# Truth, ω-inconsistency and Harmony

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Truth theories like FS, Friedman-Sheared's truth theory, have two rules: the T-in rule, P infers TP, as the introduction rule of the truth predicate T and the T-out rule, TP infers P, as the elimination rule of it [H]. They look like the introduction rule and the elimination rule of a new logical connective T. From proof theoretic semantics (PTS) viewpoint, one can think that the truth predicate is a logical co nnective which is governed by these two rules; this has been suggested by some proof theoretic sema ntics people [Hjo]. From this viewpoint, the nature of truth is like deflationist's nature of truth and on e of the most important properties of truth is that the truth predicate does not disturb the traceability of the argument from the premises to the conclusion.

However, a crucial problem has been known: any criteria to be a logical connective, known as a "har mony" of the introduction rule and the elimination rule, is not satisfied because of the ω-inconsistenc y of FS. For example, Belnap and Dummett said the harmony means that adding the connective is a conservative extension to a proof system results in a conservative extension. However, in FS case, a dding T to PA results FS and FS is ω-inconsistent (any of its model must contain a non-standard natu ral number) though PA is ω-consistent. This seems to violate the conservativity.

Such ω-inconsistency is caused by the fact that the truth predicate enabled us to define paradoxical f ormulae of seemingly infinite-length. These formulae can be regarded as coinductive objects, or pote ntially infinite objects, in terms of computer science. The reason of the failure of the harmony is that the criteria of harmony are defined not for coinductively defined paradoxical formulae but for induct ively defined formulae. In this presentation, we examine how we can extend the criteria for harmony for coinductive formulae along the line of meaning explanation of coinductive objects in computer s cience [Sz], and investigate the nature of truth with respect to a constructor of a coinductive data typ e.

[H] Halbach, Volker, 2011, Axiomatic Theories of Truth, Cambridge University.

[Hjo] Ole Hjortland, 2012, Harmony and the Context of Deducibility, in Insolubles and Consequenc es: Essays in honour of Stephen Read, Catarina Dutilh Novaes and Ole Thomassen Hjortland (eds.), College Publications

[Setzer] Anton Setzer, 2012, Coalgebras as Types determined by their Elimination Rules, in Epistem ology versus Ontology.