Inferentialism about logic contradicts the thesis of meaning as use*

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1 Overview

In this article I argue that inferentialism about logic, which is roughly the same as what is called proof-theoretic semantics, contradicts the thesis of meaning as use, although many proponents of inferentialism about logic such as Belnap (1962) and Dummett (1991) have considered that it is in harmony with the thesis. I mean by the thesis of meaning as use the following: The meaning of a word in a language is its use in the language. I assume throughout the article that the thesis of meaning as use is true. This thesis is said to originate in the later Wittgenstein's philosophy.

Inferentialists about logic such as Dummett (1991), Read (2004), and Murji and Hjortland (2009) claim that the meaning of a logical constant in a logic can be given by some or all of the rule(s) of inference governing the logical constant in a deductive system for the logic (usually, its introduction rule(s) in the standard system of natural deduction). In the following two sections, I shall show that the inferentialist claim leads to a contradiction via the thesis of meaning as use.

As for when deductive systems represent the same logic, the following convention is employed in the article: Deductive systems represent the same logic (or are equivalent) if and only if they have the same deducibility relation, where it is supposed that those deductive systems have the same

*This manuscript will be revised in the near future. Any (critical) comments and questions are highly welcom. Note that I do agree with several forms of inferentialism about logic, though I disagree with the form of it discussed in this manuscript. For example, I agree with the claim that the meaning of a logical constant in a logic (resp. deductive system) is its use in the logic (resp. deductive system), which seems to be a consequence of the thesis of meaning as use.

language. According to Straßburger (2007), this view on the equivalence between deductive systems is considered as folklore among logicians. Indeed, for example, various deductive systems for classical logic or intuitionistic logic represent the same logic; any deductive system for classical logic does not represent the same logic as any deductive system for intuitionistic logic.

2 The first argument

Let us consider the meaning of a logical constant 'or' in classical logic. According to inferentialism about logic, the meaning of 'or' in classical logic is given by some rule(s) governing it in a deductive system for classical logic. However, there are many deductive systems for classical logic, which have the same deducibility relation. Which deductive system then determine the true meaning of 'or' in classical logic in the inferentialistic way?

One cannot say that the meaning of 'or' in the sense of inferentialism about logic is the same in all deductive systems for classical logic. This is just because meaning is use. For instance, the use of 'or' in the standard system of natural deduction for classical logic is different from that in the standard Hilbert-style system for classical logic, since the proof procedures of the two systems are different (recall that some formulae containing 'or' can be much more easily proved in the system of natural deduction than in the Hilbert-style system). This argument holds if either the system of natural deduction or the Hilbert-style system is replaced with the standard sequent calculus for classical logic. Indeed, any two of the proof procedures of the three systems are significantly different.

Consequently, it follows from the thesis of meaning as use that those deductive systems for classical logic give 'or' different meanings (note that the same thing holds true of the other logical constants in classical logic). Now, they are legitimate deductive systems for classical logic and represent the same logic. In addition, we usually accept that there is only one meaning of a logical constant in classical logic. Hence, the meaning of a logical constant in classical logic cannot be given by any rule(s) of inference governing it in any particular deductive system. Inferentialism about logic thus leads to a contradiction via the thesis of meaning as use.

In a nutshell, a logic is not a particular deductive system, whence, by the thesis of meaning as use, the meaning of a logical constant in a logic cannot be given in any way which essentially depends on a particular deductive system, since its use varies from one deductive system for the logic to another. Note that the rules of inference in a deductive system essentially depend on the deductive system and are not invariant with respect to the equivalence between deductive systems.

Logic is like manifold in differential geometry and then a deductive system for a logic is like an atlas on a manifold, which is a collection of local coordinate systems which covers the manifold. A manifold can conveniently be studied by means of an atlas on it such as a so called good atlas (the notion of good atlas is mathematically defined). Likewise, a logic can conveniently be studied by means of a deductive system for it such as a cut-free sequent calculus. However, just as a manifold is not an atlas (note that there are different atlases representing the same manifold), so a logic is not a deductive system (note that there are different deductive systems representing the same logic), which is just a way to represent the logic. As a manifold can be represented using the notion of structure sheaf rather than that of atlas, there are many other ways to represent a logic, including semantical, algebraic and topological ones, which are all legitimate.

As is often remarked in an introduction to manifold theory, we have to distinguish carefully between properties of an atlas on a manifold, which may be extrinsic to the manifold, and properties of the manifold itself, which are intrinsic to the manifold and are independent of any particular choice of atlas. Likewise, we have to be careful to distinguish between properties of a logic such as its consistency and properties of a deductive system for the logic such as the cut-eliminability of a sequent calculus for the logic, which is just a way to establish the consistency or other properties of the logic. The meanings of logical constants in a logic concern properties of the logic itself, not properties of a particular deductive system for the logic, whence they cannot be given by any rules in a particular deductive system, which concern properties of the particular deductive system, not properties of the logic itself.

