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THE 28-XIU CONSTELLATIONS IN EAST ASIAN CALENDARS AND ANALYSIS OF THEIR OBSERVATION DATES

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Abstract: In traditional calendrical systems of East Asia, the 28-Xiu (lunar lodge) constellations have the earliest origin to indicate seasons as 中星 (*Zhong xing*, mid-star), which were already completed before the fifth century BC in ancient China. However, it is shown that the significance of the 28-Xius had gradually faded away towards the modern ages, from the calendrical point of view. Nevertheless, star positions of the 28-Xius measured in old star maps and catalogs are vitally important from the astronomical viewpoint, in order to know correctly when and how the stars in these sources were observed. We have recently developed a statistical method for dating analysis of the 28-Xiu observation epochs, which is applicable to many historical star maps and catalogs in a unified way. In this paper we introduce summarized results on star maps in the Kitora tumulus of Nara in Japan, the Korean stone-inscribed star map of the fourteenth-century 天 象列次分野之圖 (*Cheonsang Yeolcha Bunyajido*), the Chinese star catalog in the *Shi-shi Star Manual* 石氏星経 *(Shi shi xingjing*) and Ptolemy's star catalog in the *Almagest*, thereby showing the validity of our approach.

Keywords: East Asia, calendars, 28-Xiu constellations, dating analysis, statistical method, star maps, Kitora tumulus, Korean stone-inscribed star map, the *Shi-shi Star Manual*, the *Almagest*

1 HISTORICAL BACKGROUND

The traditional luni-solar calendar systems used in East Asia originated from ancient China, and consist of various components and items, such as solar or seasonal dating, lunar phase dating, motions of the Sun, Moon and five planets (called *Qizheng* 七政 in Chinese, seven celestial objects), solar and lunar eclipse predictions, the seasonal variations the lengths of day and night, and so on. Among them, the 28-Xiu constellations (which are sometimes referred to as 'asterisms' in order to distinguish them from Western constellations) have the earliest origin. In ancient China they originally were the only means that could be used to recognise the seasons, through their meridian transits.

The 28-Xius 宿 (literally mean lunar lodges or mansions) are fundamental constellations located approximately along the Celestial Equator and the Ecliptic. They are believed to have been invented initially for the purposes of astrology, and at a later point in time were used as a reference-frame system on the sky to specify the positions of the stars, the Sun, the Moon and the planets.

The first part of this paper discusses how the 28-Xiu constellations evolved in the calendrical history of East Asia, and the second part briefly introduces the results of statistical estimations on when positions of the 28-Xiu stars were observed using data from a few representative historical star maps and catalogs. Further details of the latter section, including the methodology involved, will be published elsewhere.

2 HISTORICAL EVOLUTION OF THE 28-XIU CONSTELLATIONS IN EAST ASIAN CALENDARS

2.1 Earliest Use of the 28-Xius in Ancient Chinese Literature

The origin of the 28-Xiu constellations is considered to go back to four stars that were adopted as four seasonal indicators. The earliest names of these four stars appear in one of the oldest Chinese classics on history, *Shangshu Yaodian* 尚書堯典 (*The Canon of Yao*) as:

> 日中星鳥，以殷仲春，
> The day is at medium length and the star is *Niao* (鳥, Bird), thus this determines exact mid-spring (仲春),
>
> 日永星火，以正仲夏，
> The day is at its maximum and the star is *Huo* (火, Fire), thus this determines exact mid-summer (仲夏),
>
> 宵中星虛，以殷仲秋，
> The night is at medium length and the star is *Xu* (虛, Void), thus this determines exact mid-autumn (仲秋),
>
> 日短星昴，以正仲冬.
> The day is at its minimum and the star is *Mao* (昴, Pleiades), thus this determines exact mid-winter (仲冬).

The text has generally been regarded as a description of actual observations for the four stars (or an asterism in the case of the Pleiades) at their southern culmination at sunset or dusk. Using the above descriptions and the star names, a number of historians have attempted to estimate the observational epochs of the stars (Gaubil, 1723; Iijima, 1921; Noda, 1937; Sun and Kistemaker, 1997; Zhu, 1926). Their estimated observational dates range widely from c. 2300

BC down to c. 400 BC. This dating diversity seems to depend upon mainly when and how each author assumed the timing of the observations after sunset, although all the results unanimously point to very ancient times.

The earliest use of the complete set of the 28-Xiu names can be seen on the cover of a treasure box, which was excavated in 1977 from the ancient pit grave (17m x 21m) of King Zeng Hou Yi (曾侯乙) at Sui County, Hebei, in China (Sun and Kistemaker, 1997; Wang et al., 1979). Chinese literature tells us that King Yi governed the district of Zeng state during the early Zhanguo period (475–221 BC), and died in 433 BC. This material thus shows that the system and names of the 28-Xius had already been established before the fifth 5th century BC. Now let us compare the age of the 28-Xiu constellations with the oldest calendar systems adopted officially by the Chinese dynasties.

The first scientific calendar of ancient China was the *Taichu Calendar* 太初暦 (and later called the *Santong Calendar* 三統暦) which was used by the early Han Dynasty during 104 BC–84 AD. The next-oldest calendar was the *Sifen Calendar* 四分暦 of the late Han Dynasty, used from AD 85 to 263. Both *Calendars* are characterized by the fact that they adopted the length of the mean solar year as 365 and a quarter days, which is the same as that of the *Julian Calendar*. Therefore, it is understood that the origin of the 28-Xiu constellation system is much older than the earliest luni-solar calendrical theories developed in China.

