What Is Essential to Constitute a Physical Realm? Hints from a Spatiotemporal Structure [*]

Sho Fujita[†]

Abstract

Space and time are fundamental factors in describing physical phenomena. Metrical features such as distance and angle are necessary and sufficient to describe physical spacetime. Contemporary philosophy of spacetime regards spacetime as a metric field. Structural realism applied to spacetime supports this interpretation, viewing a spatiotemporal structure as concrete in the physical world. In General Relativity, the spatiotemporal structure has mathematical substructures at different stages, with abstract structures like topology being defined mathematically without metric. Although topology is independent of locality and can *arise from* the metric, it may be essential for physical reality in quantum gravity theories. Abstract features are more important for physics as a spatiotemporal structure given by metric emerges from more fundamental entities in micro regions.

Key Words: Structural realism, identity, hole argument, FLRW metric, emergence

1 Introduction: Contemporary Physics Deals with Physical Realms Abstractly

When pondering the ontological question of the reality of space and time, the focus is usually on physical spacetime. Both physics and philosophy have primarily been studying this concept of spacetime, initially exploring it as the physical realm in which concrete entities like ourselves exist.

According to physics, spacetime is described as being curved by Riemannian non-Euclidean geometry in the context of the General Theory of Relativity (GTR). Contemporary philosophy of spacetime has also considered what can be considered as spacetime. In GTR, spacetime is described by a complex mathematical structure known as a manifold, which is a global topological space that includes relational local features such as connection and metric. Philosophy of spacetime must deal with abstract entities. In fact, the relationship between mathematical models and physical spacetime is a key issue for philosophy to address. This demonstrates how mathematical objects and structures exist in the physical world.

Recently, the concept of space has become broader and is influencing the ontology of spacetime. In mathematics, there are other types of abstract space,

---

[*] email:fujita.sho.p6@a.mail.nagoya-u.ac.jp

[†] Department of Complex Systems Science, Nagoya University Graduate School of Informatics, Furo-cho Chikusa-ward, Nagoya-City, 464-8601, Aichi, Japan

such as topological space and vector space, which are derived from sets where each element is constituted by specific relations. Additionally, physical worlds are now understood to be more extensive than in the past. In contemporary physics, theories such as GTR and string theory introduce high-dimensional realms beyond four-dimensional spacetime, and quantum theory uses phase spaces and configuration spaces to describe N-particle systems. These types of space play important roles in explaining various physical phenomena, including micro regions, but it is important to note that their roles are distinct from that of spacetime.

Simply put, Hilbert space and Fock space in quantum theory are not real as physical entities. This is because these spaces are not physical realms, but rather useful tools for explaining physical observables in quantum phenomena. They are used by physical theories, not by mathematical theories. Nevertheless, we cannot locate these spaces anywhere in the world.

But the situation is more complex. Paul argues that these tool-like spaces are more fundamental than spacetime (Paul 2012). According to him, the 3-N dimensional configuration space occupied by N-particle systems is the true reality, and spacetime is not a fundamental constituent of this physical world. In some quantum gravity theories, where micro regions are dominated by quantum effects, spacetime can be seen as emergent or derived from more fundamental entities (Wüthrich 2012, 2018; Huggett & Wüthrich 2013), such as causal sets or spin networks, which I will discuss in Section 3. This emergent nature of spacetime presents a new worldview that challenges the traditional metaphysics of spatiotemporalism.

Therefore, the above ontological question encompasses various interpretations, leading to numerous forms of realism and anti-realism about spacetime (Slowik 2015). However, I aim to clarify the mathematical concept that plays a crucial role in determining physical spacetime and to demonstrate what constitutes a physical realm. This clarification is directly linked to the traditional debate about realism in spacetime, namely substantivalism versus relationism, which has its roots in the historical discussions between Newton and Leibnitz about spacetime as a container for matter.

Furthermore, when it comes to spacetime in GTR, a structural interpretation (Esfeld & Lam 2008; Lam & Esfeld 2012) can be very useful in understanding how a mathematical structure corresponds to a physical one. This correspondence relation (Psillos 2010, 2011; Pincock 2007; Suppe 1989) and discussions about contemporary philosophy of spacetime (Earman & Norton 1987; Maudlin 1988; Teller 1991; Hoefer 1996; Dorato 2000) will be addressed in section 2. The main point of discussion in GTR is the interpretation of the gravitational field (2.2 and 2.3). In mainstream discussions, the gravitational field is considered to be spacetime as it is also a metric field in GTR and expresses local properties of spacetime as spatiotemporal features. This view will be explored further.

From the possible interpretation of macro spacetime, I aim to explore the aspect of spatiotemporal structure that is retained in more fundamental entities in micro regions where spacetime emerges. Some quantum gravity theories abandon ordinary properties of spacetime such as distance and angle, and quantizing gravity according to these theories may lead to the conclusion that spacetime does not exist in micro regions. This is because if spacetime is comprised of gravitational fields, then quantizing gravity would mean quantizing spacetime itself. However, even if spacetime transforms into different entities through quantization, I believe these new entities must still reflect some aspects of spacetime, as long as their structures are defined by quantizing something from the spatiotemporal structure.

There must be a physical realm, no matter how small, in micro regions. This realm may be very different from ordinary spacetime, but the abstract mathematical substructure of the entire spatiotemporal structure is what is essential for constituting a physical realm, including spacetime. This raises the question again, what makes spacetime what it is? In section 3, I will demonstrate that not only local, but also global properties are necessary to identify spacetime points, even in macro GTR, using the symmetric universe model as an example. This suggests that abstract properties, rather than concrete ones such as metric, are essential for being a physical realm.

2 Abstract and Concrete Properties of Spacetime

In this section, I will examine the current state of philosophy of spacetime. To understand the existence of spacetime in the context of GTR, it is necessary to examine the relationship between physical spacetime and the mathematical models used to describe it. First, I will look at how abstract entities in scientific theories explain events in physical worlds.

2.1 Between "Theory" and "Actual World"

In philosophy of science, there is a question of the existence of theoretical objects as claimed by scientific realism. This relates to the interpretation of scientific theories and the correspondence between models and actual worlds. For example, Frigg points out that models share important aspects in common with literary fiction or pretence and give a general picture of scientific modelling (Frigg 2010). Surely theories and models may not accurately represent an actual world due to approximations or idealizations (Cartwright 1983). The phrase "theoretical objects such as electrons and mass points exist in our actual world" may not be accurate.

Psillos argues that theoretical objects such as electrons and mass points exist in models as universals, and that scientific realism that presupposes abstract theoretical objects leans towards Platonism rather than nominalism (Psillos 2010, 2011). According to Psillos, mass points described in physical theories can represent individual massive bodies in the actual world, but they do not directly refer to these physical bodies. He states that models "are not concrete and...not causally efficacious" (Psillos 2011, 4), but that they still have explanatory power.