3 The second argument

Another argument refuting the inferentialist claim proceeds as follows. Let us consider the meaning of a logical constant 'or' in intuitionistic logic. According to inferentialism about logic, the meaning of 'or' in intuitionistic logic is given by some rule(s) governing 'or' in a deductive system, which is usually considered by inferentialists about logic as the standard system of natural deduction for intuitionistic logic.

Let us recall that the standard system of natural deduction for classical logic has the same rules of inference for 'or' as those of the standard system of natural deduction for intuitionistic logic. This implies that if the meaning of ‘or’ in intuitionistic logic is given by the rules for ‘or’ in the corresponding system of natural deduction and if the meaning of ‘or’ in classical logic is given by the rules for ‘or’ in the corresponding system of natural deduction, then the meaning of ‘or’ in intuitionistic logic is the same as the meaning of ‘or’ in classical logic.

However, the meaning of ‘or’ in intuitionistic logic is shown to be different from that in classical logic by the thesis of meaning as use. For example, as is well known, the disjunction property holds of ‘or’ in intuitionistic logic, while it does not hold of ‘or’ in classical logic. Thus, the use of ‘or’ in intuitionistic logic is different from that in classical logic and therefore the meanings of ‘or’ in the two logics are different by the thesis of meaning as use. It has already been shown that, according to inferentialism about logic, the meanings of ‘or’ in the two logics become the same. Hence, inferentialism about logic leads to a contradiction via the thesis of meaning as use.

In a nutshell, by the thesis of meaning as use, the meaning of a symbol in a deductive system is not necessarily given only by the rule(s) governing it in the deductive system, since in general some larger fragment of the deductive system, which can be the whole system (especially from the holistic point of view), is needed to fully determine the use of the symbol in the deductive system.

It certainly follows from the thesis of meaning as use that the meaning of a symbol in a deductive system is given by its use in the whole deductive system or some sufficiently large fragment of the system. However, since the use of a logical constant changes depending on a variety of deductive systems containing it even if they represent the same logic, which has already been argued in Section 2, this does not imply that the meaning of a logical constant in a logic can be given by its use in the whole of a particular deductive system or some fragment of it.

I finally remark that the difference between properties of a logic and properties of a deductive system for the logic is important also in the second argument, the reason for which is that the disjunction property is an intrinsic property of a logic. Indeed, the set of all valid formulae in a logic determines whether or not the logic enjoys the disjunction property.

4 Concluding remarks

I have argued that inferentialism about logic leads to a contradiction via the thesis of meaning as use. Inferentialism about logic might succeed in giving the meaning of a symbol in a deductive system, but it fails to give the meaning of a logical constant in a logic.

I consider that the present formulation of inferentialism about logic cannot be maintained, but there is still a possibility that it can be fixed (1) by weakening the notion of use in such a way that the equivalence between deductive systems preserves the use of logical constants in the weakened sense, where note that the equivalence between deductive systems do not necessarily preserve the use of a logical constant (for instance, as already argued in Section 2, the major three deductive systems for classical logic are equivalent, while the uses of 'or' in the three systems are different), and (2) by reformulating the major claim of inferentialism about logic as follows: The meaning of a logical constant in a logic can be given by its use in such weakened sense in the whole of a deductive system for the logic.

It is crucial that the use of a logical constant in such weakened sense does not vary depending on different deductive systems as far as they represent the same logic (i.e., it is an invariant with respect to the equivalence between deductive systems), which is a consequence of (1). Thus, for example, the use of 'or' in such weakened sense is the same among all the deductive systems for classical logic and so is the meaning of 'or' if the thesis of meaning as use in such weakened sense is true.

However, it appears that such weakened notion of use is obscure at present and that the use of the word 'use' in such weakened sense deviates from the use of the word 'use' in our ordinary language because of its property described in (1) above. Indeed, it would be an artificial notion invented in order to save inferentialism about logic from the crisis. Moreover, whereas the thesis of meaning as use is plausible, it is dubious whether the thesis of meaning as use in such weakened sense is true, without which it seems difficult to justify the reformulated claim of inferentialism about logic in (2).

I wish that this article would contribute to our understanding of the nature of logic.

References

[1] Belnap, N. D. 1962. Tonk, plonk and plink. *Analysis* 22: 130-134.

[2] Dummett, M. 1991. *Logical Basis of Metaphysics*. London: Duckworth.

[3] Murji, J., Hjortland, O. T.. 2009. Inferentialism and the categoricity problem: reply to Raatikainen. *Analysis* 69: 480-488.

[4] Read, S. 2004. Identity and harmony. *Analysis* 64: 113-119.

[5] Straßburger, L. 2007. What is a logic, and what is a proof? In *Logica Universalis: Towards a geneneral theory of logic second edition*, 135-152. Basel: Birkhäuser Verlag.