2.2 Decline in the Importance of the 28-Xiu Constellations in Chinese Calendars

During the early Han Dynasty, more than 2000 years ago, meridian transits of the 28-Xius were almost exclusively used as seasonal indicators. They were called *Zhong xing* 中星, meaning mid-stars. But around the same era, more advanced seasonal indicators began to be used in Chinese calendars. They were called the 24-Qi 二十四気, some of which have very remote origins, and date back as far as the 28-Xiu constellations. The 24-Qis were indispensable in the luni-solar calendar. The reason is that, in luni-solar calendars, the leap months are needed every few years, to adjust the possible difference of more than 30 days between the natural seasons and the calendrical ones.

The first complete set of the 24-Qis appeared in the official book on the history of the Han Dynasty, namely the *Han shu* 漢書, which was compiled by the historian Ban gu 班固 (AD 32–92). After the Han period, calculation of the dates and times of the 24-Qis in the Chinese calendars become more sophisticated, because of refinements in the theories of the motion of the Sun and the Moon. On the other hand, in parallel with the development of the 24-Qis, the role of the 28-Xiu constellations gradually lost its significance, and in later times, meridian transit predictions of the 28-Xiu constellations became only nominal or symbolic, rather than practical.

Figure 1 shows an early example of the 28-Xiu description in the *Hou Han Shu* 後漢書 (*History of the Later Han Dynasty*), which covers the periods from the first to third century AD. The uppermost line of the table represents the 24-Qi timings, every 15 days from the winter solstice 冬至 until the vernal equinox 春分. The following lines show the ecliptical longitudes of the Sun, the gnomon length of the solar shadow, and lengths of day and night expressed by percentages. The last two lines list the meridian transits of the 28-Xiu constellations at dusk and dawn. Notice that the value for each 28-Xiu is given to a fraction of a Chinese degree.

Figure 2 is the calendrical information written in the Song Shi 宋史, the official record of the history of Song Dynasty (tenth to twelfth centuries), about one millennium after the times of the Later Han Dynasty shown in Figure 1. The lines from the top respectively show the lengths of day and night, and sun-rise and sun-set times calculated to the fraction (minutes) of a *ke* 刻, one hundredth of a day. One can see that the meridian transit predictions of the 28-Xiu constellations at dawn and dusk written in the last line are shown only in degrees and fractions are not given. This fact indicates that, during the Song Dynasty the 28-Xiu constellations had already lost their significance from the calendrical point of view, and they only played a formal or symbolic role.

However, from the astronomical viewpoint, the 28-Xiu constellations are still very important, because the analysis of their positions provides us with information on when those stars were observed. Thus in the next section, we focus on a statistical estimation of the observational times for the 28-Xiu constellations.

2.3 Characteristics of the 28-Xiu Constellations

It is no doubt that the number ‘28’ for the 28-Xiu constellations is attributable to the sidereal period of the Moon in 27.3 days, so this constellation system is likely to have initially been invented for the purpose of assigning daily positions of the Moon on the sky. Here using Figure 3 we first summarize some characteristics of quantities relating to the 28-Xiu constellations, preparing for the following analysis.

(1) Each Xiu 宿 has a standard (or reference) star called the *Ju*-star 距星 in it (each numbered star in Figure 3), from which the relative latitudes

Figure 1: The meridian transit of the 28-Xiu constellations given in the calendrical part of the *Hou Han Shu* (*History of the Later Han Dynasty*).

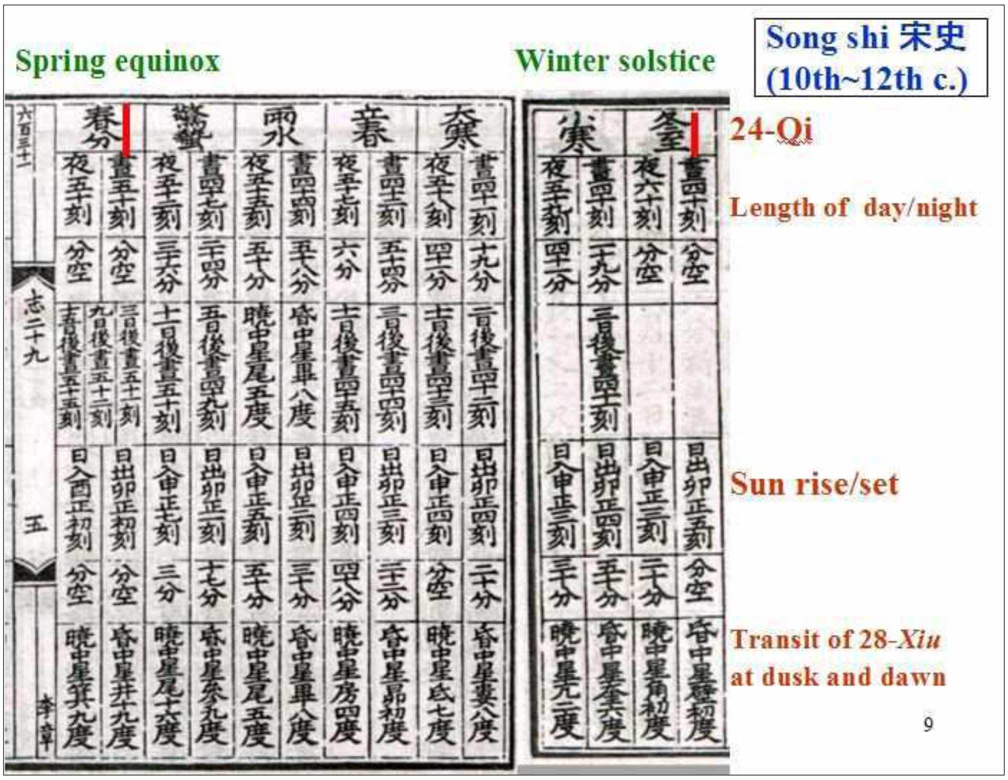

Figure 2: The Meridian transit of the 28-Xiu Constellations given in the calendrical part of *Song Shi* (*History of the Song Dynasty*).

and longitudes of other stars are measured.