> The processes of idealization and abstraction are such that the description of the model isolates the explanatorily relevant features of the represented system with respect to the behaviour under study. It specifies the basic or more central explanatory mechanism or regularity. (Psillos 2011, 16)

Theoretical objects and structures as abstract universals correspond to something concrete. The Linear Harmonic Oscillator (LHO), for example, can accurately explain various calculated values of the swing of specific pendulums. This interpretation can also be applied to unobservable entities, such as electrons, which are discussed within the context of scientific realism versus antirealism. Unlike macro bodies, electrons cannot be directly observed, but their motion can be influenced and intervened with, according to Hacking (1983). Even if the concrete entities that instantiate or exemplify the theoretical electron are different from those electrons, there must be a causal entity in the actual world for theories to be verified through experiments and observations. In this sense, scientific realism presupposes that specific electrons or their counterparts exist in the actual world.

Models of data and structures are described mathematically in theories, but they also exist in the actual world through a correspondence relationship, as Psillos acknowledges.

> For now, the question is whether the model of the data itself (let us fix our attention on this to make things easier) is n-adequate [nominalistic adequate] vis-a-vis the phenomena, and answering this question presupposes either a direct confrontation of the model with the (unstructured) phenomena or the comparison of the model with another—one that (presumably) captures the causal structure of the phenomena. The first option does not seem to make much sense. The second option requires that the phenomena (or the world) have a built-in causal structure. (Psillos 2010, 956)

This interpretation suggests that theoretical entities and structures in models and theories correspond to structures in the actual world, based on isomorphism, and so on. From the perspective of limited scientific realism, models only need to capture certain aspects of the world, as Giere points out.

> We can also imagine that the two models are equally endowed with any supposed superempirical virtues such as simplicity or unity. Here I am strongly inclined to say that there can be no scientific basis for claiming that one model better fits the overall structure of the universe. Again, we have a limit on realist claims. (Giere 2004, 751)

If we want to adopt a nominalistic position, "[i]t is a further and separate claim that the model of the data (or the theoretical model for that matter) adequately represents concrete (causal) physical systems (or patterns). For the theory to be n-adequate, it is the latter claim that has to be true" (Psillos 2010, 956). Ketland says that structures are "nominalistically equivalent iff their concrete parts are isomorphic" (Ketland 2010, 208) meaning that the concreta behave 'as if' the theory is true.

It is important to interpret abstract entities or structures introduced in theories correctly. Nominalists might eliminate or reduce them, partly influenced by Field's belief that there is no abstract entity (Field 1980). Pincock's fictionalist view holds that mathematical systems are essential for theorizing physical worlds ("theoretical indispensability"), but not necessary for determining what exists ("metaphysical dispensability"). According to Pincock, when doing physical science, entities or structures in mathematical systems provide us with accurate knowledge, but this knowledge should be supported by empirical evidence, not theoretical content. Only with physical facts can these existences and properties in a theory be verified, and pure mathematical content in the theory can be disregarded.

> it is coherent and sensible to maintain that *the actual bottom-level physical facts* render the nominalistic content of empirical science true and the platonistic content of empirical science is fictional. (Pincock 2007, 268[1])

As an example in his paper (2007, 267), if a temperature theory (T) is incomplete and the physical basis for temperature is still unresolved, with T leaving

---

[1] his refined nominalistic position is devised from Balaguer's approach (Balaguer 1998) to challenge Quine's ontological commitment.

the crucial interpretive question of the existence of the lowest temperature open, facts about whether temperature has a lowest value should be determined based on the empirical evidence available to us. Not until we gain evidence, will the lowest temperature come into existence and its theoretical counterpart will be eliminated.

This eliminative approach supports qualitative parsimony. Adopting Pincock's perspective means we don't have to accept the existence of an excess of theoretical structures that are mathematically described. In this case, models are merely epistemological tools used to super-empirically interpret the world, rather than ontological entities (Suppe 1989). In addition, if we were to separate the causal structures from other theoretical elements that are not supported by empirical evidence, we could avoid making reference to others (van Fraassen 1980, 2006). Theoretical abstract entities have a different status from that of physical entities.

What is a causal structure? Besides the argument between realism and nominalism, if we accept the distinction between a model or language-like entity in scientific theories and the actual physical world, the latter refers to a causal world in which phenomena occur and can be directly experienced through touch and observation.

That is to say, the causal world is an actual physical world where particular objects and structures exist. This concrete realm differs from abstract ones and may refer to a spacetime realm, namely our universe[2].

Surely, the world constituted by spacetime has special features. At least, a spacetime realm is a part of the physical world, whether it is our world or another parallel world, as long as it is not in fiction, for example, novels. Philosophy of science, which deals with the realism of unobservable micro physical entities such as electrons, also connects causality strictly to a spacetime realm. Chakravartty establishes criteria of realism as entities having causal effects through detections (Chakravartty 1998, 2007), namely in the spacetime realm. In the next subsection, I will concentrate on this spacetime realm and traditional discussions of spacetime itself.

2.2 Privileged Spacetime Realm

It is true that many of philosophers and physicists consider spacetime to be the clearest realm. As mentioned in the previous subsection, physics originally deals with natural phenomena that occur in spacetime where matter exists with a particular position referred to by four-dimensional coordinate values (t, x, y, z). Philosophy, on the other hand, has addressed the realism of not only matter but also mind, mathematical numbers and sets, which do not exist in spacetime. The problem of universals is about whether these abstract entities exist in realms other than spacetime. In essence, do physicists focus on matter and spacetime while philosophers of physics, mathematics, and metaphysicians focus on abstract entities and realms?

But especially in contemporary physics since the 20th century, physical phenomena have been extended beyond spacetime. GTR presupposes a high-dimensional spacetime as a new physical realm in which black holes exist. In addition, quantum gravity theories and quantum cosmology are developing more fundamental entities from which spacetime derives or emerges, namely the origin of spacetime itself. Of course, physics before the 20th century also used abstract concepts such as "action" and "Lagrangian" described in analytical mechanics, but their mathematical behaviors based on fundamental laws are

---

[2] Accurately, the realm we can interact causally with is limited to only within the light cone emitted from us.

merely theoretical explanations for phenomena in the spacetime realm, rather than physical phenomena themselves. At least in the past, physical phenomena were limited to the spacetime realm.

Spacetime ontology has been a topic of discussion for centuries, but it is only relatively recently that structural interpretations have been incorporated into the philosophy of spacetime based on GTR. These new interpretations have partially influenced the traditional debate about the realism of spacetime, substantivalism and relationism, which dates back to the Newtonian era.

In Newtonian physics, spacetime can be considered as a background for matter. It is assumed that spacetime acts as a container for matter. However, a key question still remains: Does spacetime exist independently of matter? This question deals with whether or not the container, made up of space and time, remains even in the absence of matter. This raises the issue of the realism of empty spacetime, or a series of momentary vacuums. From a metaphysical perspective, this empty container is considered as absolute spacetime, which according to Newton, requires God's perception of matter (Newton, based on Alexander 1956, p.15), representing an early substantivalist interpretation.

Spacetime consists of infinite spacetime points, and each point may be considered real according to the substantivalist viewpoint in the context of Newtonian mechanics. These spacetime points are mathematically represented as elements of a set that spans the entire spacetime. It is still possible for substantivalists to hold a belief that spacetime points don't have intrinsic properties, despite the fact that such a belief supports Leibniz-shift being an effective counterargument against substantivalism. This viewpoint is in line with Newton's own belief, as stated below.

