Parental Transfers Under Ambiguity

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Abstract

This note introduces parental uncertainty into parent–child monetary transfers. A parent questions the probability distribution of a child's future economic success. As a result, the parent endogenously tilts his/her subjective probability model away from an approximating probability model. In this case, parental transfers increase with model uncertainty, thereby reducing the child's effort and probability of economic success. This theoretical result raises several empirical questions, of which two are as follows. For one thing, informed parents (e.g. those who hold the same job as their child) transfer less money, and their child exerts more effort. Another is that economic uncertainty (e.g. recessions or pandemics) prompts higher parental transfer payments and reduces the child's effort.

Keywords: Bequest; Ambiguity; Knightian uncertainty; Robust control; Inheritance

JEL Classification: D13; D81; E20

1 Introduction

Standard economic models assume rational expectations to simplify analysis. In that tradition, studies of parental transfers generally assume parents hold rational expectations, meaning they perfectly estimate the probability distribution of their child's future economic success. Perfect estimation might hold for parents who understand their children's

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jobs. Among parents for whom that is not the case, assuming rational expectations is a mistake because parents' ignorance might determine the amount of parental transfers.

This note examines how parental ambiguity regarding a child's probability distribution affects the amount of parental transfers. The probability of the child's financial success is the sum of the child's future effort and luck, and parents can observe neither. Following the multiplier preferences of Hansen and Sargent (2001), we study altruistic parents who subjectively estimate the probability. Consequently, the parent chooses amount of transfer which performs well under the worst scenario.

Our theoretical results suggest that parents' model uncertainty result in larger parental transfers. The parent equates the marginal utility of his/her consumption and the subjective expected utility of a child's future consumption. For many reasons—fear of macroeconomic collapse, poor understanding of children's jobs or prospects—parents' subjective expected utility might defy the rationally expected one. If so, parents might make larger transfers to their children. Larger transfers might disincentivise children's efforts, whereas smaller transfers have the opposite effect.

That theoretical result provokes several empirical questions regarding parental transfers. For one, the model predicts that parents familiar with their children's occupations (e.g. hold the same job as their children) transfer less wealth to them. Anticipating fewer gains in the future, those children have higher probabilities of occupational success. This interpretation suggests a positive aspect of inheriting parents' jobs. The model also predicts that parents' anticipation of macroeconomic uncertainty (e.g. during recessions or pandemics) adversely impacts children's future economic success.

Most studies of parental transfers adopt an altruistic model (e.g. Becker 1974; Becker and Tomes 1979; Becker 1981) in which parents transfer their wealth to gain utility from the increase of the child's utility. Exchange models such as Bernheim et al. (1985) and Cox (1987) extend altruistic models to grant parents to value their children's consumption of particular goods or activities.[1]

Citing discrepancies between theory and evidence, several theoretical and empirical studies reject the assumption of rational expectations (e.g. Ellsberg 1961; Mehra and Prescott 1985) and adopt ambiguity aversion models, primarily in the disciplines of asset pricing (e.g. Epstein and Wang 1994; Barillas et al. 2009), monetary policymaking (e.g. Walsh 2004; Woodford 2010; Dennis 2010), precautionary saving (e.g. Gollier 2011; Peter 2019), and fiscal policymaking. (e.g. Svec 2012; Karantounias 2013 ; Ferriere and Karant-

[1] Another large branch of the literature studies the consequences of accidental bequest (See Yaari (1965) and Davies (1981)).

ounias 2019)[2]

2 Model and Results

We consider a household containing one altruistic parent and one selfish child. The child's income is y_h with probability $p \in [0,1]$, and is y_l with probability $1-p$, where $y_l < y_h$. We suppose that p is a function of child's effort and a random variable ϵ, i.e. $p = p(e;\epsilon)$, where $p_e > 0$, $p_{ee} < 0$, and $p_\epsilon > 0$. Let $\mu(\epsilon)$ be the probability measure of ϵ and $\int_{\underline{\epsilon}}^{\bar{\epsilon}} \mu(\epsilon)d\epsilon = 1$ where $\bar{\epsilon} \in \mathbb{R}_+$ and $\underline{\epsilon} \in \mathbb{R}_-$ be the maximum and minimum values of ϵ, respectively.[3] The timing of events is as follows.