(2) Coordinates of each <u>*Ju*</u>-star itself are given by:

(a) Quji-angle 去極度 (polar declination) = 90 deg – δ, and

(b) Xiu-angle 宿度 (adjacent relative R.A.) $\Delta\alpha_i = \alpha_{i+1} - \alpha_i$, where α and δ mean right ascension and declination. From equation (b), one can understand that Xiu-angle 宿度 gives no information about where and how each Xiu 宿 locates on the sky (see Figure 3).

3 STATISTICAL DATING ANALYSIS OF HISTORIC STAR MAPS AND CATALOGS

As we plan to publish details of statistical procedure for our dating analysis elsewhere, here we will restrict ourselves to mentioning only the motives for this work, an outline of our analytical method, and a summary of the main results obtained.

3.1 The Motives for our Study

In 1998, a star map was discovered on the ceiling in the Kitota tumulus (burial mound), Asuka village, Nara, Japan. From archaeological studies, the construction period of this tumulus was estimated to be between ~AD 680 and ~710. Although during the examination of the tomb made in 1998 image calibration of the star map

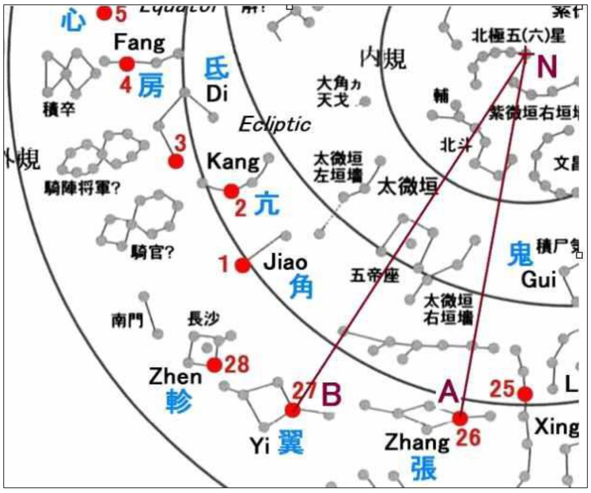

Figure 3: The Quji-angle 去極度 and Xiu-angle 宿度 for each 28-Xiu constellation. For example, the Quji-angle of No. 26 *Ju* star in the Zhang constellation 張 is the angle AN (= 90° – $\delta 26$), and the Xiu-angle of the same star is the angle ANB ($\Delta\alpha 26 = \alpha 27 - \alpha 26$), where N is the North Celestial Pole. This figure is taken from a part of Figure 5. Since in ancient Chinese astronomy, there was no concept of mathematical projections such as the Greek stereo projection, the Quji-angle and Xiu-angle are similar to a kind of modern polar coordinates.

was insufficient because a medical-type fiberscope with illumination was inserted into the dark tumulus, two Japanese astronomers, Hashimoto (1998) and Miyajima (1998) made a preliminary investigation and analysis of the star map.

From 2004, the Japanese Agency for Cultural Affairs (JACA) started repair and restoration work on the Kitora tumulus, to protect it from damage by mold and erosion. In parallel with this restoration work, precise digital images of the star map were produced, corrected for distortion effects due to camera lenses. In 2014 a group study of the Kitora star map began by the above JACA, the National Astronomical Observatory of Japan, and the NHK broadcasting company. The author had a chance to participate in this research project as a NAOJ-OB member. His primary interest was to estimate observational dates for the 28-Xiu constellations.

3.2 The Ceiling Star Map in the Kitora Tumulus

Figure 4 shows the overall appearance of the Kitora star map. Each star image is depicted with a circular gold plate that is 6mm in diameter and 0.2–0.3mm in thickness. The diameter of the Celestial Equator measures ~40cm, much smaller than those on similar stone-inscribed Chinese and Korean star maps. For unknown reasons, the circle of the Ecliptic is drawn in a completely wrong orientation. The break running horizontally in the middle was caused by a long-term shift in stone walls of the tumulus. Nevertheless, our measured positions (in celestial latitude and longitude) of the 28-Xiu stars were consistent with each other between the upper and lower parts of the stone break, so this did not influence the dating analysis of the times of the observations for the 28-Xiu stars. Each star image was connected by red lines, sometimes by double and fairly inexact lines. This fact strongly suggests that the majority of the star positions were copied free-hand from an original star map.

Figure 5 is a computer output reproduced from the Kirota photographs and digital data. Red circles with numbers are the *Ju*-stars (standards stars) of the 28-Xiu constellations, identified by the author after referring to Chinese star maps and the literature (Tsuchihashi and Chevalier, 1911). The identification of several stars is tentative, because their constellations are drawn with heavy distortion. In addition, data for the constellations 9Niu 牛, 10Nu 女 and 11Xu 虛 are missing because the original images were damaged.

3.3 Outline of the Dating Analysis: (1) The Least-Squares Approach

In this subsection we simply overview our statistical approach to estimate observational dates of stars that appeared in historical star maps and catalogs by taking the Kitora star map as an analysis example. Then the same method is applied to other historic materials, such as the Chinese star catalog in the *Shi-shi Star Manual*, the Korean stone-inscribed star map of the fourteenth century and Ptolemy's star catalog in the *Almagest*, to see how our approach works.