> For just as the parts of duration derive their individuality from their order, so that (for example) if yesterday could change places with today and become the later of the two, it would lose its individuality and would no longer be yesterday, but today; so the parts of space derive their character from their positions, so that if any two could change their positions, they would change their character at the same time and each would be converted numerically into the other. The parts of duration and space are only understood to be the same as they really are because of their mutual order and position, nor do they have any hint of individuality apart from that order and position which consequently cannot be altered. (Newton 1962, p.136)

That is to say, spacetime points should be identified based on their specific positions in the whole. Even in Newtonian physics, a spacetime realist like Newton can avoid presupposing haecceities (I will discuss this further in 3.1) for spacetime points, instead opting for structural interpretations. Spacetime points are geometric elements that are a priori indistinguishable from each other, not algebraic elements that can be distinguished from each other based on their properties (Stachel 2002).

This worldview also extends to spacetime in GTR. The 20th century field theory introduced a groundbreaking perspective, where spacetime is non-flat and has a non-Euclidean geometry described by metric field functions. In GTR, these fields are considered physical gravitational ones, and the spacetime realm mathematically consists of infinite spacetime points. Despite the shift in spacetime theories from Newtonian physics to Einstein's GTR, the interpretation of spacetime points remains similar.

In GTR, geometrically each point has its own properties, including metrical relations with neighbouring points defined by a metric tensor g_{ik} throughout the spacetime realm. The metric fields give all spacetime points spatiotemporal relational properties, and they locally describe how a specific realm of spacetime is curved.

> However, the metric tensor at a space–time point cannot strictly speaking be understood as an intrinsic property, since it involves infinitesimally neighbouring space–time points through the notion of tangent space on which it is defined. (...) [T]he fundamental space–time properties would be relational only in an infinitesimal sense so that the fundamental relations are only infinitesimal relations (Lam & Esfeld 2012, 248-249)

Metric plays a crucial role in establishing the relational properties of each spacetime point.

In recent discussions of the philosophy of spacetime, both substantivalism and relationism widely acknowledge that spacetime should be seen as a metric field (Slowik 2004, 2015). The former argues that this dynamic spacetime exists independently of other physical fields, such as metric field substantivalism (Hoefer 1996), while the latter claims it is just a property of or can be reduced to other physical fields, like dispositions (Teller 1991). In light of this ambiguous realism of metric fields, the conventional distinction between substantivalism and relationism has become outdated (Rynasiewicz 1996) and structural realism about spacetime is considered a third perspective (Dorato 2000). All of these discussions are based on the assumption that the essence of physical spacetime is the metric field.

For ontological structural realism (OSR), especially for the moderate versions proposed by Esfeld and Lam, this picture of spacetime points is very consistent, even though the spacetime in Newtonian physics differs radically from that in GTR. They view spacetime as “a mind-independent physical structure whose basic constituents have no fundamental intrinsic properties independently of the structure they are part of” (Esfeld & Lam 2008, 44). They regard these constituents as spacetime points at least in classical GTR, and they maintain that spacetime can exist independently, together with its relational properties, which are not reducible to the properties and relations of matter. “Moderate structural realism claims that the spacetime structure exists as a mind-independent physical network of spatiotemporal relations among spatiotemporal constituents (such as spacetime points) that do not possess any intrinsic properties” (Esfeld & Lam 2008, 42-43).

However, these structural interpretations make the realism of spacetime more complicated. In the next subsection, I will examine what these structural interpretations mean.

2.3 What Does Spacetime Refer to?

Of course, there are geometric properties other than metric about spacetime. In particular, manifold substantivalism claims that gravitational fields are kinds of matter and that spacetime points should have intrinsic properties as a manifold or primitive identities, namely haecceity, independent of relational ones given by metric (Hoefer 1996). I think that this idea, called manifold substantivalism, is strongly connected with a traditional conviction that spacetime is a container and exists independently of physical fields.

However, manifold substantivalism faces a serious problem. If we stick to the difference between spacetime and matter even in GTR, metric (gravitational) field may be included in the matter side for a manifold substantivalist. This view implies that a spacetime realm can exist without the metric field.

This view leads to fatal indeterminism, namely the contemporary hole argument (Earman & Norton 1987).

The hole argument is concerned with the question of what orbit test particles move on inside H, a small hole with no other fields in a manifold M. Suppose a particle moves in M passing through H, and there are some orbit models in which diffeomorphisms from one to another manifold point are applied to a domain only inside H. In the same hole H, the particle passes through mathematically different points by diffeomorphisms. Therefore, only inside H, it seems as if there were different orbits and metric tensors in each model. If these apparently different models show physically different situations, geodesic solutions of the particle and g_{ik} can be obtained infinitely. This induces indeterminism for the geodesic equation and the Einstein equation.

Of course there are only passive coordinate transformations to realize diffeomorphisms based on general covariance in GTR between these models and we cannot observe a bare point without metrical properties. In fact, in order to observe which point the particle passes through, we use these properties, for example, through a ruler and a clock. What coordinate value given to a point depends on these items. That is to say, apparently different orbits are identified observationally with metrical information.

The metric field is necessary for identifying spacetime points. Earman and Norton argue that substantivalists "must either (a) accept that there are distinct states of affairs which are observationally indistinguishable, or (b) deny their substantivalism" (Earman & Norton 1987, 522). If we believe that a bare point in M has an identity independent of physical fields, including the metric field, as manifold substantivalism suggests, we cannot avoid fatal results. Contemporary substantivalism and relationism have developed by emphasizing the importance of the metric field in this contemporary version of the hole argument.

Apparently different metric fields diffeomorphically related as a group refer to a spatiotemporal structure. Stachel says that spacetime points in a manifold inherit all their chronogeometric (and inertiogravitational) properties and relations from metric fields. With that assumption, an entire equivalence class of diffeomorphically related mathematical solutions represents just one physical solution (Stachel 2002, p.233). Each solution is a different form to describe one common structure.

Intuitively, a structure absent from metric is so abstract that it runs short of physical spatiotemporal concepts such as distance, angle, and causality of light cones. In fact, it is very strange to call it spacetime. Einstein says as below.

> There can be no space nor any part of space without gravitational potentials; for these confer upon space its metrical qualities, without which it cannot be imagined at all. (Einstein 2007, p. 618)

Spacetime cannot be detached from a gravitational field in it and spacetime cannot be accounted independently of the gravitational field. Einstein himself explains this worldview as this.

> If we imagine the gravitational field, i.e. the functions g_{ik} to be removed, there does not remain a space of the type (I) [Minkowski spacetime], but absolutely nothing, and also no "topological space". For the functions g_{ik} describe not only the field, but at the same time the topological and metrical structural properties of the manifold. (...) There is no such thing as an empty space, i.e. a space without field. Space-time does not claim existence on its own, but only as a structural quality of the field. (Einstein 1961, pp.155-156)

This dynamical field interacts with others via the Einstein equation, and so spacetime is no longer a passive background. Before I conclude that metric field substantivalism is more refined than manifold substantivalism, I want to emphasize two points here as follows:

1. The worldview that "spacetime is a structural quality of the field" is another aspect of structural interpretations different than the discussion continuing from the previous subsection, namely how spacetime points are identified.
2. g_{ik} describes not only metrical properties but also topological properties of the manifold.

