Categorical Approach to Conflict Resolution: Integrating Category Theory into the Graph Model for Conflict Resolution

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Abstract—This paper introduces the Categorical Graph Model for Conflict Resolution (C-GMCR), a novel framework that integrates category theory into the traditional Graph Model for Conflict Resolution (GMCR). The C-GMCR framework provides a more abstract and general way to model and analyze conflict resolution, enabling researchers to uncover deeper insights and connections. We present the basic concepts, methods, and an application of the C-GMCR framework to the well-known Prisoner's Dilemma and other representative cases. The findings suggest that the categorical approach offers new perspectives on stability concepts and can potentially lead to the development of more effective conflict resolution strategies.

Index Terms—category theory, conflict resolution, Graph Model for Conflict Resolution

I. INTRODUCTION

Category theory is a branch of abstract mathematics that provides a powerful and unifying framework for understanding and analyzing various mathematical structures. It was introduced by Eilenberg and Mac Lane [1] as a means to study algebraic topology. Since then, category theory has grown to become a fundamental tool in many areas of mathematics, including algebra, geometry, logic, and computer science. At its core, category theory deals with objects and the relationships (called morphisms or arrows) between them. A category consists of a collection of objects and a set of morphisms that satisfy specific axioms. The morphisms capture the structure-preserving relationships between objects, making category theory particularly suited for studying and comparing different mathematical structures.

One of the key concepts in category theory is the notion of a functor, which is a mapping between two categories that preserves their structure. Functors allow mathematicians to relate and compare different categories, leading to deeper insights and understanding of the underlying structures. Another important concept is natural transformations, which describe relationships between functors. Natural transformations are used to express the idea of "naturality" or coherence in mathematical structures and provide a way to study how different constructions in mathematics relate to each other. In recent years, category theory has gained increasing attention and relevance in various fields, especially in computer science and artificial intelligence. It has been applied to programming languages, type theory, semantics, and the design of functional programming languages like Haskell. Moreover,

category theory has played a significant role in the development of homotopy type theory, a promising new foundation for mathematics that unifies aspects of logic, topology, and algebra. The growing interest in category theory is due in part to its ability to provide a unifying language and framework for diverse areas of mathematics and science. As new connections and applications continue to be discovered, category theory is expected to remain an influential and essential tool in the ongoing development of mathematical understanding.

GMCR [2], [3] is a decision-analytic approach for modeling and analyzing complex conflicts among multiple decision-makers (DMs). It provides a systematic framework for representing the preferences, options, and strategic interactions among DMs involved in various conflicts, including environmental disputes, international trade negotiations, and resource management problems [4]. Recent research on GMCR has focused on refining its theoretical foundations, improving its computational efficiency, and exploring its applications to various domains, including the development of advanced software implementations for automated conflict analysis and visualization [5], [6]. Moreover, GMCR research has integrated other mathematical and computational methods, such as game theory, optimization algorithms, and artificial intelligence techniques, to enhance its modeling capabilities and analytical power [7]. This interdisciplinary research has expanded the application of GMCR in addressing increasingly complex and dynamic conflict situations, thereby reinforcing its significance in the field of decision analysis and conflict resolution.

The expressive power of GMCR bears a resemblance to the cognitive mechanism of consciousness underlying human decision-making. Building upon the framework's inherent flexibility and abstraction, we have introduced the notion of representing preferences as binary values of permissible and impermissible [8], as well as a novel methodology for assessing the feasibility of states based on four values: T, F, B, and N [9]. This paper emanates from our awareness of the same question.

The impetus for this paper was prompted by the recognition of conceptual parallels between the GMCR framework, which delineates a conflict situation using a digraph composed of vertices and edges, and category theory. Our contention is that merging the mathematical depictions of these two theories

could enhance the range of possibilities for modeling and analysis. Section II of this paper will examine the fundamental concepts and definitions, while Section III will elucidate the potential benefits of incorporating category theory into GMCR through the use of illustrative examples.

II. CONCEPTS AND METHODS

A. Category Theory

The basic definitions include categories, objects, morphisms, and functors.