In order to do the dating analysis of measured star positions, first we assume the following two basic prerequisites, (a) precession is the only cause in positional changes of the 28-Xiu stars (the proper motion of each star is neglected because it is much smaller than precession), and (b) all the 28-Xiu stars were observed nearly simultaneously, say within a few years. Since the 28-Xiu constellations have the earliest original [missing text] the fundamental framework,

Figure 4: The Kitora tumulus star map. The equator diameter drawn in red measures 40.1cm.

It will be reasonable to adopt assumption (b). Here we used the precession theory by Simon Newcomb (Mueller, 1969), and star coordinates were taken from the *Bright Star Catalogue*, *Fourth Edition* (Hoffleit, 1982).

In order to reduce the influences of errors in the measured star positions, we used a statistical approach, that is, we estimated the time when the mean error (not errors of individual stars), E, became minimum, where E was expressed by the equation:

$E^2 = \Sigma(O - C)^2/n$

where O is the measured position of a star, C the one calculated by precession theory, and n the number of stars used in the analysis. As is well known, such an approach is called the least-squares method.

Measured quantities of the 28-Xiu stars are:

(1) the Xiu-angle 宿度 ($\Delta\alpha_i = \Delta\alpha_{i+1} - \alpha_i$), and

(2) the Quji-angle 去極度 ($= 90° - \delta$); these two quantities are independent each other.

In addition, we adopted the following quantity for our analysis:

(3) Positional shift (PS)

$PS^2 = (\delta_O - \delta_C)^2 + (\alpha_O - \alpha_C)^2\cos^2\delta$.

The reason for using this PS will be explained in the next paragraph. Then, the mean errors to be analyzed by the least squares method are the following three quantities:

(i) $E^2 = \Sigma(\Delta\alpha_o - \Delta\alpha_c)^2/n$
(ii) $E^2 = \Sigma(\delta_o - \delta_c)^2/n$
(iii) $E^2 = \Sigma PS^2/n$

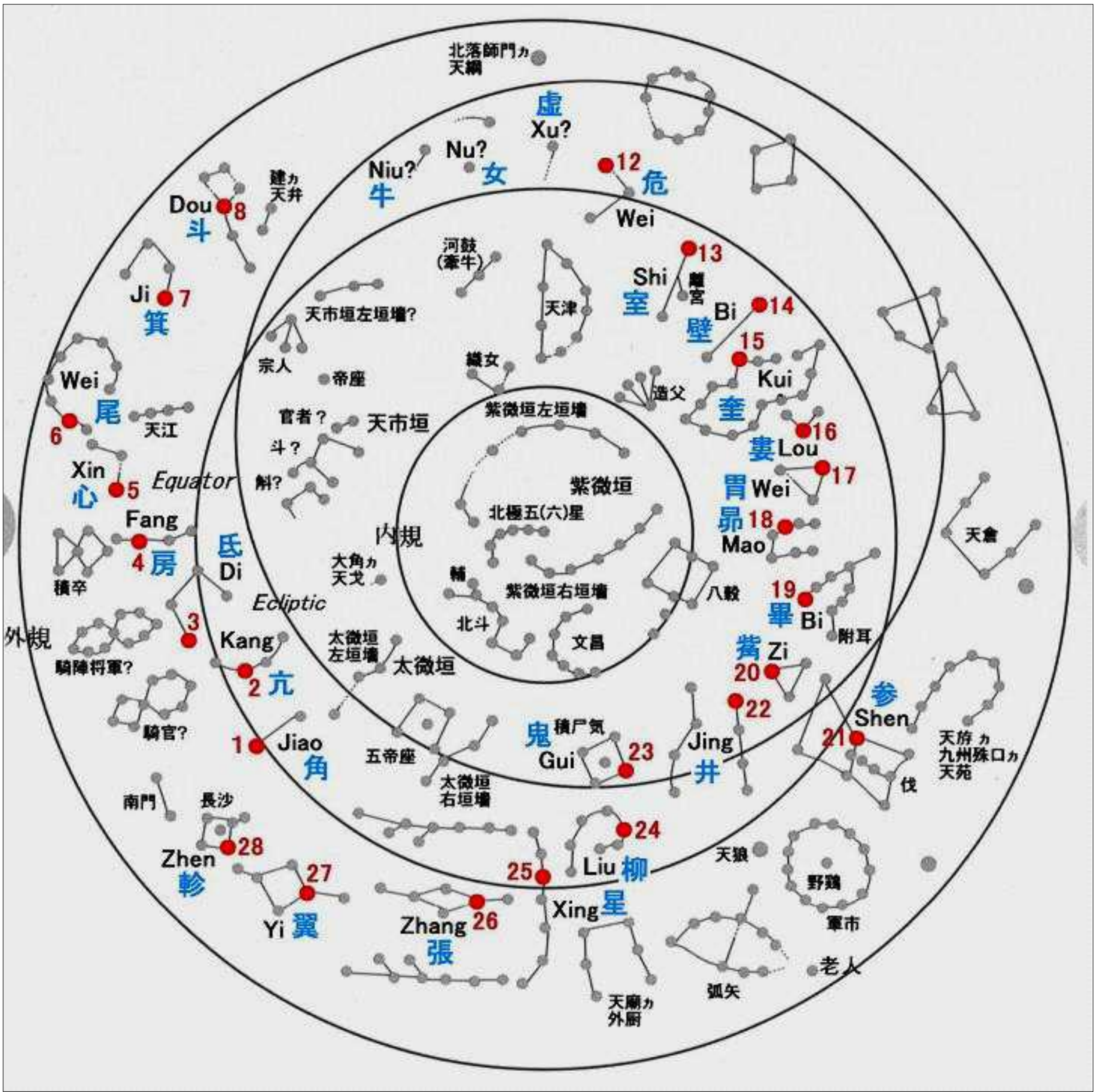

Figure 5: Kitora star map, computer-generated output from the calibrated digital data (Nara National Research Institute for Cultural Properties, 2016). Numbered stars represent each *Ju*-star, as the origin of each celestial longitude and latitude measurement.