Spacetime and matter may be just different aspects of one structure. Regarding point 1, the question of what spacetime is becomes blurred and the division between spacetime and matter becomes more and more ambiguous, leading to super-substantivalism, which claims that spacetime refers to the whole universe, including not only the metric but also other material entities. This interpretation is influenced by Einstein's intention to unite gravity and the electromagnetic field.

Locality given by the metric field tensor leads to topology. As for point 2, I will focus on it in this paper. Einstein acknowledged that without the metric, it is impossible to describe not only physical spacetime itself, but also a topological space. It is difficult to imagine what would occur in the universe without gravitational fields (Maudlin 1990).

Is topology included in the metric? Hoefer argues that spacetime or part of spacetime can be described by the metric field, not the global topology (Hoefer 1996, 24-25). Maudlin claims that "the topology flows from the metric rather than the metric being imposed on the topological space" (Maudlin 1990, 554). Dorato also emphasizes that the topology of spacetime cannot be determined prior to the Einstein equation (Dorato 2000, 1610). I would like to suggest that topology, which is claimed by the three of them, is not a property previously possessed by spacetime, but rather emerges from all local metrical information. I think this is a doctrine of contemporary philosophy of spacetime.

3 Extended Physical Realms

While metrical features play an important role for spacetime and a spacetime realm, there is also a gap between metric and topology. A topological space is already a differentiable manifold consisting of continuous points and on which ordinary vector and tensor fields are defined.[3].

I wonder whether topology is included in metric in 2.3 or not, but mathematically, topology without metric is possible. In other words, even without local detailed properties, physical theories can be described to some extent with only global properties. Fundamental laws can be defined, and geodesic equations can be written without a metric because even the covariant differentiation of contravariant, covariant, and mixed tensors to establish differential equations can be defined on the condition that the concept of connection is introduced:

$$v^{\mu}\nabla_{\mu}v^{\lambda} = \frac{d^2x^{\lambda}}{dt^2} + \Gamma^{\lambda}_{\mu\nu}\frac{dx^{\mu}}{dt}\frac{dx^{\nu}}{dt}. \tag{1}$$

$\Gamma^{\lambda}_{\mu\nu}$ is the Christoffel symbol, and this equation shows the parallel transport of a body for all orbits in a manifold or a topological space, leaving various possibilities about how the velocity of the particle is kept in moving along a

---

[3] For details, refer to Hawking & Ellis (1975).

geodesic line. Of course, although written formally, a connection without a metric is so abstract that we cannot specify the physical significance of geodesic equations.

This equation is an abstract geodesic equation. In addition, Maxwell's equations of electromagnetism can also be partially given without a metric. That is to say, abstract spatiotemporal properties can be described without a metric, and therefore, we should be careful about what is essential for physical spacetime or a spacetime realm.

3.1 Give-and-Take Relation Between Topology and Metric

In this subsection, I suggest that the hole argument does not imply that the spacetime manifold is not physical spacetime. Sophisticated substantivalism and relationism argue that in order to describe our ordinary spatiotemporal concepts such as distance and angle, there must be a metric. I do not intend to challenge this worldview, but does the hole argument deny properties as a manifold by determinism?

In the discussion of the hole argument, the core idea for holding determinism is that the points inside H in a manifold are not individuated independently of the g_{ik} field. Given some arbitrary coordinate systems, each orbit of a test particle is mathematically different in each coordinate system. If we want to interpret these orbits as physically identified, different coordinate points in each system must be identified through diffeomorphisms. This identification is based only on maps between different values as functions of each coordinate system. Inside H, it is not the coordinate value (t, x, y, z) but the geometrical structure given by the metric tensor or Ricci tensor that determines the spacetime points as they are.

This view surely abandons the coordinate values as intrinsic or prior properties to identify spacetime points, but it never gets rid of properties as a manifold. The hole argument states that only with topological information, we cannot determine what orbit a particle follows inside H. For example, all orbits in Figure 1 are the same from topological viewpoints. They inherit common properties from the topological space and never be contradictory with each other. Their differences are seen in more concrete properties, but information about topology is common in all coordinate systems. That is to say, *topology is insufficient to specify how a body moves in spacetime*

This means that spacetime points, which cannot be distinguished from each other only with topological properties, can be distinguished with additional features. These features should be about neighboring points included in continuous orbits a particle follows inside H, namely local relations between spacetime points. Hence, a metric is necessary to distinguish between these points, and we can specify orbits clearly in a manifold. All points have local features in the manifold, and this denies the fact that spacetime is a point manifold. However, at least discussions of the hole argument based on Earman & Norton deal with points in a manifold as if these points were devoid of metric and labeled by determinate coordinate values (t, x, y, z) in one universal coordinate system. Manifold substantivalism identifies spacetime points only with coordinate labels rather than with topological properties or positions in a manifold, leading to indeterminism. Conversely, without universal coordinate values covering a manifold, indeterminism does not occur, and the models of orbits are identified topologically. By possessing a metric, a manifold becomes so abundant that apparently different candidates with the same topological properties can also be identified as one.

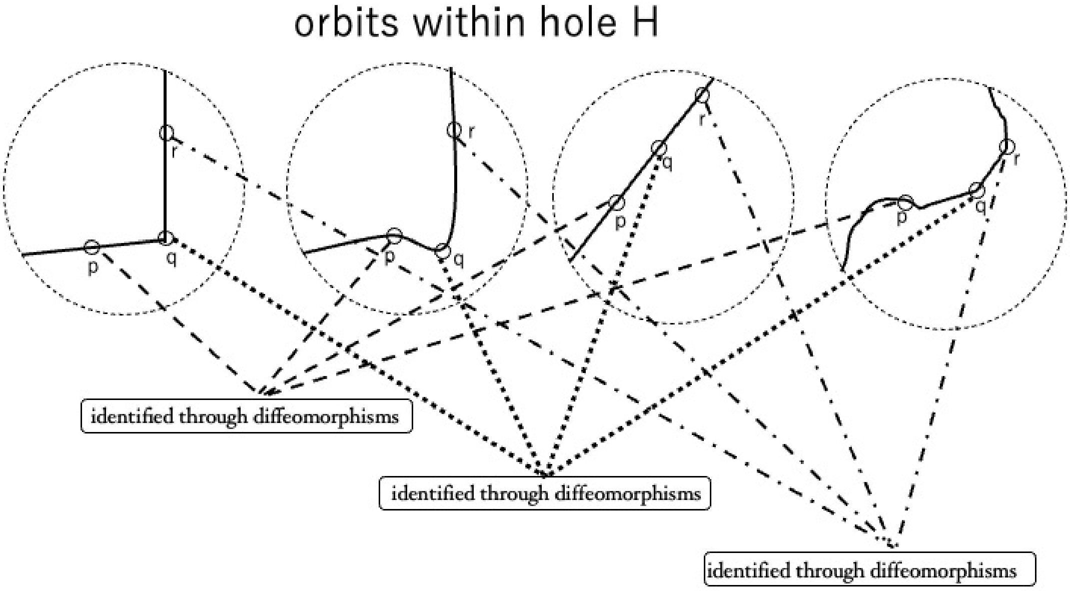

Figure 1: Apparently different orbits: This figure shows that an arbitrary coordinate system has each orbit of a test particle and seems different mathematically. But different coordinate points in each system can be related through diffeomorphisms, and if we interpret related points as the same physical spacetime points labeled p, q, r, indeterminism does not occur. This interpretation implies that all orbits are the same not only metrically but also topologically. They are different only in each coordinate value given to each point within H.