1. The parent consumes c and bequeaths monetary transfer b (hereafter we call it as bequest) to the child.
2. ϵ is realised.
3. The child expends work effort e;
4. The child's income is determined, and the child consumes.

There are two differences between the traditional model and ours: we include ϵ, and parents are unsure about their probability function. We solve our model by backward induction.

2.1 Child's Decision

Given ϵ and b, the child chooses e, which maximises the following expected utility:

$$E_y\left[U(e;\epsilon,b)\right] = p(e;\epsilon)u(y_h+b) + (1-p(e;\epsilon))u(y_l+b) - v(e), \tag{1}$$

where u is utility from consumption and $-v$ denotes the disutility of effort. The expectation operator represents the realisation of y. Both functions are continuously differentiable; u is strictly concave ($u' > 0$, $u'' < 0$); v is strictly convex ($v' > 0$, $v'' > 0$).

The first order condition is written as:

$$\frac{dE_y\left[U(e;\epsilon,b)\right]}{de} = \left(u_h - u_l\right) p'(e;\epsilon) - v'(e) = 0, \tag{2}$$

[2]SeeHansen and Sargent (2008) for more examples of applications.

[3]Note that our model coincides with the standard altruistic parent model if $\bar{\epsilon} = \underline{\epsilon} = 0/$

where $u_i = u(y_i + b)$. Eq. (2) implies that $e^*(b;\epsilon)$ sets expected marginal utility of the child's consumption equal to the marginal disutility of his or her effort. Eq. (2) yields that:

$$\frac{de^*(b,\epsilon)}{db} = \frac{(u_h^{'} - u_l^{'})p'(e;\epsilon)}{-(u_h - u_l)p''(e;\epsilon) + v''(e)} < 0. \tag{3}$$

Eq.(3) shows that the parent's bequest disincentivises the child's effort.[4]

2.2 Bequeathing without Ambiguity

As a benchmark, we analyse the situation in which the parent accurately assesses the true probability $p(e;\epsilon)$. Let W be the parent's utility. Given the budget constraint: $c + b = I$ and the child's decision function $e^*(b;\epsilon)$, the parent selects the amount of bequest that maximises the following rationally expected utility:

$$E_\epsilon \left[E_y \left[W\left(b\right) \right] \right] = u(I - b) + \beta \int_{\underline{\epsilon}}^{\overline{\epsilon}} E_y \left[V(b;\epsilon) \right] \mu(\epsilon) d\epsilon, \tag{4}$$

where V is the child's value function: $V(b;\epsilon) := U\left(e^*(b,\epsilon);\epsilon, b\right)$ and β is the intercohort discount factor (or degree of altruism) such that $0 < \beta < 1$. Using the envelop theorem, the first order condition is given by:

$$-u^{'}(I-b) + \beta \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left(p(e^*;\epsilon)u_h^{'} + (1 - p(e^*;\epsilon))u_l^{'} \right) \mu(\epsilon) d\epsilon = 0, \tag{5}$$

which yields b^*—the amount bequeathed absent parental ambiguity. Eq. (5) balances the marginal utility of reducing the parent's consumption with the rationally expected marginal utility of increasing the child's consumption, implying that a relatively economically suffering child, such as unemployed, student, or single parent, will receive more transfers.

2.3 Bequeathing under Ambiguity

Suppose the parent suffers ambiguity about the true distribution of random variable ϵ and believes it spans numerous alternatives. We apply the robust control theory to this issue.