Positions of the 28-Xiu stars observed by an armillary sphere have been expressed traditionally by the Xiu-angle (relative R.A.) and Quji-angle (90° – declination). In Figure 4 (Section 3.2), we have seen the evidence that the majority of the constellations in the Kitora star map were drawn freehand. In this case we believe that the analysis using the PS is more appropriate, rather than the longitude and latitude, because when we look at a shape in images or pictures it is very likely that our brain does not recognize the shape by resolving it into one-dimensional components such as longitude and latitude, but rather we perceive the shape two-dimensionally, as a whole.

However, when we attempt to adopt the PS for analysis, we encounter a serious difficulty, namely that the longitudes of the 28-Xiu stars are not measured from a common cardinal direction like the vernal equinox, so we can neither define the orientation of the polygon consisting of the 28-Xiu stars in space, nor calculate the value of $E^2 = \Sigma PS^2/n$. After trial-and-error, we eventually chose an approach that uses the direction of each star seen from the North Celestial Pole as the origin of longitude, instead of the vernal equinox. How successfully this approach works is explained later.

3.4 Dating Procedure Using the Longitude of each Xiu Star

For explanatory simplicity, here we consider a three-star model instead of the actual 28-Xiu stars. We also hypothesize that positions of all the stars in question were observed, say exactly in AD 300, and they do not include any errors at all. Then the behavior of the mean error, E, for each star as a function of time will look like the graphs shown on the left-hand side of Figure 6.

Next if small random errors are artificially added to the position of each star, the time when the E for each star becomes minimum will deviate from AD 300, such as T1 for star-1 as shown on the right hand side, T2 for star-2, etc. But we were able to demonstrate through extensive simulations that the mean of Ti, ΣTi/n still stays fairly close to the original value of AD 300. If 28 stars are used in such simulations with various hypothetical measuring errors, we could always recover the assumed observational epoch of AD 300.0 within uncertainties of ~10–20 years. Therefore, we made full use of this finding in the following analysis of the 28-Xiu data.

3.5 Outline of the Dating Analysis: (2) Interval Estimation

We adopt here a strict inference approach based on the modern statistics (initiated by, e.g., Ronald Fisher). As far as the author knows, such an approach has never been attempted before in the dating analysis of star observations. This use of the 28-Xius as a common 'probing tool' will open up a way to apply our method to a wide variety of historical star maps and catalogs in a unified way.

In modern statistics, the interval estimation of a 'population parameter' proceeds like this (e.g., Conover, 1971):

(1) Specify a confidence level β (e.g., 90%: we adopted the value throughout our work);
(2) With given measured data, obtain their mean and SD (this is called classical point estimation); and
(3) Using the above two estimates and β, calculate the confidence interval, through numerical techniques such as a random-noise simulation, the bootstrap method (invented by Bradley Efron in 1979), etc.

The random-noise simulation generates pseudo-random noises in a personal computer, to simulate the distribution of residuals (O-C) for measured data. Using them, the technique calculates least-squares estimates for the observation dates of the stars at least 100 times, and determines the confidence interval from all of the results. As easily anticipated, this procedure is very laborious and time-consuming. Because of the inconvenience, more efficient approaches have been devised. Among them, we have found that the bootstrap method is best for our purposes.

The bootstrap method is conducted like this from sampled (measured) data: we pick at random 100–150 sets of pseudo data allowing re-sampling (this allows us to pick the same measured sample more than once), and determine the confidence intervals through the boot-strap procedures, which have several options for calculating the confidence interval (Chernick 1999). We found that the bootstrap method was generally 50–100 times more efficient than the random noise simulations, and could still provide accurate results.

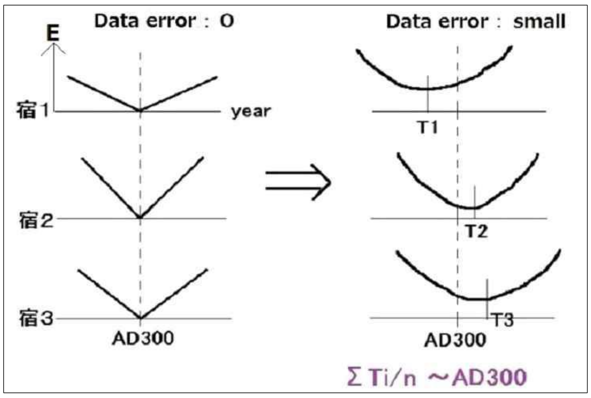

Figure 6: Schematic graphs for the dating procedure, using the longitude of each Xiu star.

3.6 Results of the Kitora Dating Analysis

Table 1 shows Quji angles (90° – δ) and Xiu angles (Δα) of 25 measured *Ju* stars for the Kitora star map (see Figure 5). Note that No. 9, 10, and 11 stars could not be measured due to image loss. The third column is star identification, using the names in Bayer's *Uranometria*.