1) Category: : A category $\mathcal{C}$ consists of:
- A collection of objects: $\mathrm{Ob}(\mathcal{C})$
- A collection of morphisms (arrows): $\mathrm{Hom}(\mathcal{C})$

The morphisms in $\mathrm{Hom}(\mathcal{C})$ are associated with a domain (dom) and a codomain (cod), such that for every morphism $f \in \mathrm{Hom}(\mathcal{C})$, there exist objects $A, B \in \mathrm{Ob}(\mathcal{C})$ with $f: A \to B$, where A is the domain and B is the codomain.

The following axioms must hold:
- **Composition:** For any two morphisms $f: A \to B$ and $g: B \to C$, there exists a morphism $g \circ f: A \to C$ (the composition of f and g).
- **Associativity:** For any three morphisms $f: A \to B$, $g: B \to C$, and $h: C \to D$, the composition is associative: $(h \circ g) \circ f = h \circ (g \circ f)$.
- **Identity:** For each object $A \in \mathrm{Ob}(\mathcal{C})$, there exists an identity morphism $\mathrm{id}_A: A \to A$, such that for any morphism $f: A \to B$, $f \circ \mathrm{id}_A = f$, and for any morphism $g: B \to A$, $\mathrm{id}_A \circ g = g$.

2) Morphisms: Morphisms (also called arrows) are structure-preserving maps between objects in a category. Given objects A and B, the set of all morphisms from A to B is denoted by $\mathrm{Hom}(A, B)$ or $\mathcal{C}(A, B)$.

3) Functors: A functor F is a mapping between two categories $\mathcal{C}$ and $\mathcal{D}$ that preserves their structure. It consists of two mappings:
- $F: \mathrm{Ob}(\mathcal{C}) \to \mathrm{Ob}(\mathcal{D})$, which maps objects in $\mathcal{C}$ to objects in $\mathcal{D}$.
- $F: \mathrm{Hom}(\mathcal{C}) \to \mathrm{Hom}(\mathcal{D})$, which maps morphisms in $\mathcal{C}$ to morphisms in $\mathcal{D}$.

The functor must satisfy the following conditions:
- For any morphism $f: A \to B$ in $\mathcal{C}$, $F(f): F(A) \to F(B)$ in $\mathcal{D}$.
- For any composition of morphisms $f: A \to B$ and $g: B \to C$ in $\mathcal{C}$, $F(g \circ f) = F(g) \circ F(f)$ in $\mathcal{D}$.
- For each object $A \in \mathrm{Ob}(\mathcal{C})$, $F(\mathrm{id}_A) = \mathrm{id}_{F(A)}$ in $\mathcal{D}$.

B. GMCR

GMCR is a framework consisting of four tuples: $(N, S(A_i)_{i \in N}, (\succsim_i)_{i \in N})$. N is the set of all DMs, S denotes the set of all feasible states. (S, A_i) constitutes DM i's graph G_i, where S is the set of all vertices and $A_i \subset S \times S$ is the set of all oriented arcs. (S, A_i) has no loops; $(s, s) \in A$ for each $s \in S$. The preferences of each DM are presented as $(\succsim_i)$, where the set of all DMs $N : |N| \geq 2$, set of all states $S : |S| \geq 2$, and preference of DM i satisfy reflectiveness,

completeness, and transitivity. $s \succsim_i s'$: s is equally or more preferred to s' by DM i; $s \succ_i s'$: s is strictly preferred to s' by DM i; $s \sim_i s'$: s is equally preferred to s' by DM i.

We assume that a rational decision-maker (DM) aims to transition to a more favorable state and attempts to do so by repeatedly making unilateral moves, over which the DM has control. For $i \in N$ and $s \in S$, DM i's reachable list from state s is defined as the set $\{s' \in S \mid (s, s') \in A_i\}$, denoted by $R_i(s)$. $R_i(s)$ represents the set of all states where DM i can move from s to s' in a single step. A unilateral improvement for DM i from state s is defined as an element of the reachable list of DM i from s (i.e., $s' \in R_i(s)$), where i strictly prefers state $s'(s' \succ_i s)$. Consequently, the set of unilateral improvement lists for DM i from state s is described as $\{s' \in R_i(s) \mid s' \succ_i s\}$ and denoted by $R_i^+(s)$. $\phi_i^+(s)$ represents the set of all states that are more preferential for DM i than s, described as $\{s' \in S \mid s' \succ_i s\}$, and $\phi_i^{\approx}(s)$ denotes the set of all states that are at most equally preferential to state s, described as $\{s' \in S \mid s \succsim_i s'\}$. Additionally, $R_{N-\{i\}}(s)$ is defined as the set of all states that can be achieved through sequences of unilateral moves by DMs other than DM i. Similarly, $R_{N-\{i\}}^+(s)$ is defined as the set of all states that can be achieved through sequences of unilateral improvements by DMs other than DM i.

