.

Conclusion

- Weyl‘s idea of Symbolic Construction is very close to the idea of Leibniz’s theory of expression (cf. *Dialogus,* 1677; *Quid sit idea?,* 1677). Since Leibniz’s theory of ideas defines the truth as a symbolic order between relations.
- For Weyl, Leibniz is seem to be used to relativize Hilbert’s Formalism and Brouwer’s (or Weyl’s own) Intuitionism; or more widely, to reconcile Idealism with Realism (of Space and Time).

Conclusion

- Though several metaphysical thesis of Leibniz were not so much convincing to Weyl, he respected the mathematical structure of Monadology, and updated it to a mathematized-symbolic-constructive structure of the world.
- Or we can say that Weyl also updated his concept of continuum by Leibniz's philosophy. It is very Leibnizian that Weyl tried to synthesize Idealism and Realism by a symbolic method of science which explores the "relationship between the part and the whole" for the case of continuum.

References

- [LC]: *The Labyrinth of the Continuum* (The Yale Leibniz Series), tr., ed. and intro. By R.T.W. Arthur, Yale University Press, New Haven and London, 2001.
- [LDeV]: *The Leibniz-De Volder Correspondence: With Selections from the Correspondence Between Leibniz and Johann Bernoulli* (The Yale Leibniz Series), tr., ed. and intro. by Paul Lodge, Yale University Press, New Haven and London, 2013.
- [NE] *Nouveau essais sur l'entendement Humain*, éd. par Jacques Brunschwig, Paris : GF-Flammarion, 1990; *New Essays on Human Understanding*, trans. by Peter Remnant and Jonathan Bennett, Cambridge University Press, 1996.
- *Correspondance Leibniz-Clarke*, 2e édition, éd. par André Robinet, Presses Universitaires de France, 1991; *The Leibniz-Clarke correspondence*, ed. by H. G. Alexander, Manchester University Press, 1956.

References

- Hermann Weyl [1994], *The Continuum: A Critical Examination of the Foundation of Analysis*, tr. by Stephen Pollard and Thomas Bole, The Thomas Jeffersson University Press, 1987, reprinted by Dover in 1994. (Originally pulishded in German in 1918 under the title *Das Kontinuum: Kritische Untersuchungen über die Grundlagen der Analysis*; Japanese tr. by Hisao Tanaka and Sakaé Fuchino, 2016.)
- Hermann Weyl [1949], *Philosophy of Mathematics and Natural Science*, Princeton University Press. (Originally pulishded in German in 1927 in *Handbuch der Philsophie* under the title “Philosophie der Mathematik und Naturwissenschaft”.)

References

- Herbert Breger [1986], “Leibniz, Weyl und das Kontinuum.” *Studia Leibnitiana Suppl*. 26: 316–33.
- Erhard Scholz [1995],“Hermann Weyl‘s “Purely Infinitesimal Geometry”, Proceedings of the Internation Congress of Mathematics, Basel: Birkhäuser, 1995, pp. 1592-1603.
- Erhard Scholz [2000],“Hermann Weyl on the concept of continuum”, In Hendricks et .al. [2000], *Proof Theory: History and Philosophical Significance*. Dordrecht: Kluwer, pp. 195–220.
- Erhard Scholz [2012], “Leibnizian Traces in H. Weyl’s Philosophie der Mathematik und Naturwissenschaft“ in Ralf Krömer, Yannick Chin-Drian (eds.) *New Essays on Leibniz Reception: In Science and Philosophy of Science 1800-2000*, Springer, 2012, pp. 203-216.