Metrical information is necessary and sufficient to describe physical spacetime in this case and includes topological information. Ultimately, many orbits are identified not only by topology but also by metric. A spacetime manifold is not to be labeled by a universal coordinate system, but rather by an arbitrary one with a metric tensor defined at each point to enable us to distinguish between points.

If topology were compared to fruits, metric would be, for example, an apple, and apparently different orbits in the hole would correspond to different pictures of the same apple drawn from various directions. Even if spacetime were found to be an apple, it would not deny the fact that spacetime is a fruit. It only means that in order to draw pictures of spacetime, more concrete information of being an apple must be added.

Does metrical information always uniquely determine spacetime points in a manifold? The answer is no, and this is a weak point of structural interpretations, sophisticated substantivalism, and relationism. Even if all metrical features are given to all spacetime points in the universe, there is a case in which we cannot distinguish each of these points, namely, when some symmetries are imposed on spacetime. Wüthrich points out that if a metrical structure given to spacetime in the universe is homogeneous and isotropic, leading to the so-called Friedmann-Lemáitre-Robertson-Walker (FLRW) metric, all spacetime points will be identified (Wüthrich 2009). In other words, we cannot distinguish one spacetime point from another with metrical information depending on the symmetries imposed on spacetime. Let us now examine Wüthrich's suggestion.

The FLRW metric is derived from the cosmological principle that the universe is approximately the same everywhere and possesses translational and rotational symmetries described as this in polar coordinates:

$$ds^2 = c^2dt^2 - a^2(t)[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)]. \tag{2}$$

$a(t)$ is the scale factor of space and K is the space curvature which is constant through all spacetime points. In this metric, as Wüthrich shows, there exists a one-parameter family of spacelike hypersurfaces Σ_t, or a preferred foliation labeled by a cosmological time t (Wüthrich 2009, 1044). In addition, for any of the points $p, q \in \Sigma_t$, there exists an isometry f of the metric g_{ik} on M with $f(p) = q$. Isometries of g_{ik} on M form a group of automorphisms called the isometry group of M onto itself. This group implies a mapping between any points p and $q \in \Sigma_t$ with "all (metrical) relations fixed, that is, the ascriptions of metrical properties to "places" in the structure" invariant. By mapping, metrical properties of p can be taken over by metrical properties of q and if we follow a moderate version of OSR for spacetime points as Esfeld and Lam claim, p can be identified with q. Since spatial homogeneity enables isometries between two arbitrary points, all points can be identified in eq. 2.

> Any point on any spacelike hypersurface is thus equivalent to any other point on the same hypersurface. In particular, the symmetries imply that the spatial curvature of all the spacelike hypersurfaces Σ_t of the preferred foliation is constant. (Wüthrich 2009, 1044)

According to Leibniz's Principle of the Identity of Indiscernibles (PII), objects should be distinguished only in terms of their properties:

$$\forall F(F(a) \leftrightarrow F(b)) \rightarrow a = b. \tag{3}$$

It is important to consider how wide the range included in F is, for example, whether F is limited to physical properties or to intrinsic ones, etc. French and Redhead suggest that PII has two versions, a strong one and a weak one. In the weak version, F includes properties of spatial location, while in the strong version, F excludes properties of spatial location (French & Redhead 1988, 234)[4]. Although there are similar objects that share many properties, in classical physics, more than one rigid body cannot occupy the same space points, and they cannot be identified by the criteria of the weak PII.

Spatial location is of significant importance related to spatiotemporal properties. Structural realists consider F to consist only of automorphically invariant relational properties, not intrinsic ones. If objects in spacetime refer to spacetime points, all of them must share the same relational properties in the FLRW metric, namely 'places' in the structure. Spatial locations for spacetime points may be such invariant relational properties. Hence, it is concluded that p=q for any two arbitrary points, which is disastrous as it suggests that the (spatial) universe consists of nothing but one lonely point (Wüthrich 2009, 1040).

However, this conclusion is controversial even if we keep on being a structural realist. Aside from PII, one way to evade this conclusion is to admit primitive "numerical distinction (diversity)" such that there is more than one object

---

[4] As Wüthrich tells in his paper, French also divides PII into three versions–(i)∀F ranges over all possible properties, (ii)∀F ranges over all possible properties except spatiotemporal ones, and (iii)∀F ranges only over intrinsic properties. (French 2006; Wüthrich 2009, 1045)

with the same properties (Esfeld & Lam 2008, 33[5]). This view supports the realism of some objects in quantum entanglement and can dispense with the idea of haecceities, in other words, primitive thisness or primitive identity given to all individuals beyond the same intrinsic and extrinsic properties to ultimately distinguish each of them (Adams 1979), which opposes OSR. Intuitively, it is natural that in the FLRW metric, there are infinitely indistinguishable spacetime points rather than just one point in the universe.

Another way is to search for other properties for each object that are different from common extrinsic ones. As an example of this, Saunders proposes "weak discernibility", which shows that an object has an irreflexive relation with itself even in a symmetric structure (Saunders 2003, 2006). He uses Max Black's example of two spheres of iron positioned in an otherwise empty universe, one mile apart in space (Black 1952), and points out that they are weakly discerned by the symmetric and irreflexive relation "one mile apart in space" (Saunders 2006, 57). If two spheres consist of the same ingredients and have the same size, shape, or color, they may surely share all intrinsic properties. In addition, both of them possess an extrinsic property of being one mile apart from the other, but they do not have a distance of one mile from themselves. So this extrinsic property is an irreflexive relation. Similarly, metrical properties are relations for a spacetime point with other points and not with itself. Spacetime points with no intrinsic properties in symmetric solutions of the field equation such as the FLRW metric are weakly discernible (Lam & Esfeld 2012, 254).

However, I would like to argue that spacetime points in a symmetric geometric structure can be distinguished by using properties from the manifold or topology. As I mentioned earlier, two spheres in Black's example, which are rigid bodies in the sense of classical physics, cannot occupy the same place and can be distinguished from each other, even if we do not appeal to weak discernibility. But if we take an extremely relationist stance, such as Leibniz, this distinction of location becomes nonsensical (Leibniz 1981). This distinction presupposes space as "a fixed background of topology $\mathbb{R}^3$" (Wüthrich 2009, 1045). The difference implies the difference in where points are put in a manifold. Although I stated that a metric field is necessary and sufficient to describe physical orbits, it does not mean that a spacetime manifold does not exist.