Based on the DMs' state transitions, we can derive standard stability concepts: Nash stability (**Nash**Nash) [10], [11], general meta-rationality (**GMR**) [12], symmetric meta-rationality (**SMR**) [12] , and sequential stability (**SEQ**) [13], [14].

The alignment of the individual components of the GMCR with category theory can be established as in the following subsection.

C. C-GMCR

1) Correspondence of Elements: States as Objects: In the category-theoretic framework, states in GMCR can be considered as objects. Each state represents a distinct configuration of decision variables for the DMs involved in the conflict.

State Transitions as Morphisms: State transitions can be represented as morphisms between states (objects). These morphisms preserve certain structures, such as the relationship between decision variables and DMs.

Define a category C-GMCR, with states as objects and state transitions as morphisms. This category should satisfy the axioms of composition and identity:

Composition: If there is a morphism (state transition) from state A to state B and another morphism from state B to state C, there must exist a composite morphism (state transition) from state A to state C. Identity: For each state, there exists an identity morphism (no state transition) that, when composed with any other morphism (state transition) leading to or from that state, yields the same morphism. Preference Relations and Functors: DM preferences can be captured using functors, which are structure-preserving mappings between categories. In this case, we can define a functor F for each DM that maps states in C-GMCR to an ordered set of preference levels, preserving the preference structure of the DMs. By

incorporating category theory into GMCR, you can gain new insights into the structure and properties of conflict resolution models, allowing for a more sophisticated analysis of conflicts and their resolutions.

2) C-GMCR: To introduce category theory into GMCR, we can define a new framework, "Categorical GMCR" (C-GMCR), by establishing basic definitions that incorporate category-theoretic concepts. Here is a proposal for these definitions:

- C-GMCR: C-GMCR is a model consisting of six tuples including overlapping definitions, $C = \langle N, S, (A_i)_{i \in N}, (\succsim_i)_{i \in N}, \mathcal{C}, \mathcal{M} \rangle$.
- N: A nonempty finite set representing the DMs involved in the conflict. Each DM is denoted by $i \in N$.
- C-State (Object) S: A C-State is an object in a category that represents a specific configuration of decision variables, corresponding to the outcome of actions each DM can take in a conflict situation. Let S be the set of all states representing decision variable configurations in a conflict situation. Each state $s \in S$ is an object in the category C_{conflict}.
- A_i: For each DM $i \in N$, $A_i \subseteq S \times S$ represents the set of oriented arcs reflecting the movements controlled in one step by DM i. The symbol $\times$ denotes the Cartesian product.
- C-State Transition (Morphism), $\mathcal{M}$: A C-State Transition is a morphism between states (objects) that represents a structure-preserving change in the decision variables, corresponding to a DM moving from one action to another. A state transition is a morphism $f : s_1 \to s_2$ between states (objects) $s1, s2 \in S$. Denote the set of all state transitions as $\mathcal{M}$.
- C-Preference: C-Preference for a DM $i \in N$ is defined over pairs of morphisms (state transitions) and their compositions. Let f, g, h be morphisms in the category $\mathcal{C}$, such that:
  $f : s_i \to s_j \ g : s_j \to s_k \ h = g \circ f : s_i \to s_k$
  The C-Preference relation $\succ_i^c$ for DM i is a binary relation on the set of morphisms, where $f \succ_i^c g$ means that DM i prefers the state transition represented by the morphism f to the state transition represented by the morphism g.
  Similarly, the C-indifference relation $\sim_i^c$ for DM i is a binary relation on the set of morphisms, where $f \sim_i^c g$ means that DM i is indifferent between the state transitions represented by the morphisms f and g. The C-Preference and C-indifference relations should satisfy the same properties as the preference relations in GMCR:
  - $\sim_i^c$ is reflexive and symmetric: $\forall f, g \in \mathcal{M}, f \sim_i^c f$ and $f \sim_i^c g \Leftrightarrow g \sim_i^c f$.
  - $\succ_i^c$ is asymmetric: $\forall f, g \in \mathcal{M}, f \succ_i^c g \Leftrightarrow \neg(g \succ_i^c f)$.
  - $\sim_i^c, \succ_i^c$ is strongly complete: $\forall f, g \in \mathcal{M}$, either $f \sim_i^c g$, $f \succ_i^c g$, or $g \succ_i^c f$.
- C-Conflict (Category), $\mathcal{C}$: A C-Conflict is a category $C\_conflict$, consisting of a collection of states (objects)

