Geometric invariants associated with the discrete series representations

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The discrete series representations form a family of square-integrable irreducible unitary representations of a real semisimple Lie group $G$. They are fundamental in representation theory and harmonic analysis on $G$. In this talk, we discuss some geometric invariants: associated variety, associated cycle and isotropy representation, for the discrete series representations. These invariants play an essential role to understand irreducible representations of $G$ in connection with nilpotent orbits in the Lie algebra. The talk will start with explaining the above geometric invariants explicitly by using three dimensional simple Lie group $SU(1,1)$. Then, the associated variety and isotropy representation for the discrete series will be described by means of the principal symbol mapping of certain differential operator of gradient-type on the Riemannian symmetric space $G/K$. We may discuss also a relationship between the isotropy representation and the moment map defined on a conormal bundle on the flag variety.