Let $\tilde{E}_\epsilon$ be the parent's subjective expectation of the random variable ϵ and suppose the absolute continuity with respect to p. Then the Radon-Nikodym theorem indicates

[4] The Online Appendix contains the detailed derivation of each equation.

that there is a measurable function $m(\epsilon)$ such that $\tilde{E}_\epsilon[\epsilon] = E[m(\epsilon)\epsilon]$ and $E_\epsilon[m(\epsilon)] = 1$. Following the literature of robust control, we measure the distance between the actual and approximating models by relative entropy: $E_\epsilon\left[m(\epsilon)\ln m(\epsilon)\right]$.[5]

Given $e^*(b;\epsilon)$, the parent chooses b, which maximises:

$$\tilde{E}_\epsilon\left[E_y\left[W\left(b, m(\epsilon);\theta\right)\right]\right] = u(I-b) + \min_{m(\epsilon)} \Bigg\{ \beta \int_{\underline{\epsilon}}^{\bar{\epsilon}} m(\epsilon)\left(V(b;\epsilon) + \theta \ln m(\epsilon)\right)\mu(\epsilon)d\epsilon - \beta\theta\lambda \Bigg(\int_{\underline{\epsilon}}^{\bar{\epsilon}} m(\epsilon)\mu(\epsilon) \quad (6)$$

where λ is the Lagrangian multiplier of the legitimate constraint: $\int_{\underline{\epsilon}}^{\bar{\epsilon}} m(\epsilon)\mu(\epsilon)d\epsilon = 1$ which assures each approximating model expresses a legitimate probability. The penalty parameter θ measures the degree of concern for robustness; a higher θ implies the parent is more confident about the approximating model. As $\theta \to \infty$, the parent has full confidence in the approximating model, and it coincides with the rational expectations model in Section 2.2. Using the envelope theorem, the first-order condition of the inner minimisation problem is given by

$$\beta\left(V(b;\epsilon) + \theta\left(1 + \ln m(\epsilon)\right) - \lambda\theta\right) = 0. \quad (7)$$

With the legitimate constraint and Eq. (6), we obtain the optimal distortion $\tilde{m}(\epsilon)$:

$$\tilde{m}(\epsilon) = \frac{\exp\left(-\frac{V(b;\epsilon)}{\theta}\right)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}} \exp\left(-\frac{V(b;\epsilon)}{\theta}\right)\mu(\epsilon)d\epsilon}. \quad (8)$$

Note that the optimal distortion $\tilde{m}(\epsilon)$ puts a higher probability on a bad scenario (low realisation of ϵ) than the actual probability. The size of the optimal distortion increases as the penalty parameter θ decreases, and vice versa.[6]

With $\tilde{m}(\epsilon)$, the parent's objective function is written as:

$$\tilde{E}_\epsilon\left[E_y\left[W\left(b, \tilde{m}(\epsilon);\theta\right)\right]\right] = u(I-b) - \beta\theta\ln\left(\int_{\underline{\epsilon}}^{\bar{\epsilon}} \exp\left(-\frac{V(b;\epsilon)}{\theta}\right)\mu(\epsilon)d\epsilon\right). \quad (9)$$

[5] Note that relative entropy is convex and grounded.

[6] Note that the optimal distortion converges to 1 as $\theta \to \infty$ for all ϵ.

Using the envelop theorem, the first order condition is given by:

$$\frac{d\tilde{E}_{\epsilon}\left[E_y\left[W\left(b,\tilde{m}(\epsilon);\theta\right)\right]\right]}{db} = -u^{'}\left(I-b\right) + \beta\int_{\underline{\epsilon}}^{\overline{\epsilon}} \tilde{m}(\theta) \underbrace{\left(pu_h^{'} + (1-p)u_l^{'}\right)}_{V'} \mu(\epsilon)d\epsilon = 0, \quad (10)$$

which yields $\tilde{b}(\theta)$**—**the amount of the bequest under parental ambiguity. Eq. (10) balances the marginal utility of reducing the parent's consumption with the subjectively expected marginal utility of increasing the child's consumption.

Proposition 1

1. *Parental ambiguity aversion increases their bequests:* $\frac{d\tilde{b}(\theta)}{d\theta} < 0$*, where* $\tilde{b}(\theta) \geq b^*$.
2. *Greater parental ambiguity aversion reduces effort expended by the child:* $\frac{de^*(b;\epsilon)}{d\theta} > 0$*, where* $e^*(\tilde{b}(\theta);\epsilon) \leq e^*(b^*;\epsilon)$.

Proof.