and state transitions (morphisms) that satisfy specific axioms related to composition and identity. A C-Conflict is a category $C_{\text{conflict}} = (S, \mathcal{M})$, where S is the set of states (objects) and $\mathcal{M}$ is the set of state transitions (morphisms). The category must satisfy the following axioms:
  - Composition: For any $f : s_1 \to s_2$ and $g : s_2 \to s_3$ in $\mathcal{M}$, there exists a composite morphism $g \circ f : s_1 \to s_3$ in $\mathcal{M}$.
  - Identity: For each state $s \in S$, there exists an identity morphism $1\_s : s \to s$ in $\mathcal{M}$ such that for any $f : s_1 \to s_2$, $1_{s_2} \circ f = f$ and $f \circ 1_{s_1} = f$.

These axioms ensure the coherence of the state transitions, capturing the dynamics of the conflict.
- C-GMCR Preference Functor: A C-GMCR preference functor is a structure-preserving mapping between the category C_{conflict} and an ordered set of preference levels for each DM. The functor maps states (objects) to their corresponding preference levels while preserving the preference structure of the DMs involved in the conflict. For DM i, we define a preference functor $P_i : C_{\text{conflict}} \to C\_i$, where C_i is an ordered set representing preference levels for DM i. The functor P_i maps each state $s \in S$ to a preference level $p \in C\_i$, preserving the preference structure:
  - $P_i(s_1) \precsim P_i(s_2)$ if and only if DM i prefers s_2 over s_1 or is indifferent between them.
  - For any state transition $f : s_1 \to s_2$, $P_i(f) = P_i(s_1) \to P_i(s_2)$.

We can define the stability concept in C-GMCR as follows:
  - C-GMCR-Nash : A state s^* in the C-Conflict is a Nash Equilibrium if for all DMs, there is no state transition $f : s^* \to s$ such that $P_i(s) \succ P_i(s^*)$.
  - C-GMCR-GMR: A state s^* is a GMR Equilibrium in the C-Conflict if for DMs, there is no sequence of state transitions $f_1 : s^* \to s_1, f_2 : s_1 \to s_2, ..., f_n : s(n-1) \to s$ such that $P_i(s) \succ P_i(s^*)$ and each DM j in the sequence has $P_j(s_j) \succsim P_j(s_{j-1})$ for $j = 1, ..., n$.
  - C-GMCR-SMR: A state s^* is an SMR Equilibrium in the C-Conflict if for all DMs, there is no sequence of state transitions $f_1 : s^* \to s1, f_2 : s1 \to s2, ..., f_n : s(n-1) \to s$ such that $P_i(s) \succ P_i(s*)$ and each DM j in the sequence has $P_j(s_j) \succ P_j(s_{j-1})$ for $j = 1, ..., n$.
  - C-GMCR-SEQ: A state s^* is sequentially stable in the C-GMCR conflict if there exists a sequence of state transitions $f_1 : s_0 \to s_1, f_2 : s_1 \to s_2, ..., f_n : s(n-1) \to s^*$ such that each DM j in the sequence has $P_j(s_j) \succ P_j(s_{j-1})$ for $j = 1, ..., n$, and there is no sequence of state transitions starting from s* that strictly improves the preferences of all decision-makers involved in the sequence.
- C-GMCR Equilibrium: Equilibrium concepts in GMCR, such as Nash Equilibrium, GMR, SMR, and SEQ, can

be redefined in C-GMCR. A C-GMCR equilibrium is a state or a collection of states in the C_{conflict} category where no DM can unilaterally improve its preference level through state transitions (morphisms). This concept can be formalized using the preference functors for each DM.

By redefining these equilibrium concepts in the C-GMCR framework, we can leverage the category-theoretic perspective to analyze and understand the conflict resolution process, potentially uncovering new properties and strategies for addressing conflicts. Several propositions can be derived from the definitions in C-GMCR.