I think we should conclude that different points p and q in the same manifold differ only because there is a topology of spacetime, even in the FLRW metric universe. p and q are indistinguishable from a viewpoint of metric owing to isometries, but at least they are referred to by different coordinate values in the same coordinate system, which means that they are already in different places. All spacetime points in M are placed with a definite order, called topology, independently of locality. As Figure 1 shows, p, q, and r in each coordinate system are, at least, different points in the same hole H, independently of metrical information. Here, we want to remember Newton's quotation in 2.2.

- For the spacetime realists, the parts of space, that is to say, space points derive their character from their positions, or order.

Our universe, consisting of different infinite continuous points, has global topological features, whether a homogeneous and isotropic geometric structure is imposed on spacetime or not. Hence, two points can be distinguished from each other since they have different topological properties.

Again, I want to analogize the relation between topology and metric to the relation between fruits and apples. In this case, two objects cannot be

---

[5] in Wüthrich, numerical distinction is expressed as "numerical plurality" (Wüthrich 2009)

distinguished because they are very similar apples. But originally, they were regarded as two different fruits until it was discovered that they were the same kind of fruit. This is a clear example that concrete information is not always sufficient to distinguish objects from each other.

In short, topology and metric are in a give-and-take relation to describe physical spacetime. In order to determine an orbit in a hole, topological features are not enough to know which points a particle passes through. Given metrical features, we can understand which point in a manifold corresponds to which physical spacetime point. Conversely, to tell one space point from others, which are indistinguishable from each other only with metric, topological features are needed. To describe macro spacetime in all cases, both metric and topology are necessary and sufficient.

Of course, as I mentioned in section 2, metrical properties eventually lead to topological ones, and global topology is naturally included in the structure of spacetime. However, if we regard spatiotemporal properties as features limited to locality, we cannot distinguish points that are different globally in a structure. For the structural realism of spacetime, spacetime exists as a geometric structure of a manifold/topology with metric, rather than only of metric[6].

Spacetime points may exist independently of metrical relations between them. Except for cases involving symmetric geometric structures, a spacetime point is identified only by metric, but this does not imply that a point is ontologically given by these local relations. Esfeld and Lam, following moderate structural realism, argue that "the relations and the objects that stand in the relations are on the same ontological footing and are also conceptually interdependent" (Esfeld & Lam 2008, 37). They hold that the metric tensor field defines spatiotemporal relations between spacetime points, which are necessary for the definition of the field.

Topology is *independent of locality, but it arises from metric*. Esfeld and Lam only consider metrical relations as spatiotemporal relations, and they reject topological relations as intrinsic properties. However, I believe that topological relations should also be included in spatiotemporal relations. In other words, even for structural realists, not only spacetime points but also topology or point manifold as a substructure of the spatiotemporal structure are ontologically posited without reference to local properties.

3.2 More Abstract in Micro Regions

I have shown that metric is an essential factor for defining physical spacetime, but topological features also contribute to making spacetime what it is. Roughly speaking, I adhere to a worldview in which a spacetime point or a point manifold is real independent of the locality given by the metric, but arises from the metric, and we do not need to assume primitive or intrinsic identities beyond a spatiotemporal structure. At least as far as the information about the spacetime realm is concerned, it is clear that local metrical relations are more concrete than global topological ones. However, physical theories can still be formulated without a metric, as I have discussed earlier in this section. There-

---

[6] Sophisticated substantivalism and relationism consider spacetime to be metric rather than a manifold, and Hoefer puts forward metric field substantivalism as a refined version after he raises manifold plus metric substantivalism as an intermediate position from manifold substantivalism to metric field substantivalism (Hoefer 1996). However, he intends manifold plus metric substantivalism to refer to spacetime whose points possess primitive thisness with metrical properties. So, this position falls into hole arguments and differs from my suggestion. What I want to emphasize using the words "a manifold/topology with metric" is that not only local but also global properties for each point should be included in a structure of spacetime.

fore, abstract features play an important role in defining physical spacetime as well.

A mathematical structure can approach a physical structure by adding information to itself. In the case of four-dimensional spacetime, it starts with a set of four-dimensional continuous points. Next, to form a geometric structure, these points are ordered to create each topological space covering a manifold from charts to atlases, subject to certain restrictions. At this point, global properties of spacetime are already expressed, and we can create a map of spacetime drawn on $\mathbb{R}^4$. While this structure may be abstract and insufficient to describe physical spacetime, a spacetime manifold reflects fundamental aspects of macro spacetime or the universe, which is the physical realm.

Metrical information is so concrete that it can specify physical spacetime. As noted earlier, with equation (1), there are multiple ways to move a velocity vector depending on how the Christoffel symbol $\Gamma^{\lambda}_{\mu\nu}$ is defined. Assuming metric connection, one way is fixed, but there are still infinite geodesic solutions sharing the same spatiotemporal structure. Using metrical information, we can only determine the geodesic line after choosing a specific coordinate system, which then labels each spacetime point with coordinate values. General covariance allows us to use arbitrary coordinate systems to relate a point manifold with metric to physical spacetime.

If we turn our eyes to quantum theory, the physical realm becomes more ambiguous. Some quantum gravity theories suggest that spacetime emerges from more fundamental entities (Wüthrich 2018; Huggett & Wüthrich 2013, and others). This view suggests that in micro regions, spatiotemporal features break down or no longer hold. However, how do we decide what to include in these spatiotemporal features, and what criteria should we use to claim the emergence of spacetime?

More fundamental entities vary depending on the quantum theory being used due to different methods of quantization. Quantum gravity theories are still incomplete as candidates to unify GTR and quantum mechanics. What can sophisticated substantivalism, relationism, and structural realism say about these theories? The worldviews of spacetime in a micro-region are clearly different from those in a macro-region described by GTR.

The dominant theory is super string theory, which considers units of matter to be one-dimensional strings rather than point-like particles in a fixed background of 11-dimensional spacetime. However, there is a dimensionality difference between macro spacetime and the micro background, and correspondence relations are complicated because super string theory quantizes a classical theory different from GTR, which is just a limited case only applicable under certain conditions.

The continuity of spacetime is not always maintained in other theories. Causal set theory, for example, focuses on causality within a spacetime realm but deprives spacetime of continuity. In this case, spacetime is essentially discrete and consists of many causal sets that are embedded in a manifold, rather than spacetime points. Additionally, loop quantum gravity directly quantizes the metric and arrives at a graph called a spin network, which has nodes and edges that quantify discrete volumes and discrete areas of edges corresponding to the surface of adjacency of the connected volumes, as if they describe atoms of space. In this case, spacetime is also essentially discrete. Properties that are even more fundamental than topology and metric, such as continuity, may not be necessary for a micro spacetime realm.

Local properties given by metric are not seen in micro-regions as they are in a macro-region. In loop quantum gravity, as Huggett and Wüthrich show (Huggett & Wüthrich 2013, 279; Wüthrich 2018, 7-8), locality is broken in spin

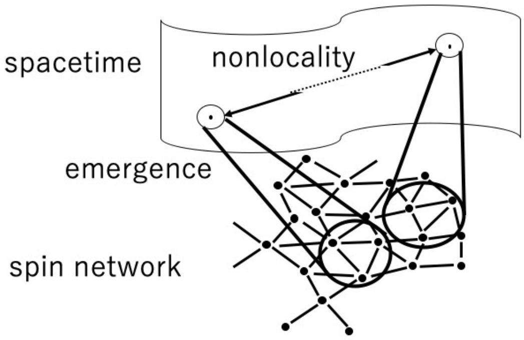

Figure 2: Emergence of spacetime from spin network: This figure shows that a spin network has locality between two nodes in a state space but exhibits non-locality as spacetime when macro spacetime emerges. I drew this figure from Fujita (2020, 7) based on the one in Wüthrich (2018, 8).

networks. They raise two differences between spin networks and ordinary lattices regarding locality. Spin networks are born as quantization of metric, namely spacetime, and they should express quantities about spacetime through a quantum superposition of different spin networks. Each state has its locality, and locality is not fundamental!