The graph (a) in Figure 7 shows behavior of the mean error $E^2 = \Sigma(\Delta\alpha_o - \Delta\alpha_c)^2/n$ for the Xiu-angle as a function of year. As a result of a least squares fitting, we obtained 79 BC ± ~450 corresponding to the minimum E (see Section 3.3). However, if we look at the confidence inter-

Table 1: Measured positions of Kitora 28-Xiu stars and estimated observational dates with each Xiu star based on their longitudinal origins. For the meaning of each column entry see the text. **[Replace table: quality very poor]**

		Bayer	90−δ	δ		Δα	
No.	28宿名	距星	去極度	赤緯	相対赤経	宿度	推定年
1	角	α Vir	92.2	2.2	0.0	11.0	−64
2	亢	κ Vir	84.6	−5.4	11.0	9.0	−23
3	氐	α Lib	95.5	5.5	20.0	15.0	−82
4	房	π Sco	104.3	14.3	35.0	6.8	−255
5	心	σ Sco	112.2	22.2	41.8	7.8	−76
6	尾	μ Sco	126.3	36.3	49.6	19.1	8
7	箕	γ Sgr	117.4	27.4	68.7	12.9	19
8	斗	φ Sgr	119.5	29.5	81.6	54.4	150
9	牛	β Cap					
10	女	ε Aqr					
11	虚	β Aqr					
12	危	α Aqr	98.1	8.1	136.0	16.1	−79
13	室	α Peg	83.6	−6.4	152.1	16.1	−137
14	壁	γ Peg	81.7	−8.3	168.2	5.1	−118
15	奎	ξ And	67.3	−22.7	173.3	19.8	42
16	婁	β Ari	71.4	−18.6	193.1	7.9	−102
17	胃	35Ari	73.1	−16.9	201.0	11.9	−61
18	昴	17Tau	61.7	−28.3	212.9	15.7	−274
19	畢	ε Tau	68.3	−21.7	228.6	17.2	−105
20	觜	λ Ori	67.7	−22.3	245.8	1.9	−2
21	参	δ Ori	94.6	4.6	247.7	8.0	−69
22	井	μ Gem	64.8	−25.2	255.7	29.6	165
23	鬼	θ Cnc	63.7	−26.3	285.3	4.3	−185
24	柳	δ Hya	77.7	−12.3	289.6	16.1	−88
25	星	α Hya	86.5	−3.5	305.7	10.3	−9
26	張	υ Hya	94.6	4.6	316.0	23.4	−313
27	翼	α Crt	109.1	19.1	339.4	11.9	
28	軫	γ Crv	114.4	24.4	351.3	8.7	−279

val (for β = 90%) reaching almost 900 years for the full span (a horizontal bar attached below to the graph), one can understand that this estimate does not actually give any useful information in the dating analysis. This is attributable to the fact that the Xiu-angle is very insensitive to the time variation. Figure 7(b) is a similar graph for declination. The minimum mean error $E^2 = \Sigma(\delta_o - \delta_c)^2/n$ provides us with the time 73 BC, a fairly close value to that for the Xiu-angle, 79 BC. But the uncertainty interval for this estimate is still large, at about ±~300 years.

Figure 7(c) is the result of a least-squares fit for the mean error $E^2 = \Sigma PS^2/n$ with respect to the 24Liu 柳 stars. The confidence interval of β = 90% was calculated by a pseudo-random num-

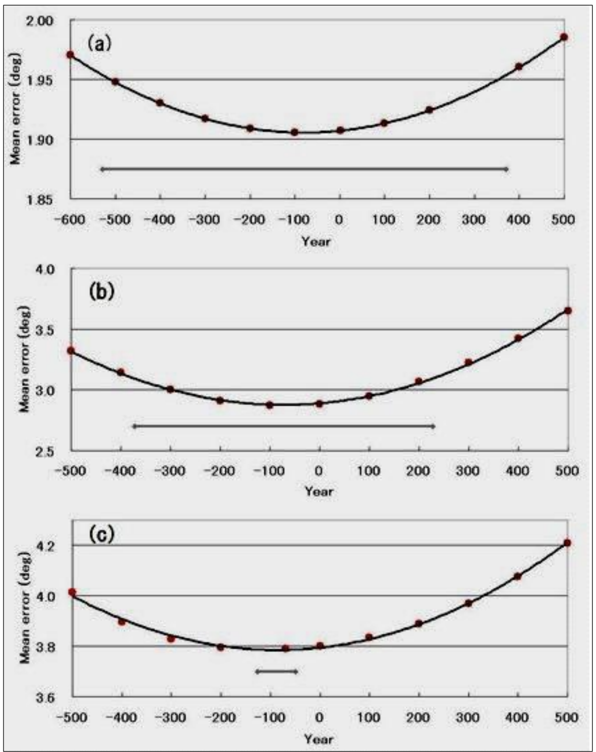

Figure 7: Time variations of mean errors Es for (a) Xiu-angle (Δαi), (b) declination, and (c) the PS with respect to the 24Liu 柳 star. A horizontal bar attached to below each graph represents its confidence interval.

ber simulation. The most likely observation time was calculated to be 88 ± ~40 BC. Notice the smallness of the confidence interval, compared with those for the previous Xiu-angle and Quji-angle. But if we take No.8 Dou 斗 star as the origin of the longitudes for all the 28-Xiu stars we get an observational epoch of AD 150, which is very different from that for the 24Liu 柳 stars, although the confidence interval is almost the same at about ±~40 years.

Therefore, we estimated all of the observation epochs when each Xiu star was taken to be the origin of the longitude, and the results are shown in the last column in Table 1. One can see that the estimated times scatter over a considerably wide range. Their mean and standard deviation were found to be 80 BC and 122 years. These values correspond to the situation explained at the bottom in the right-hand part of Figure 6.

Hence using these point-estimation results and the bootstrap method, we eventually estimated the final observation epoch with a confidence interval of β = 90%, as [123 BC, 39 BC], or 80 BC ± ~40 year (mean residual = 3.8°).