Proposition 1. If a state transition $f : s_1 \rightarrow s_2$ exists such that $P_i(s_1) \prec P_i(s_2)$ for DM i, then there exists no state transition $g : s_2 \rightarrow s1$ with $P_i(s_2) \succ P_i(s_1)$ for the same DM.

Proof. This proposition follows from the definition of the C-GMCR Preference Functor. Since P_i is a functor, it preserves the preference structure of the DM. If $P_i(s_1) \prec P_i(s_2)$, it means DM i prefers s_2 over s_1. Consequently, there cannot be a state transition $g : s_2 \rightarrow s_1$ where $P_i(s_2) \succ P_i(s_1)$ for the same DM i, as it would contradict the preference ordering. $\square$

Proposition 2. If a state s^* is a C-GMCR equilibrium, then for all DMs, there is no state transition $f : s^* \rightarrow s$ such that $P_i(s) \succ P_i(s^*)$.

Proof. This proposition is a direct consequence of the definition of C-GMCR Equilibrium. A state s^* is a C-GMCR equilibrium if no DM can unilaterally improve its preference level through state transitions. Therefore, there cannot exist a state transition $f : s^* \rightarrow s$ with $P_i(s) \succ P_i(s^*)$ for any DM i. $\square$

Proposition 3. In a C-GMCR conflict, if there exists a state s^* such that $P_i(s^*)$ is the maximum preference level for all DMs, then s^* is a C-GMCR equilibrium.

Proof. Assume s^* is a state with the maximum preference level for all DMs. Suppose there exists a state transition $f : s^* \rightarrow s$ for some DM i. Since $P_i(s^*)$ is the maximum preference level, it must be the case that $P_i(s) \precsim P_i(s^*)$. By the definition of a C-GMCR equilibrium, if there is no state transition from s^* that improves the preference level for any DM, then s^* is a C-GMCR equilibrium. Hence, s^* is a C-GMCR equilibrium. $\square$

These propositions provide insights into the properties and behaviors of conflicts modeled using the C-GMCR framework, laying the foundation for further analysis and potential conflict resolution strategies.

III. APPLICATION AND ANALYSIS

The present section utilizes the C-GMCR framework to explicate conventional conflicts.

A. Prisoner's Dilemma

Here is a textual representation of the diagram of the Prisoner's Dilemma in the style of C-GMCR:

$$s_1(C,C) \xrightarrow{\ f_1\ } s_2(C,D)$$
$$\downarrow{\scriptstyle f_2} \qquad\qquad \downarrow{\scriptstyle f_4}$$
$$s_3(D,C) \xrightarrow{\ f_3\ } s_4(D,D)$$

This diagram shows the four states (objects) in the C-GMCR conflict representing the decision variable configurations:

- s_1: A cooperates, B cooperates (C, C)
- s_2: A cooperates, B betrays (C, D)
- s_3: A betrays, B cooperates (D, C)
- s_4: A betrays, B betrays (D, D)

The diagram also shows the non-identity state transitions (morphisms) between the states:

- $f_1 : s_1 \rightarrow s_2$ (A keeps cooperating, B changes to betraying)
- $f_2 : s_1 \rightarrow s_3$ (A changes to betraying, B keeps cooperating)
- $f_3 : s_2 \rightarrow s_4$ (A changes to betraying, B keeps betraying)
- $f_4 : s_3 \rightarrow s_4$ (A keeps betraying, B changes to betraying)

B. Elmira Conflict

The Elmira conflict, which transpired in Canada, provides an instance of an environmental dispute that showcases an analytical resolution approach utilizing the GMCR framework [15]. To delineate the Elmira conflict using the C-GMCR, the first step is to establish the objects in the C-State category, which are the feasible states. The morphisms in C-Transition embody the state transitions controlled by each DM. For instance, DM **M**(Ministry of Environment) has the authority to modify the control order, DM **U**(Uniroyal)has the discretion to either delay, accept, or abandon the order, while DM **L**(local government) can insist on strict enforcement.

The objects in C-Preference represent the preferences of each DM: M: $s_7 \succ s_3 \succ s_4 \succ s_8 \succ s_5 \succ s_1 \succ s_2 \succ s_6 \succ s_9$; U: $s_1 \succ s_4 \succ s_8 \succ s_5 \succ s_9 \succ s_3 \succ s_7 \succ s_2 \succ s_6$; L: $s_7 \succ s_3 \succ s_5 \succ s_1 \succ s_8 \succ s_6 \succ s_4 \succ s_2 \succ s_9$.