> Except perhaps for very special states, local beables can thus not be part of the fundamental reality, but must instead emerge in some limit—presumably the same as that in which locality emerges. How such local, i.e., topological, structures like relativistic spacetimes emerge from spin networks is at present little understood. (Huggett & Wüthrich 2013, 279)

Secondly, they point to a gap between a fundamental structure of spin network and a spatiotemporal structure about locality from a viewpoint of empirical coherence. Even in one spin network structure rather than a superposition of different spin networks, "two fundamentally adjacent nodes will not map to the same neighbourhood of the emerging spacetime...Hence the empirically relevant kind of locality cannot be had directly from the fundamental level" (Huggett & Wüthrich 2013, 279). (make reference to Figure 2)

Surely a spin network is a kind of state space devoid of physical features and if we read this empirical coherence with physical coherence, how does a spin network function as a physical entity?

We are now in a phase where we admit there are phenomena not presupposing a spacetime realm. Even without clear spatiotemporal features, can we talk about a physical realm? Maudlin calls this aspect "physical salience" and I think he is conscious that we must extract physical contents from a mathematical structure of a fundamental entity (Maudlin 2007). Especially for (moderate) structural realism, this question leads to ontology about whether a fundamental structure is physically real or not[7].

Many differences between a more fundamental structure and a spatiotemporal one are due to discontinuities based on correspondence relations. For example, in interpreting causal sets, as Wüthrich says, a symmetric structure in causal set theory cannot correspond to a symmetric spatiotemporal structure

---

[7] Wüthrich refers to emergence of spacetime from structural viewpoints (Wüthrich 2012, 2018), but only admits that a traditional spatiotemporal structure is no longer in circulation.

such as the FLRW metric in GTR, although a concept of causality is common to both theories and realms (Wüthrich 2012). In loop quantum theory, geometric features are not seen in a spin network, which is not a physical space as I mentioned above. Through quantization, many original features of a spin network form a structure, and there is no isomorphism between it and a metrical structure, even though loop quantum gravity quantizes metric.[8].

I think it is crucial to recognize the common aspects between a spatiotemporal structure and a more fundamental structure when discussing the ontology of a micro physical realm. While there may be many discontinuities between these structures, as one emerges from the other, recognizing the commonalities between them can provide a more comprehensive answer to what constitutes a physical realm beyond spacetime and what factors are the most significant in a spatiotemporal structure.

It is not true that all information about spacetime is erased in quantum physical realms. In fact, many parameters of spin networks may be isomorphic to those featuring boundary conditions imposed on the topology. Thus, a spin network can reflect a part of the information about the topology of spacetime.

To summarize, both causal sets and spin networks abstract a spatiotemporal structure. The spatiotemporal structure that arises from the metric to the manifold in classical macro spacetime is partially transferred to the more abstract description of the quantum physical realms. In contemporary physics, including quantum gravity theories, essential aspects of spacetime consist in a pregeometric structure.

4 Conclusion: A New Way of Interpretations About Spacetime Through Macro and Micro Region

In this paper, I have focused on how a physical realm is described by physical theory in relation to the philosophy of spacetime. In this discussion, I have touched upon what scientific realism wants to claim about theoretical entities described by mathematics. Scientific realists have been studying how these abstract entities, including a spatiotemporal structure, can be said to exist. There is no doubt that we live in a concrete physical realm, namely a spacetime realm, which is very close to being "real". I have revealed the realism of this privileged realm and have claimed that the abstract features of it are also real.

In section 2, I explored the meaning of realism concerning theoretical entities by discussing the metaphysical debate between realism and nominalism. In general, causality and intervention are important factors in defining specific entities that exist in physical phenomena in our actual world, with positions in spacetime, rather than as universals in abstract models[9]. Additionally, I showed that various mathematical aspects are used to describe this privileged physical bent spacetime in GTR. In the philosophy of spacetime, spacetime is referred

---

[8] I provide details based on Butterfield's formulation about emergence (Butterfield 2011) and claim that there are structural discontinuities in quantizing gravitational field while there is an isomorphism between quantized electromagnetic field and classical electromagnetic field (Fujita 2020).

[9] In this paper, I associated causality (an empirical property) with the spacetime realm due to the context of scientific realism. However, there is a causal realm beyond the spacetime realm in quantum theory, which is a phase space where virtual particles like photons move (Fujita 2021), even if we do not bring up quantum gravity theories as I discussed in section 3.2. Additionally, semi-particles defined in solid-state physics may be considered not real, although they have causal inertia (Gelfert 2003; Falkenburg 2007).

to by the metric field, rather than a point manifold, for empirical (physical) reasons. This has led to sophisticated substantivalism, relationism, and structural realism, which aim to solve problems such as the hole argument.

In section 3, I argue that although spacetime is metric as established by previous studies, it also has topological properties as it is a set of all points in a manifold that *arises from* the metric, independent of locality. This is important because it emphasizes that there are more abstract properties given to spacetime points than only the metric. In symmetric universes, such as the FLRW metric, spacetime points are identified not only by the metric but also by topology. In micro regions, the macro spatiotemporal structure becomes more abstract because of more fundamental entities in each quantum gravity theory. These entities reflect some features of spacetime, and a spatiotemporal structure emerges from them. Therefore, in considering quantum gravity, what is essential for being a physical realm in general is more abstract.

Through this paper, I am conscious of a realm where entities live. In a macro region, physical particular entities are in spacetime and it is presupposed in a spacetime realm that physical phenomena occur. If we call this view spatiotemporalism, to believe in micro physical realms grounding spacetime itself such as spin networks is against spatiotemporalism. However, quantum theories defend rejecting spatiotemporalism as ontology (Paul 2012) and the time has come when we should interpret physical phenomena in a broad sense.

Acknowledgment

This work was supported by JSPS KAKENHI Grant Number JP21J01573 and Research Grants of Suntory Foundation in Japan. Additionally, I want to express my gratitude to native English speakers, including my lovely wife, for checking my poor English.

References

Adams, Robert. M. (1979). Primitive Thisness and Primitive Identity. *Journal of Philosophy*, 76, 5–26.

Alexander, Horace. G, (Ed)., (1956). *The Leibniz-Clarke Correspondence: Together with Extracts From Newton's Principia and Opticks.* New York: Barnes & Noble.

Balaguer, Mark. (1998). *Platonism and Anti-Platonism in Mathematics.* Oxford: Oxford University Press.

Black, Max. (1952). The identity of indiscernibles. *Mind* 61, 153–164.