The following is a summary of the dating analyses for the Kitora star map (using a confidence level of β = 90%):

- Xiu-angle 宿度： 79 BC ± ~450 (simulation)
- Declination 赤緯： 73 BC ± ~300 (simulation)
- Positional Shift PS (relative R.A.):
  24Liu柳： 88 BC ± ~40 (simulation)
  8Dou斗： AD 150 ± ~40 (simulation)
- Final result using the bootstrap method for all 28-Xiu stars:
  [123 BC, 39 BC], or 80 BC ± ~40 (mean residual = 3.8°).

3.7 Results of Dating Analysis for the 28-Xiu Data in *Shi-shi Xingjing* 石氏星経

In the previous section the dating analysis of the Kirota star map provided us with a very old observation epoch, about 900 years before the construction of the Kitora tumulus. In those ancient times, Japan was still in a very primitive state. It was not a unified country, but instead was divided into about 100 local tribes. So it is very likely that the original data used to create the Kitora star map came from China. In order to investigate this hypothesis, we analyzed the typical 28-Xiu data of the ancient China, using the *Shi-shi Xingjing* (*Mr. Shi's Star Manual*), which is cited in the Kaiyuan Zhanjing 開元占経 (AD 714–725) by an Indian astronomer who worked in China during the Tong Dynasty (Ren, ca.1993). The data are believed to be the oldest that were obtained by observing with the aid of an armillary sphere.

The following is summary of the dating analyses for the 28-Xiu data in the *Shi-shi Xingjing* with a confidence level of β = 90% (see Table 2):

- Xiu-angle 宿度: AD 134 ± ~270 (simulation)
  Mean residual = ~0.5° (for the Kitora star map ~3.0°)
- Declination 赤緯: AD 21 ± ~250 (simulation)
- Positional Shift PS (relative R.A.):
  e.g., 24Liu 柳: 59 BC ± ~20 (simulation)
- Final result using the bootstrap method for all the 28-Xiu stars:
  [65 BC, 43 BC], or 54 BC ± ~11.

Table 2. 28-Xiu data in *Shi-shi Xingjing* 石氏星経 (*Shi's Star Manual*) cited in the *Kaiyuan Zhanjing* 開元占経 (AD 714–725).

宿	1 角	2 亢	3 氐	4 房	5 心	6 尾	7 箕	8 斗	9 牛	10 女	11 虚	12 危	13 室	14 壁
宿度	12	9	16	5	5	18	11	26.25	8	12	10	17	16	9
去極度	91		94	108	109	124	118	116	110	106	104	99	85	86
宿	15 奎	16 婁	17 胃	18 昴	19 畢	20 觜	21 参	22 井	23 鬼	24 柳	25 星	26 張	27 翼	28 軫
宿度	16	12	14	11	16	2	9	33	4	15	7	18	18	17
去極度	70	80	72	74	78	84	94.4	70	68	77	90	97	99	99

3.8 Results of Dating Analysis for the 28-Xiu Data on the Korean stone-inscribed Star Map 天象列次分野之圖 (*Cheonsang Yeolcha Bunyajido*)

Next we applied our bootstrap method to the well-known Korean stone-inscribed star map 天象列次分野之圖 (*Cheonsang Yeolcha Bunyajido*) which dates to AD 1395. This stone inscription consists of a star map in the upper part, and astronomical explanations in the lower part.

First, we tried to measure the 28-Xiu stars in the star map, but we could not identify several *Ju*-stars (reference stars) of the 28-Xius, perhaps because of inaccuracy of their positions. Therefore, instead, we mainly analyzed the data provided in the table in the lower part.

Here is a summary of dating analyses for the 28-Xiu data in the Korean stone-inscribed star map (with a confidence level of β = 90%):

- Declination 赤緯 (map): 53 BC ± ~100
- 赤緯 (table): 51 BC ± ~100
- Positional Shift PS (w.r.t. 24Liu 柳): 71 BC ± ~30
- Final result using the bootstrap method for all 28-Xiu stars: [78 BC, 54 BC], or 66 BC ± ~15.

3.9 Results of Dating Analysis for the 28-Xiu Stars in the *Almagest*

In order to confirm the validity of our approach using the 28-Xiu stars and the bootstrap method, we attempted to analyze star positions included in the *Almagest* by Claudius Ptolemy (Ptolemaeus, ca 90–ca168). As is well known, Ptolemy is *the* representative astronomer of ancient Greece, and is most famous for his book the *Almagest* (*Mathematike Syntaxis*), a grand compilation of Greek astronomy based on Geocentrism. In Chapters 2–4 in Book 7 in the *Almagest*, there is a star catalog consisting of 48 constellations (1028 stars).

Ptolemy maintains in his book that stars in the catalog were observed by himself in AD 138–139 from Alexandria, Egypt, although he refers also to Hipparchus' (ca 190 BC–ca 120 BC) systematic observations of stars in 128 BC from the island of Rhodes (Grasshoff, 1990). Hence we carried out a dating analysis of Ptolemy's catalog by picking out those stars that corresponded to the Chinese 28-Xiu stars. Because the positons of the *Almagest* stars were measured from the vernal equinox, their analysis was much easier and straightforward than for the Chinese 28-Xiu stars.

For Xiu-angle 宿度 data, our simulation method gave the observation epoch as AD 13 ± ~200. Thus, one can understand that the Xiu-angle is a very insensitive quantity for the precessional change of positions, and so is inappropriate for the dating analysis of star maps and catalogs. On the other hand, the bootstrap method using the right ascension for all the 28-Xiu stars provided us with the following observation epoch: [AD 63, AD 84], or AD 74 ±~10. But Ptolemy's observation date mentioned in the *Almagest* is AD 138–139, which is obviously inconsistent with our result. So is our approach wrong?