We represent the Elmira conflict using a C-GMCR diagram. In this diagram, nodes represent the objects of C-State (states), while arrows represent the morphisms of C-Transition (state transitions) and C-Preference (preferences). The arrow colors and labels indicate which DM is responsible for the transition or preference. The diagram shows the states and transitions controlled by each DM, with M in blue, U (Uniroyal) in red, and L in green.
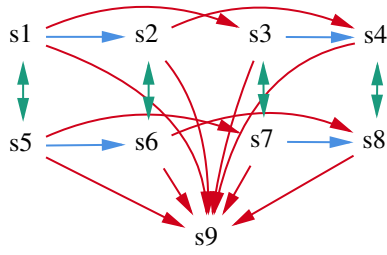
Fig. 1. Elmira Conflict : C-GMCR

Fig. 2. International Traded Conflict-Base Case : C-GMCR

The application of category theory to the Elmira conflict allows us to explore higher-order strategies by introducing the concept of morphisms and composition of state transitions. In traditional GMCR, the analysis focuses on the immediate consequences of each DM's actions. However, in C-GMCR, we can examine not only the immediate transitions but also sequences of actions and their compositions. This is achieved by employing morphisms, which represent relationships between state transitions, and the composition of morphisms, which allows for the chaining of transitions. Let's consider an example in the Elmira conflict. Suppose we have a sequence of actions (morphisms) that leads to the state s_i for **M**, followed by a sequence of actions for **U**, which transitions to state s_j. By examining the composition of these morphisms, we can gain insights into how the combined actions of **M** and **U** might result in different strategic outcomes.

The chaining of state transitions can be represented using the composition of morphisms: $f : s_i \rightarrow s_j$, $g : s_j \rightarrow s_k$, $h = g \circ f : s_i \rightarrow s_k$.

In this example, h represents the overall effect of the sequence of actions taken by **M** and **U**. By analyzing these higher-order strategies, we can better understand the strategic interdependencies and the long-term effects of different actions on the Elmira conflict.

The use of category theory in C-GMCR allows for the exploration of these complex sequences of actions and their compositions, thereby revealing higher-order strategies and interactions that might not be apparent in the traditional GMCR analysis.

C. International Trade Conflict

1) Base case: Let us contemplate a simplified instance of an international trade conflict that involves three nations: Country A (**A**), Country B (**B**), and Country C (**C**). The conflict concerns the negotiation of trade tariffs and quotas on a specific product. DMs in this conflict are the governments of the aforementioned nations, each of which possesses distinct preferences based on their economic interests. Herein, we provide a succinct depiction of the position of each nation: **A**: Interested in free trade with low tariffs to increase exports, **B**: Seeking higher tariffs to protect domestic industries, **C**: Prefers moderate tariffs to balance between domestic industry protection and export promotion. We can represent the possible actions for each country as follows: **A**: Low Tariffs (L),

Moderate Tariffs (M), **B**: High Tariffs (H), Moderate Tariffs (M), and **C**: Moderate Tariffs (M), Balanced Approach (B).

Let's assume there are six feasible states (S) that represent the different combinations of actions by the countries: $S = s_1, s_2, s_3, s_4, s_5, s_6$, where s_1: (L, H, M), s_2: (L, H, B), s_3: (L, M, M), s_4: (L, M, B), s_5: (M, H, M), s_6: (M, H, B). By employing a GMCR-based methodology, it is feasible to articulate the preferences of each country in the following manner: **A**: $s_3 \succ s_4 \succ s_1 \succ s_2 \succ s_5 \succ s_6$, **B**: $s_5 \succ s_6 \succ s_1 \succ s_2 \succ s_3 \succ s_4$, **C**: $s_4 \succ s_6 \succ s_1 \succ s_3 \succ s_2 \succ s_5$.

The diagram depicts the states as ellipses, and the transitions between states as arrows with the corresponding DMs' actions (**A**, **B**, **C**) indicated in green, red, and blue, respectively.