Since $\frac{d\left(\frac{d\tilde{E}_{\epsilon}\left[E_y\left[W(b,\tilde{m}(\epsilon);\theta)\right]\right]}{db}\right)}{d\theta} = \beta\int_{\underline{\epsilon}}^{\overline{\epsilon}} \frac{d\tilde{m}(\theta)}{d\theta}\left(pu_h^{'} + (1-p)u_l^{'}\right)\mu(\epsilon)d\epsilon > 0$*, the strict concavity of* u *and Eq.* (10) *imply the first result. The second result follows from the first result and Eq.* (3).

Both equalities in Proposition 1 hold as $\theta \to \infty$. Facing ambiguity, parents leave a larger bequest than in the case absent ambiguity. Since the bequest disincentivises the child's effort (Eq. 3), the parent's ambiguity has a negative effect on the child's effort.

Figure 1 illustrates the implication of Proposition 1. A higher θ implies less parental ambiguity aversion. If $\epsilon = 0$, implying no ambiguity, penalty parameter 0 does not affect bequest b. If $\epsilon > 0$, a higher penalty parameter θ indicates less parental aversion to ambiguity, and parents reduce their bequest. The effect of ϵ on bequests diminishes as θ increases. As $\theta \to \infty$, the outcome coincides with benchmarked rational expectations.

3 Discussion and Conclusion

This note has studied a robust parental transfer problem by a Beckarian altruistic parent. The theoretical prediction suggests avenues for empirical research that expands understanding of parental transfers. For instance, Figure 2 describes the Economic Policy Uncertainty Index and the share of inherited wealth in private wealth in the US and the

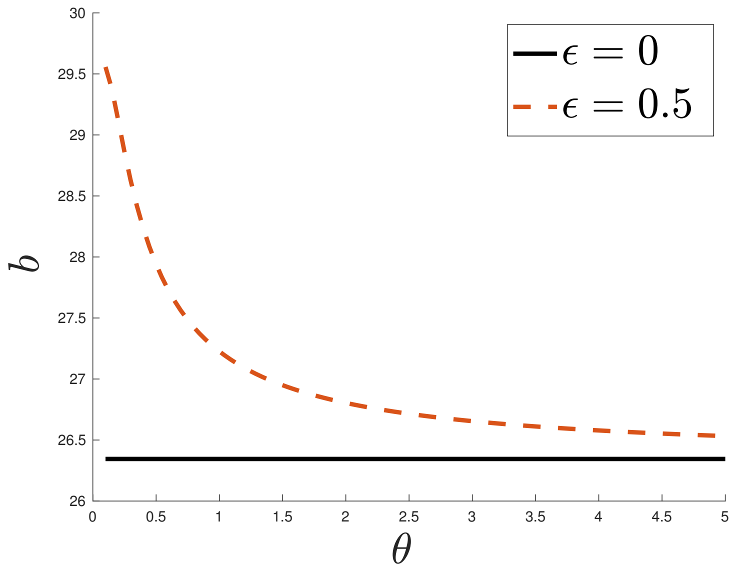

Figure 1: Model Uncertainty and Bequest

Note: $u(c) = \ln c$, $-v(e) = \ln(1-e)$, $p(e;\epsilon) = e + \epsilon$, $\beta = 0.985$, $y_h = 150$, $y_l = 50$, and $y_p = 100$. The shock takes either ϵ (with probability probability 0.5) or $-\epsilon$ (with probability probability 0.5).

UK from the 1900s to 2010s.[7] The data show that uncertainty and inheritance had similar trends—high (low) degree of uncertainty and high (low) amount of inheritance, which is consistent with our findings.

[7]For more details about the construction of the Historical Economic Uncertainty Indexes, see https://www.policyuncertainty.com/us_historical.html and https://www.policyuncertainty.com/uk_historical.html.