Butterfield, Jeremy. (2011). Emergence, reduction and supervenience: A varied landscape. *Foundations of Physics,* Volume 41(6), 920–959.

Cartwright, Nancy. (1983). *How the Laws of Physics Lie*. Oxford: Oxford University Press.

Chakravartty, Anjan. (1998). Semirealism. *Studies in History and Philosophy of Science*, 29(3), 391–408.

.... (2007). *A Metaphysics for Scientific Realism:Knowing the Unobservable*. Cambridge: Cambridge University Press.

Dorato, Mauro. (2000). Substantivalism, Relationism, and Structural Spacetime Realism. *Foundations of Physics*, 30, 1605–1628.

Earman, John & John Norton. (1987). What price substantivalism:The hole story. *British Journal for the Philosophy of Science*, 38, 515–525.

Einstein, Albert. (1961). Relativity and the Problem of Space. In *Relativity: The Special and the General Theory* New York: Crown.

.... (2007). Ether and the theory of relativity. In Michel Janssen, John D. Norton, Jürgen Renn, Tilman Sauer, John Stachel (Eds.), *The Genesis of General Relativity. Boston Studies in the Philosophy of Science vol 250*. Dordrecht: Springer, Volume 3, pp.613–619.

Esfeld, Michael & Vincent Lam. (2008). Moderate structural realism about space-time. *Synthese*, 160(1), 27–46.

Falkenburg, Brigitte. (2007). *Particle Metaphysics: A Critical Account of Subatomic Reality (The Frontiers Collection)*. Springer.

Field, Hartry. (1980). *Science Without Numbers*. Princeton: Princeton University Press.

French, Steven & Michael Redhead. (1988). Quantum Physics and the Identity of Indiscernibles. *British Journal for the Philosophy of Science*, 39, 233–246.

French, Steven. (2006). Identity and Individuality in Quantum Theory. In Edward N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*. Spring 2006 edition. Stanford, CA: Metaphysics Research Lab, Center for the Study of Language and Information, Stanford University.

Frigg, Roman. (2010). Models and Fiction. *Synthese*, 172(2), 251–268.

Fujita, Sho. (2020). Emergence of Spacetime from Structural Interpretations: Why is Spacetime's atom not Spacetime itself? *Journal of the Japan Association for Philosophy of Science "Kagaku Kisoron Kenkyu" in Japanese*, 48(1), 1–19.

.... (2021). Abstractness for Nominalists: Physical Worlds Outside of the Spacetime Realm (translated from Japanese) *Contemporary and Applied Philosophy*, 13: 141–173. https://doi.org/10.14989/266868

Gelfert, Axel. (2003). Manipulative success and the unreal. *International Studies in the Philosophy of Science*, 17(3), 245–263.

Giere, Ronald. (2004). How models are used to represent reality. *Philosophy of Science*, 71(5), 742–752.

Hacking, Ian. (1983). *Representing and Intervening: Introductory Topics in the Philosophy of Natural Science*. Cambridge: Cambridge University Press.

Hoefer, Carl. (1996). Absolute versus Relational Spacetime: For Better or Worse, the Debate Goes On. *British Journal for the Philosophy of Science*, 49, 451–467.

Hawking, Stephen W & George F. R. Ellis. (1975). *The Large Scale Structure of Space Time*. Cambridge: Cambridge University Press.

Huggett, Nick & Christian Wüthrich. (2013). Emergent spacetime and empirical (in)coherence. *Studies in the History and Philosophy of Modern Physics*, 44(3), 276–285.

Ketland, Jeffrey. (2011). Nominalistic Adequacy. *Proceedings of the Aristotelian Society New Series*, 111, 201–217. Oxford University Press.

Lam, Vincent & Michael Esfeld. (2012). The Structural Metaphysics of Quantum Theory and General Relativity. *Journal for General Philosophy of Science*, 43(2), 243–258.

Leibniz, Gottfried. W. (1981). *New Essays on Human Understanding*. Translated and edited by Peter Remnant and Jonathan Bennett. Cambridge University.

Newton, Isaac. (1962). *Unpublished Scientific Papers of Isaac Newton*. Translated by A. Rupert Hall and Marie Boas Hall. Cambridge: Cambridge University Press.

Maudlin, Tim. (1988). The essence of space time. *Proceedings of the Biennial Meeting of the Philosophy of Science Associationin* Vol.2, 82–91. Cambridge University Press.

.... (1990). Substances and space-time: What Aristotle would have said to Einstein. *Stud. Hist. Phil. Sci*, 21, 531–561.

.... (2007). Completeness, supervenience, and ontology. *Journal of Physics A: Math- ematical and Theoretical*, 40(12), 3151–3171.

Paul, Laurie. A. (2012). Building the world from its fundamental constituents. *Philosophical Studies*, 158(2), 221–256.

Pincock, Chris. (2007). A Role of Mathematics in the Physical Sciences. *Nous*, 41(2), 253–75.

Psillos, Stathis. (2010). Scientific Realism: Between Platonism and Nominalism. *Philosophy of Science*, 77(5), 947–958.

.... (2011). Living with the Abstract: Realism and Models. *Synthese*, 180(1), 3–17.

Rynasiewicz, Robert. (1996). Absolute Versus Relational Space-Time: An Outmoded Debate? *Journal of Philosophy*, 93(6), 279–306.

Saunders, Simon. (2003). Physics and Leibniz's Principles. In Katherine Brading and Elena Castellani (Eds.), *Symmetries in Physics: Philosophical Reflections*. Cambridge: Cambridge University Press, pp. 289–308.

———————— (2006). Are Quantum Particles Objects? *Analysis*, 66, 52–63.

Slowik, E . (2004). Spacetime, Ontology, and Structural Realism. *International Studies in the Philosophy of Science*, 19 (2), 147–166.

———————— (2015). The 'Space' at the Intersection of Platonism and Nominalism. *Journal for General Philosophy of Science*, 46(2), 393–408.

Stachel, John. (2002). "The Relations between Things" versus "The Things between Relations": The Deeper Meaning of the Hole Argument. In David B. Malament (Eds.), *Reading Natural Philosophy: Essays in the History and Philosophy of science and Mathematics*. Chicago and La Salle, Illinois: open court, pp. 231–266.

Suppe, Frederick. (1989). *The Semantic Conception of Theories and Scientific Realism*. Urbana: University of Illinois Press.

Teller, Paul. (1991). Substance, Relations, and Arguments about the Nature of Space-Time. *Philosophical Review*, C, 362–397.

van Fraassen, Bastiaan. C. (1980). *The Scientific Image.* Oxford: Oxford University Press.

———————— (2006). Structure: Its Shadow and Substance. *British Journal for the Philosophy of Science*, 57, 275–307.

Wüthrich, Christian. (2009). Challenging the Spacetime Structuralist. *Philosophy of Science*, 76(5), 1039–1051.

———————— (2012). The Structure of Causal Sets. *Journal for General Philosophy of Science* 43(2), 223–241.

———————— (2018). The emergence of space and time. In Sophie Gibb, Robin Finlay Hendry and Tom Lancaster (Eds.), *Routledge Handbook of Emergence*. London: Routledge; 1 edition (March 18, 2019).