2) Intricate case: Let us examine a more intricate relationship that arises from complex interdependencies among DMs and state transitions. Suppose there exist trade agreements or regulations that constrain the actions of DMs based on the actions taken by other DMs. DMs are **A** (Country A), **B** (Country B), and **R** (Regulatory Body). Assume the following states for this conflict: s_1: No tariffs imposed s_2: Country A imposes tariffs on Country B s_3: Country B imposes tariffs on Country A s_4: Both countries impose tariffs on each other s_5: Regulatory Body imposes new trade rules The state transitions could be as follows: **A**: $(1, 2), (3, 4)$ **B**: $(1, 3), (2, 4)$ **R**: $(1, 5), (2, 5), (3, 5), (4, 5)$ The additional morphisms representing the regulations could be the following:

If the Regulatory Body imposes new trade rules (state 5), Country A and Country B must lift their tariffs: $(5, 1)$ If both countries impose tariffs on each other (state 4), the Regulatory Body must intervene and impose new trade rules (state 5): $(4, 5)$.

Regarding a framework that incorporates factors beyond the primary DM, we have previously demonstrated a concept whereby the DM's capacity to undertake new state transitions
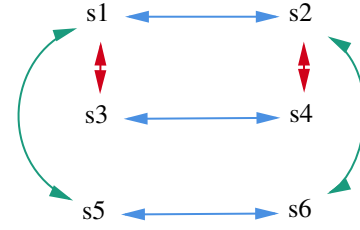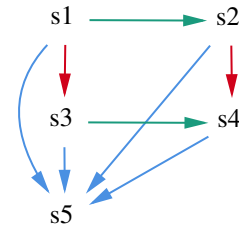

Fig. 3. International Traded Conflict-Intricate Case : C-GMCR

is affected by external factors [16]. Fig 4 illustrates the graph with multi arcs, which can be gained or lost due to external factors. The graphical representation depicts the behavioral responses of prisoners to rumors of a prospective pardon, with extra arcs in solid line indicating the influence of external factors beyond the prisoners' control. As a future research direction, we aim to extend this framework in a more generalized manner using C-GMCR.

IV. Conclusion and Future Research

In this paper, we have demonstrated the advantages of incorporating category theory into GMCR through the congruence between their respective concepts. We believe this innovative approach will offer valuable contributions in multiple domains, particularly for modeling and analyzing real-world problems using AI. Potential future research endeavors could focus on the ensuing following applications.

1) Improved Modeling and Representation: The C-GMCR framework enables more abstract and expressive representations of complex decision-making problems in AI systems, leading to enhanced modeling capabilities and a deeper understanding of conflict and decision-making processes.

2) Algorithm Development: Category theory's application in C-GMCR can inspire new algorithms for conflict resolution and decision-making in AI systems. By utilizing the abstract properties and relationships of category theory, researchers can potentially design more efficient and robust algorithms for various AI applications, such as multi-agent systems, negotiation, and game theory.

3) Knowledge Representation and Transfer: Category theory's ability to represent knowledge and relationships across domains allows AI systems to benefit from a more unified and abstract representation of knowledge, enabling effective knowledge transfer and learning across problem domains.

4) Compositionality in AI: Emphasizing the principle of compositionality, category theory benefits AI systems by facilitating the construction of complex behaviors and strategies through simpler components. This results in more modular, scalable, and maintainable AI systems that can adapt effectively to new situations.

5) Interdisciplinary Connections: Category theory's application in fields like computer science, mathematics, and physics enables AI researchers to leverage interdisciplinary connections, potentially leading to novel approaches and insights in AI systems and algorithms.
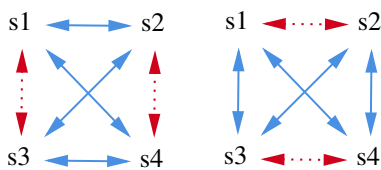
6) Formal Verification and Validation: The formalism provided by category theory in the C-GMCR framework is advantageous for verifying and validating AI systems, particularly in multi-agent settings. Category theory's rigorous mathematical foundation ensures the correct and reliable behavior of AI systems in conflict resolution and decision-making tasks.

7) Explainability and Interpretability: The C-GMCR framework can potentially enhance the explainability and interpretability of AI systems in conflict resolution and decision-making. By offering a more abstract and structured representation of conflicts, category theory assists researchers and practitioners in better understanding the underlying logic and reasoning of AI systems, leading to increased transparency and trustworthiness.

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Fig. 4. New Reachability Model of Prisoners' Dilemma