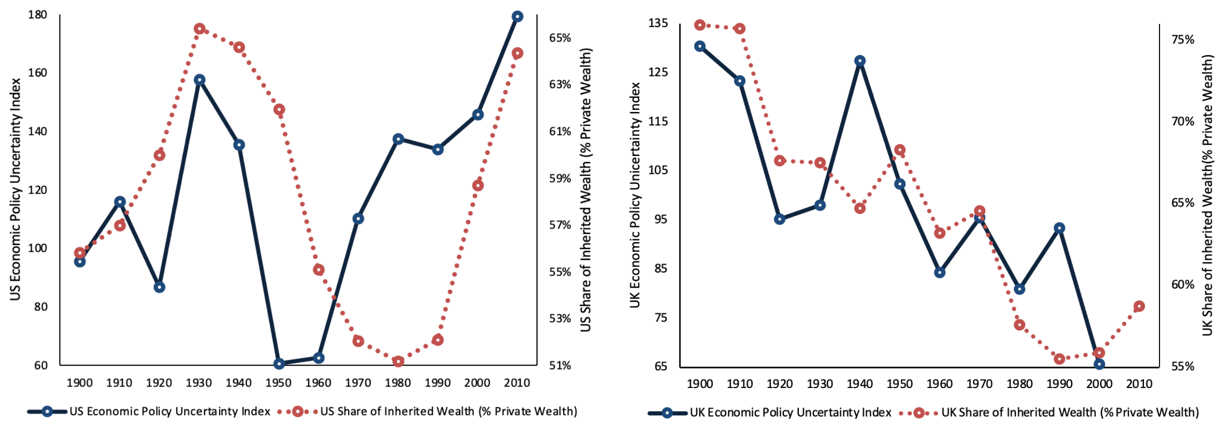

Figure 2. Policy Uncertainty and Inheritance. Note: Date for the share of inheritance is from Alveredo et al. (2011). The data on economic policy uncertainty indexes are 10 years average.

As a first pass, we have considered the simplest possible version of parent–child frame-work that clearly highlights the essential mechanism underlying the results. In the future, it would be interesting to extend the model to more realistic settings. An interesting extension is to consider dynamic parent–child models with parental ambiguity; in such models, the effects of ambiguity dynamics on parental transfers can be studied.

Another promising future research is to consider more interactive parent–child relationships. It would be possible that children may tell the probability of their success to their parents. In these situations, the parents are required to design parental transfer rules that will incentivise the children to tell their true success probability. If a child can bargain over the amount of parental transfers (Bergstrom, 1989), the child's higher bargaining power will increase the amount of bequest he/she will receive.

Furthermore, incorporating parent–child proximity into the model is an interesting extension. Our model suggests that more proximity may reduce transfers, as having more interaction might relax parental ambiguity. Moreover, the exchange models (Bernheim et al., 1985) suggest that more proximity increases parental transfers. It would be interesting to quantitatively investigate the impacts of those effects. Finally, our model can be applied to the (non)monetary supports from children to parents. Our results imply that more ambiguity on the parents' economic or health conditions during a pandemic may increase transfers from children.

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Appendix: Derivation of Equations

A1. Derivation of Eq. 3

The derivative of the first-order condition (Eq. 2) is given below:

$$\frac{\partial\left(\left(u_h-u_l\right)p'(e;\epsilon)\right)}{\partial b}-\frac{\partial v'(e)}{\partial b} = 0,$$

$$\Longrightarrow \left(u_h-u_l\right)p''(e;\epsilon)\frac{de^*(b,\epsilon)}{db}+\left(u_h^{'}-u_l^{'}\right)p'(e;\epsilon)-v''(e)\frac{de^*(b,\epsilon)}{db}=0,$$

$$\Longrightarrow \frac{de^*(b,\epsilon)}{db}=\frac{\left(u_h^{'}-u_l^{'}\right)p'(e;\epsilon)}{-\left(u_h-u_l\right)p''(e;\epsilon)+v''(e)}.\ \square$$

A2. Derivation of Eq. 5

Note that the envelop theorem implies: $\frac{\partial E_y[V(b;\epsilon)]}{\partial b} = \frac{\partial E_y[U(e^*(b,\epsilon);\epsilon,b)]}{\partial b} = p(e;\epsilon)u_h^{'} + (1 - p(e;\epsilon))u_l^{'}$. The derivative of the subjective utility function (Eq. 4) is given below:

$$\frac{\partial E_\epsilon\left[E_y\left[W\left(b\right)\right]\right]}{\partial b} = -u'(I-b) + \beta \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left(p(e;\epsilon)u_h^{'} + (1-p(e;\epsilon))u_l^{'}\right)\mu(\epsilon)d\epsilon. \square$$