In fact, there have been long-standing suspicions about Ptolemy's alleged observing date. Even in the early seventeenth century, Tycho Brahe (1546–1601) expressed doubt about Ptolemy's star observations when he compared data provided by his own observations with those derived from the *Almagest*. Then the French astronomer J.B.J. Delambre (1749–1822) voiced a similar opinion in the early nineteenth century on the basis of modern observations. More recently, a particularly aggressive accusation was made in the book, *The Crime of Claudius Ptolemy*, written by the US astronomer R.R. Newton (1977). His accusation can be summarised as: It is likely that Ptolemy did not observe the majority of stars in his catalogue himself. He simply transferred the star coordinates in Hipparchus' catalog to his own time, but used the wrong precession rate (1 degree/century, which was obtained by Hipparchus, instead of the correct rate of 1 degree/72 years), and pretended that he had observed all of the stars himself.

In order to determine whether this accusation is justified or not, let us make a simple calculation. The time difference between Hipparchus and Ptolemy is 128 + 139 = 267 years, and if we get back our obtained observation epoch [63 AD, 84 AD] to the past, with the wrong Hipparchus' precession rate of 1°/century, we arrive at [129 BC, 110 BC]. Note that this calculated interval overlaps the period of Hipparchus' observations, 128 BC (see the bottom of Figure 8). So this result confirms Newton's accusations. Further-more, it shows the validity of our methodology that was used in analysing the 28-Xiu stars.

3.10 Comparison of Observation Dates for Ancient Star Maps and Catalogs

First, here we compare the three dating analyses calculated in previous subsections for the 28-Xiu stars in the Kitora star map, the Korean stone-inscribed star map and the *Shi-shi Star Catalog*.

The three upper horizontal bars shown in Figure 8 represent confidence intervals (or uncertainty ranges). Overlapping of the three confidence intervals at 70 BC~50 BC strongly suggests the existence of a common data source. And since materials older than the *Shi-shi Star Catalog* are unknown, this catalog itself is likely to have been the original source. In addition, we note that the Kitora star map is the oldest existing one that includes scientific information—in the sense that the analysis of this star map gave us a reasonable observational epoch.

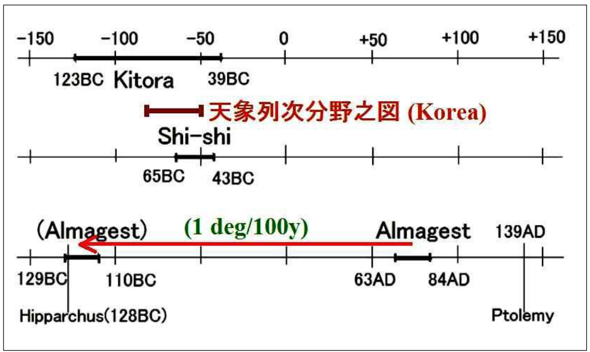

Figure 8: Comparison of observation dates for the 28-Xiu stars in old star maps and catalogs of East Asia and the Almagest.

Just for reference, at the bottom of Figure 8, our estimated confidence interval for the *Almagest's* observation time (shown with parentheses) is given, along with that calculated simply following Ptolemy's allegation. It is interesting to recognize that the oldest positional observations of the 28-Xiu stars were independently performed nearly in the same era in both Greece and China.

Finally, in order to check the accuracy of our dating analysis, we applied our method to two sets of star data from much more modern observations. They are the 28-Xiu data by Guo Shoujing 郭守敬 (AD 1231–1316), the Chinese astronomer and hydrographer of the Yuan Dynasty. Guo observed the 28-Xius in 1276 with an armillary sphere that he designed (Pan, 1989). Our bootstrap dating analysis of the 28-Xiu stars using the Xiu-angle 宿度 data provided us with Guo's observing date with a confidence interval of [1252, 1308] or 1280 ± ~25 (mean residual: 0.21°). Our dating analysis shows good agreement between the literature and derived observation times.

The second example is taken from the data by Ulugh Beg. Ulugh Beg (AD 1394–1449) was the Timurid ruler, astronomer, mathematician and Sultan of Uzbekistan. He is famous for building the enormous Ulugh Beg Observatory in Samakand between 1424 and 1429, and in 1437 for compiling the *Zij-i Sultani* (the great star catalog), which includes 994 stars (Yabuuchi, 1993).

Our dating analysis of the 28-Xius from the catalog gave 1450 ± 15 (mean residual: 0.4°) for the R.A. data, and 1425 ± 25 (mean residual: 0.3°) for the declination data. Also in this case, we find there is good agreement between observation and our analysis.

From the two examples above, one can see that our dating procedure allows us to estimate the observed epochs of historical materials within 10–20 years. Details of the analysis will be published elsewhere.

4 SUMMARY AND CONCLUSIONS

(a) The role of the 28-Xiu 宿 constellations in East Asian calendars became less significant with the passage of time, and were merely symbolic by modern times.

(b) A new statistical approach to estimate the observation times of the 28-Xius from their celestial coordinates was proposed. It can be applied to historical star maps and catalogs in a unified way.

(c) Dating analysis of the 28-Xius in the Kitoro star map, the Korean stone-inscribed star map and the *Shi-shi Star Catalog* all showed overlapping observation times (~70 BC–~50 BC), strongly suggesting the existence of a common data source (probably the *Shi-shi Star Catalog* itself).

(d) Our analysis of the *Almagest* stars supports the century-long suspicion that Ptolemy did not observe those stars by himself, but simply adopted the star coordinates observed by Hipparchus and used a wrong precessional rate.

(e) It is expected that our dating procedure using the 28-Xiu stars can reveal the observation times used for historical star maps and catalogs generally within uncertainties of 10–20 years.

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