A3. Derivation of Eq. 8

With the relative entropy $E_\epsilon\left[m(\epsilon)\ln m(\epsilon)\right]$, the parent's subjective expected utility is given by Eq. 6. To maximise Eq 6, we first solve the inner minimisation problem in the following manner:

$$\min_{m(\epsilon)}\left\{\beta\int_{\underline{\epsilon}}^{\overline{\epsilon}} m(\epsilon)\left(V(b;\epsilon)+\theta\ln m(\epsilon)\right)\mu(\epsilon)d\epsilon - \beta\theta\lambda\left(\int_{\underline{\epsilon}}^{\overline{\epsilon}} m(\epsilon)\mu(\epsilon)d\epsilon - 1\right)\right\}$$

The first-order condition is given below:

$$\beta\left(V(b;\epsilon)+\theta\left(1+\ln m(\epsilon)\right)-\lambda\theta\right)=0,$$

$$\Longrightarrow m(\epsilon) = \exp\left(-\frac{V(b;\epsilon)}{\theta}\right)\exp\left(-\frac{1}{\theta}+\lambda\right).$$

The legitimate constraint, $\int_{\underline{\epsilon}}^{\overline{\epsilon}} m(\epsilon)\mu(\epsilon)d\epsilon = 1$, implies the following:

$$\int_{\underline{\epsilon}}^{\overline{\epsilon}} \exp\left(-\frac{V(b;\epsilon)}{\theta}\right)\exp\left(-\frac{1}{\theta}+\lambda\right)\mu(\epsilon)d\epsilon = 1,$$

$$\Longrightarrow \exp\left(-\frac{1}{\theta}+\lambda\right) = \frac{1}{\int_{\underline{\epsilon}}^{\overline{\epsilon}}\exp\left(-\frac{V(b;\epsilon)}{\theta}\right)\mu(\epsilon)d\epsilon}.$$

By substituting $m(\epsilon) = \exp\left(-\frac{V(b;\epsilon)}{\theta}\right)\exp\left(-\frac{1}{\theta}+\lambda\right)$ into it, we derive the following:

$$\tilde{m}(\epsilon) = \frac{\exp\left(-\frac{V(b;\epsilon)}{\theta}\right)}{\int_{\underline{\epsilon}}^{\overline{\epsilon}}\exp\left(-\frac{V(b;\epsilon)}{\theta}\right)\mu(\epsilon)d\epsilon}. \square$$

A4. Derivation of Eq. 10

Note that the envelop theorem implies: $\frac{\partial V(b;\epsilon)}{\partial b} = p(e;\epsilon)u'_h + (1 - p(e;\epsilon))u'_l$.

$$\frac{d\tilde{E}_\epsilon\left[E_y\left[W\left(b,\tilde{m}(\epsilon);\theta\right)\right]\right]}{db} = -u'(I-b) - \beta\theta\frac{\int_{\underline{\epsilon}}^{\overline{\epsilon}}\exp\left(-\frac{V(b;\epsilon)}{\theta}\right)\left(-\frac{p(e;\epsilon)u'_h+(1-p(e;\epsilon))u'_l}{\theta}\right)\mu(\epsilon)d\epsilon}{\int_{\underline{\epsilon}}^{\overline{\epsilon}}\exp\left(-\frac{V(b;\epsilon)}{\theta}\right)\mu(\epsilon)d\epsilon} = 0,$$

$$\Longrightarrow \frac{d\tilde{E}_\epsilon\left[E_y\left[W\left(b,\tilde{m}(\epsilon);\theta\right)\right]\right]}{db} = -u'(I-b) + \beta\int_{\underline{\epsilon}}^{\overline{\epsilon}}\tilde{m}(\epsilon)p\left((e;\epsilon)u'_h + (1-p(e;\epsilon))u'_l\right)\mu(\epsilon)d\epsilon = 0. \square$$